Experiencing School Mathematics

Revised and Expanded Edition

Traditional and Reform Approaches to Teaching and Their Impact on Student Learning

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Phoenix Park Mathematics: Experiences and Reflections

In a similar style to the last chapter, the two main sections of this chapter describe some important features of Phoenix Park's approach and the students' responses to them. I again select the particular aspects of the teaching approach that seemed to influence students' views and understandings to the greatest extent, and that also give a representative portrayal of Phoenix Park's approach.

TEACHING AND LEARNING AT PHOENIX PARK

Open Learning

At Phoenix Park school, the curriculum was designed by the teachers. They did not use any books or work cards. Instead they brought together a collection of different open-ended projects that generally lasted for 2 to 3 weeks of mathematics lessons. Probably the most distinctive, influential, and unusual aspect of Phoenix Park's mathematics approach was the openness and freedom that this created for students.

> G: In books, it tells you everything, you read everything off the question, you read the question and you have to answer it. Here you just have to make up your own, he just tells you what you have to do and then you have to do it yourself. (Gary, PP, Year 9, JC)

The mathematics approach at Phoenix Park was open from the time projects were described to students to the time, 2 or 3 weeks later, when they gave them in. This openness manifested itself in a number of ways, including the ways in which the projects were described and defined, the ways in which teachers answered the students' questions and the ways in which teachers guided students. The students at Phoenix Park were not given specified paths through their activities; they were merely introduced to starting questions or themes and expected to develop these into extended pieces of work. When they asked the teachers questions, the teachers seemed to make deliberate efforts not to structure the work for students:

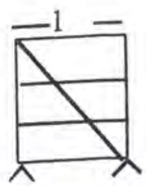
> JB: When the teachers help you here, do they talk to you generally about the topic or do they break it down and tell you bit by bit what to do?
>
> A: Very general, they hardly give you an answer.
>
> D: Usually it helps, 'cause then they don't really give you the answer, you still have to work it out for yourself. (Alex and Danny, PP, Year 10, JC)

Thus, the openness of the approach related not only to the way that mathematics was introduced, but also the way in which teachers interacted with students and supported them in their work:

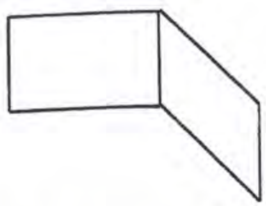
> A: Well, I think first of all you have to try and find your own methods, then if you really get stuck the teacher will come and give you suggestions for stuff and tell you how to like, progress further and then you can kind of think about it. (Andy, PP, Year 10, RT)

I have chosen the following extract because it gives a fairly representative example of the ways projects were introduced at Phoenix Park and illustrates some of the decisions teachers made as they introduced mathematical ideas and methods. In the extract, Jim is introducing a new activity called *36 pieces of fencing* to a Year 8 class.

> Twenty-five students come in and sit down. Sixteen boys and 9 girls gather around the board. A boy asks, "Sir are we gonna start a new project?" Jim says, "Yes, the title of the piece is *36 pieces of fencing* (he writes the title on the board), so you need a piece of paper—it only needs to be a scrap piece of paper at the moment, but make sure you've got something to write on." A few get up and collect paper from a stand in the room. Jim continues, "Can we have a bit of hush please? Right, you have a piece of fencing and from the side it looks like this:

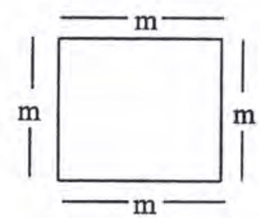


It has little legs on, like this and it is 1 meter wide. They have little hooks on and you can hook the gates together. You can put them at any angle, so those from the side would look like this":



He continues, "What we are interested in is what sort of shapes can you make with 36 pieces of fencing?" The students then start calling out shapes, a boy offers a square, a girl a hexagon. Someone asks, "do you have to use them all?" Jim says, "Yes, there are rules" and writes a heading: <u>Rules</u> on the board and then, under this, *use them all on every shape*, saying "it makes them more manageable if you have to use them all." Then, "any other shapes?" A girl says "rectangle." Jim asks, "just one?" A boy says, "a square is a rectangle." Jim says, "yes, we've already got a special type of rectangle." The students continue shouting out shapes. A boy says "rhombus," a girl says "parallelogram," and Jim is adding all of these to a list on the board. Another boy says, "pentagon," and Jim stops at this and says, "can you?" The boy says, "yeah." Jim asks, "how many sides?" A few offer "5." A girl says, "you've got 36 fences," and Jim says, "well you can have a pentagon, but what will it be like?" There is silence, so he asks, "will the sides be the same?" The students all shout "no." Jim asks, "so what will it be called?" A boy offers, "irregular." Jim writes *irregular pentagon* and then asks for more shapes. One boy suggests a quadrilateral and Jim says, "Yes, well, these are all quadrilaterals," and he points to some shape names. He puts parentheses around these on the board and writes *quadrilaterals* next to them. He then continues with "we've got a triangle but is there only one?" A girl says, "there's loads." Jim says, "yes there's loads so lets put an s on," and makes it triangles. Then Jim says, "so, we've got 4 sided, 5 sided. . . ." A boy offers "octagon" and a girl says, "yes, 8 sides." Jim asks, "yes, but what will happen?" Someone says, "there'll be some left over." Jim says, "yes, or irregular, not all the same length, so *pentagon*" and writes *(irregular) heptagon (irregular) octagon (irregular)*. A boy offers "nonagon" and Jim tells him to say it louder so that everyone can hear. Jim writes it on the board with (9) next to it, then asks, "will it be regular or irregular?" A girl says regular and Jim asks why.

She says, " 'cause 9s go into 36." Jim asks, "what other regular ones can we have?" There is silence and he adds, "well the definition seems to be if the number of sides go into the number of fences." A boy says "12" and Jim writes *dodecagon (12 sided)*. Someone offers "18" and Jim says he doesn't know what that is called, but writes up *18-sided shape*. A boy says "3 sides," and someone else says, "that's a triangle." Jim asks, "OK how many regular triangles can you make?" Someone says "one" and Jim says, "yes, where I've written regular you can also have irregular ones." He then asks, "which are easier to draw?" Someone says "irregular" and Jim says, "OK shall I make it harder and say we only want regular ones?" Some say "no" to this and some say "yes." Jim says, "we can put another rule in if you want" and writes under the rules heading *only make regular shapes*, but then adds *(you can break it sometimes)*. Justin says, "I always break the rules sir," and Jim says, "really Justin." Then "now, tell me something about a square." A girl says, "they're all the same length." Jim says, "yes so I have to go round 4 lengths all the same and if I call this m," he draws:



and says, "I'll say 4 times m equals what?" A girl and boy say "36," and Jim asks, "so how do I work out what m is?" A few say "9," one girl says "36 divided by 4." Jim responds to these saying there are two ways of looking at it: "we can say $4 \times m = 36$ by thinking about our times table, or we can say 36 divided by $4 = m$, but you can only really use the first when it's a whole number." Then "so how big is it? what is the area?" A few say "81," and Jim says, "the area is 81 meters squared. Why meters squared? because it's an area, when you work out area it's meters squared." Then "I want you to look at all of those shapes and find ones that are possible to do, and I'm interested in the area of them, why might I be interested in area? what is it useful for? I may be making a garden or a pen." Jim suddenly turns to a boy near him who has been chatting incessantly and says, "Michael, it is irritating you talking all the time, OK?" Michael looks repentant and Jim continues. "So I'm interested in area. I'd like you to explore these shapes and find areas. Now, the first thing I'd like you to do is record what I've been talking about. My writing isn't sufficient; you need to put things in your own words, your version of the problem. Expand it, write what it means, pick out shapes, decide what order you need to do them in!" As the students go back to their seats and start working, Matt, who is new to the school, says, "sir, I don't understand these shapes, I don't think I've seen them before." Jim says, "well that could be one of your tasks, find out about the shapes, look them up in your maths dictionary, or you could look in an ordinary dictionary."

> Most of the class starts work; some are talking, some have started straight in with drawing squares, some check with Jim what they are meant to be doing, two boys sitting at the back are talking about something else and not working. Matt is looking up the shapes in a dictionary, most students seem to have started doing the task, without explaining in their own words what they have to do, as Jim suggested. Three boys sitting together at the back are slow to get started; they write a few words, talk for a while, write some more, and so on. Two boys pick up their table and move it round to avoid the sun. Jim is kneeling down by the side of somebody saying, "you draw it whatever size you want to draw it." Most of the students' introductions say, "we need to see how many shapes we can make out of 36 fences" or something similar. Four girls are sitting having an animated and excited conversation about all the different shapes: "is it a quadrilateral?" (laughs). Another says, "what's that?" Another says "a trapezium?" They seem very interested. As I pass Julie, she checks with me what a regular shape is before she writes out her definition in her introduction. Some students have now written about a paragraph. Three boys at the back have only written a heading and a sentence. Most of the rest of the class have moved onto examples. None of these students is using calculators, nor do they ask for them, although they are available. One of the boys is finding out which is bigger, a rectangular area or a triangular area. Jim comes over and says, "so which was bigger?" The boy tells him and he asks, "is that what you would think, does it look bigger?" They discuss this for a while. (Jim Cresswell, Year 8)

The previous extract is a fairly typical example of an introduction to a project on which the students worked for approximately 3 weeks. The only unusual aspect was that the students were given one project to work on, rather than a choice of projects. Jim's introduction incorporated a number of features that related to the openness of the approach. Jim introduced the problem of *36 fences* by getting the students to think about the different shapes that were possible. He did not spend much time at the board telling the students information; rather he created an arena for discussion and negotiation. During the course of this discussion, the students encountered the need for certain parameters, such as 36 fences must always be used and irregular shapes are not allowed. Jim did not tell the students these constraints at the beginning, but waited for them to be raised by the students. At the end of the class discussion, Jim told the students that it was not enough to write the problem out in his words; they needed to reformulate it in their own words using their own thoughts. More important, Jim did not give them a closed question to answer; he just said, "I am interested in area. I'd like you to explore these shapes and find areas." When Matt said that he was not familiar with the shapes, Jim suggested a place that he could find out about them. He refrained from telling Matt the exact information he needed to know.

The introduction to this problem also illustrates the ways in which teachers introduced structure and guidance. At a number of points, Jim guides the students, asking, for example, whether they *could* have a 36-sided pentagon, to introduce the idea of regular and irregular shapes. Jim introduces algebraic notation to the discussion and tells students about "two ways" of looking at division. Jim deftly weaves mathematical ideas into the conversation about fences, navigating students through the mathematical terrain, drawing from the students' own comments and questions whenever possible. It is characteristic of the Phoenix Park teaching that Jim presented a problem, from which the students generated questions and through which mathematical ideas and methods were introduced. Some of the Phoenix Park problems, such as this one, were contextualized with "real-life" references, others were not, but they were all open enough to encourage different ways of thinking about the mathematics that was or could be entailed.

When the students started their work, Jim left them to their own devices. He did not "police" the room or check that they were going about things in a specific way. When he could, he interacted with students and engaged them in conversations about their work. When one of the students said that a triangular area was bigger than a rectangular area, Jim did not correct him. He asked him whether he would expect this, whether it looked bigger—he encouraged him to think about the situation. It was also typical that students completed differing amounts of work in the remainder of the lesson. Some copied Jim's introduction or another student's introduction in a fairly absent-minded way and did nothing else. Some started their work in a relaxed way, interspersing it with non-mathematical conversations; others engaged in lively debates about the problem. By the end of the first lesson, the students had produced different amounts of work that focused on different questions and problems. As time went by and more lessons were spent on the theme, the students began to diverge more and more, both in the amount of work they completed and the topics on which they worked.

Teachers introduced activities to students which they knew were mathematically rich, but the teachers did not have fixed ideas about the ways in which students would interact with the problems. In a Year 10 lesson taught by Martin Collins, Shelley was working on an investigation into the patterns that emerged from the manipulation of different sets of numbers. After investigating the patterns deriving from combinations of four numbers, Shelly moved on to investigate five:

> After working on the problem for a while, Shelley takes it over to Martin to show him. He looks at the work, laughs, and says, "golly, I didn't know it could get that complicated." Shelley says, "shall I stop?" Martin says, "no, carry on." Shelley says, "I want to carry on because I want to see what hap-

pens to the horizontals when I continue up in this direction." (Martin Collins, Year 10)

This extract is interesting not because Shelley was extending the activity in an unusual or idiosyncratic way, but because Martin had obviously not encountered the particular extensions before. Shelley also demonstrated in this extract that it was the unknown aspect of the exploration that held her interest. She was genuinely interested to know what the mathematical outcome would be of extending the work. When students showed the teachers their work, they did not seem to expect the teachers to have seen it before. They did not expect them to look and say, "yes that is right," but to look and see whether they were moving in an interesting direction. Such interactions then formed the basis for dialogue between students and teachers.

The mathematical content encountered by students during their time at Phoenix Park generally emerged from the projects on which they were working. Ideas within algebra, geometry, number, and data handling would repeatedly recur in relation to each other. The teachers would occasionally stop the class and teach them all a method or ask students to share the directions of their work, but generally the students learned about methods as they became important to the particular investigation they were pursuing. In addition, students received multiple opportunities to develop and use mathematical processes. Indeed the department's approach was designed to integrate aspects of the "using and applying" strand of the National Curriculum, which sets out processes of "application, communication, reasoning, logic, and proof" into every activity the students would meet. Over time, the students became aware of the processes they were learning:

A: It's structured so that . . . it helps with other subjects like science, the results and drawing conclusions, it helps develop those skills. (Alex, Phoenix Park, Year 10, JC)

The students gave other indications that they regarded their mathematics learning as an open experience. In interviews in Year 9, I asked the students to say whether they thought mathematics lessons were similar to any other lessons at the school. Sixteen of the 20 students said that mathematics was most similar to art, English, or humanities; nobody compared mathematics to the subjects more traditionally linked to it, such as science.

JB: Is maths similar to any other lessons at Phoenix Park, or is it different?

L: I suppose it's a bit like English and art and stuff, English, when you're left to do your own work—they explain at the beginning what to do and then you're left on your own to do it. (Lindsey, PP, Year 10, JC)

Differentiated Opportunities

The openness of the activities teachers chose to use at Phoenix Park enabled the provision of differentiated opportunities. This was important to the teachers because they strongly believed that all students should be encouraged to reach high mathematical levels and that students should not be taught in low-level groups. Teachers chose the activities carefully so that they would provide different access points for different students and enable students to work on them at different mathematical levels. The 36 fences activity enabled students to consider the areas of different shapes, draw graphs of relationships, explore combinatorial geometry, learn about trigonometry, and so on. When students began at the school in Year 8, they worked on an investigation called *consecutive numbers*. The investigation asked students to choose three consecutive numbers, square the middle numbers, and multiply the outer ones. In this early lesson, some of the students worked only with different sets of numbers; other students represented the consecutive numbers algebraically. It was commonplace at Phoenix Park for students to engage in mathematics at a variety of levels of difficulty. In one of the activities, students were asked to investigate loci. They were introduced to this concept by going into the playground and being asked to stand in different configurations. At first students were asked to stand 5 meters away from a particular student, then 5 meters from a particular line. They then had to stand at equal distance from two different points. After this introduction to the ideas, students continued to investigate relationships back in the classroom. As a homework for this activity, students were asked to imagine the path of a dot drawn on a circular piece of card that is rolled along a flat surface, then a dot on a triangle, a square, and a shape of their own. Students were told to vary the position of the dot and consider the paths formed. For some students, this was an opportunity to think about shapes and symmetry; for others, it was their first opportunity to learn about Pythagoras; for still others, it was an opportunity to learn about the major and minor axes of a parabola. This activity, like all of those introduced at Phoenix Park, enabled students to move in a number of directions around the mathematical terrain.

In the examination system used in England and Wales, students are entered for one of three levels of the same examination: higher, intermediate, or foundation. The different levels share some of the same questions, but the higher level examinations also include more difficult questions. The different levels give access to different grades. At Amber Hill, the students' examination levels were almost completely determined by the set in which they were located because the different sets were geared toward different levels of mathematics content. This meant that the students' access to different examination grades was partly determined when they were 12 or 13 years of age—3 years before the examinations. At Phoenix Park, the teachers waited to make the decision about different examination levels as late as they could to ensure that all students received opportunity to work toward the highest levels. They made decisions about examination levels when they were required to send lists of students to the examination board toward the end of the students' final year.

Schools have the option of choosing an examination option in which 20% of the students' overall grade is determined by a coursework project completed in school and graded by examination officials. Both schools chose this option. At Amber Hill, the students worked on coursework projects specifically completed for the examination. These took place during 3 weeks of Years 9 and 10, and the students enjoyed them very much. At Phoenix Park, the students and teachers chose projects from those they had worked on over the year. When the students entered their final year and were more aware of examination grades and the ways their coursework contributed to those, the teachers would tell them about the potential of different projects, saying things like, "This is an A/B-ish project and this is a C/D-ish project." Students would use this information to guide their decisions.

There is not space in this book to give many examples of the different projects the students encountered or the different mathematical opportunities the projects provided, but all of the projects shared the characteristic of being sufficiently open to enable different levels of mathematical investigation. If students finished projects or became bored with their work, the teachers would invent extensions for them or offer another idea for students to work on. The following extract, taken from a leaflet prepared by the mathematics department, demonstrates the centrality of the teachers' commitment to mixed ability, differentiated teaching, to their departmental mission:

> Mathematics is a world of powerful and beautiful structures, a way of thinking, organising, investigating and solving problems. It is also, of course, useful in everyday life.
>
> We use a wide variety of activities; practical tasks, problems to solve, investigational work, cross-curricular projects, textbooks, classwork and groupwork. Every task can be tackled by students with widely differing backgrounds of knowledge but the direction and level of learning are decided by the student and the teacher. The tasks are chosen so that each student is challenged and stretched at an appropriate level.

Phoenix Park students experienced the freedom to encounter different mathematical ideas and content levels. This meant that some students worked on high-level mathematical topics, such as calculus, that normally would not be encountered until Years 11 and 12. The teachers encouraged such high-level investigations and supported students by holding conversations with them, introducing them to new ideas, and sometimes referring them to books and other reference materials.

Learning to Learn

An important feature of Phoenix Park's approach was the careful attention teachers paid to the way students needed to learn. Corbett and Wilson (1995) argue that those working to promote educational reforms have generally overlooked the fact that students not only need to develop new ways of working in reform-oriented classrooms, but an understanding of and commitment toward the changes in their roles. They argue that, "students must change during reform, not just as a consequence of it" (p. 12). This is a simple but important point that has been given surprisingly little attention. Thus, many teachers have introduced new methods to students, such as working on open problems or having class discussions, without teaching students how to *engage* in these ways of working and how to succeed. Further, there is evidence that knowing how to succeed in relation to new and reform methods of teaching may be a form of knowledge that is inequitably distributed (Jackson, 1989; Lubienski, 2000; Pope, 1999; Zevenbergen, 1996), making it important for teachers to attend to the distribution of such knowledge. David Cohen and Deborah Ball (2000) term the different practices that students need to employ and understand in school *learning practices*. The Phoenix Park teachers seemed to pay careful attention to the learning practices that students needed to develop, teaching students how to learn in an open approach.

When students began at Phoenix Park, they had encountered more closed and traditional presentations of mathematics for the previous 8 years. The students reported that they had to make a number of adjustments when they arrived at their new school:

A: It was a big change from my last school, having the books and then just having it written on the board and being told to get on with it. (Andy, PP, Year 9, RT)

Many of the students took to the approach with ease, reporting that they were appreciative of the opportunity to work in more open ways. But some of the students, particularly boys, found the openness of the work extremely disconcerting. They said they were uncomfortable with the lack of structure or suggested direction in the problems they met, and that they preferred a more traditional approach. These students, along with the majority of students at Phoenix Park, came from homes of severe poverty, living on a housing estate (similar to a U.S. project), where police would not venture at night. When I interviewed the students at the beginning of the study, they described their motivations clearly:

S: When I go into a maths lesson I usually sit down and I think, who am I going to throw a rubber *(erasure)* at today? (Shaun, PP, Year 8, RT)

JB: Can you think of a maths lesson that you've enjoyed?

M: Messing about, that's what I enjoy doing.

JB: What would make maths better?

M: Working from books—you don't mess about if you've got a book there, you know what to do. (Megan, PP, Year 8, RT)

Although some students *blamed* their misbehavior on the openness of the work, the teachers did not give the students books or structure. This may have been the easiest option, but the Phoenix Park teachers believed that the open-ended approach they used was valuable for *all* students and that it was their job to make the work equitably accessible. They therefore developed a range of practices that served to increase the students' access to the problems they met and the methods they were expected to use.

One practice that was central to the Phoenix Park teachers' approach was that of introducing the activities to students themselves, which enabled the teachers to decide on the degree of support or structure students needed. In the 3 years that students attended Phoenix Park, they were never left to interpret text-based problems alone. The teachers always spent time with individuals, groups, or the whole class introducing ideas and making sure the students all knew how to start their explorations. At Phoenix Park, the teachers would frequently ask the students to gather around the board before leaving the class so they could all have some discussion of the homework problems being posed. M. Smith (personal communication, 2001) reported her observations of a middle-school teacher in an urban school in the United States who used a reform curriculum. She reported that the teacher would ask students to read problems aloud in class and then hold a discussion about the context of the problem and any vocabulary used in case either was unfamiliar.

Then she would ask students to discuss what they thought the task was asking them to do. After such discussions, the teacher would ask groups to work on the task and check that different individuals understood what they should do. Such practices were also employed by Phoenix Park teachers in order to make tasks equally accessible, but they contrast with many classrooms I have visited in which students are left to interpret the aim of problems from their reading of reform texts, which are often extremely wordy and linguistically demanding. The way in which work is introduced to students and the access students are given to the mathematical ideas that they are intended to explore seems extremely important for the attainment of equity.

A second important feature of the Phoenix Park teachers' practice was that they paid attention to the ways in which students communicated their understanding, as well as the students' understanding of the need for that aspect of their work. In more traditional mathematics classrooms, such as those of Amber Hill, students are required to produce correct answers. In reform-oriented classrooms, students often need to go beyond correct answers and explain their methods, justifying the approaches they have used. At Phoenix Park, the teachers paid careful attention to this aspect of the students' role and helped them understand the particular learning practices in which they needed to engage. For example, in one of the lessons I observed, the teacher asked all the students to gather round the board; then she posed the following question: "If someone new came into class and asked you what makes a good piece of work, what does Ms. Thomas like, what would you say?" The first student offered "lots of writing"; others offered suggestions such as, "have an aim," "draw a plan," and "write about patterns."

Each time the teacher came back with further questions—such as "is the amount of writing important?" "what does that mean?" "why is a plan important?", "what does a good plan look like?" "why do we record patterns?" The students struggled over many of their explanations, but they sat around the board engrossed in this discussion for some time. The students were clearly appreciative of the opportunity to learn about valued ways of working. As they talked, the teacher kept a record of the students' suggestions on the board. After approximately 40 minutes of discussion, the teacher told the students that their task was to design a poster describing the different features of "good work." She also gave them a page that the department had prepared called *hints for investigations*. It was divided into three columns headed *what to say*, *how to say it*, and *making sense of it*. These showed different suggestions for students, such as, "Can you make the problem more general?", "Make the original problem more difficult," and "Now explain how or why your algebraic rules work." The students studied the page and incorporated many of the suggestions into their posters. This lesson explicitly focused on the mathematical learning practices (Cohen & Ball, 2000) that the students needed to employ in the pursuit of their mathematical investigations.

The Phoenix Park teachers frequently encouraged individual students to explain their reasoning and communicate in more detail because the students were not used to doing so when they arrived at the school. In one of the lessons I observed, a student gave a problem on which he had been working, which showed some of his methods and a correct answer. The teacher studied it for a while and then said:

> Brilliant work John but you can't just write it down, there must be some sense to why you've done it, some logic. Why did you do it that way? Explain it. (Rosie Thomas, Year 10)

Rosie's "there must be some sense to why you've done it" typifies the sort of encouragement the students were given at Phoenix Park. The teachers strove to expand the way in which the students thought about mathematics, extending the students' value systems beyond the desire to attain correct answers. The teachers at Phoenix Park developed a range of practices aimed at helping all students understand what they needed to do to function successfully in a reform classroom, and there was considerable evidence that they were successful in that regard:

I: It's an easier way to learn, because you're actually finding things out for yourself, not looking for things in the textbook.

JB: Was that the same in your last school do you think?

I: No, like if we got an answer, they would say, "you got it right." Here you have to explain how you got it.

JB: What do you think about that?

I: I think it helps you. (Ian, Phoenix Park, Year 9)

When the Phoenix Park teachers found that some students were not communicating their thinking or interpreting numerical answers, they devoted more time to this aspect of their teaching, regarding the students' reluctance as a gap in their understanding of what was required in the work. After months of careful support from the teachers, the reluctant students started to become more engaged with their work and more comfortable with the freedom they were given. The change in some of the disaffected boys became most obvious when, in the second year of the school, they were taught by a student teacher who tried to teach mathematics in a more traditional way. In the following extract from my observation notes, the boys start to complain because of the *closed* nature of the work given to them. This was very different from the approach to which, by then, they had become accustomed:

> The student teacher starts the lesson by asking the class to copy what he is writing on the board. He is writing about different forms of data, qualitative and quantitative. The students are very quiet and they start to copy off the board. The teacher then stops writing for a while and tells the students about the different types of data. He then asks them to continue copying off the board. After a few minutes of silent copying, Gary shouts out, "Sir when are we going to do some work?" Leigh follows this up with, "Yeah are we going to do any work today sir?" Barry then adds, "This is boring, it's just copying." The teacher ignores this and carries on writing and talking about data. The boys go back to copying. The teacher looks across at Lorraine, who is looking puzzled, and asks her if she "is OK." She says, "No not really, what does all this stuff mean?" This seems to annoy the teacher or make him uncomfortable; he turns back to the board and continues writing. Gary persists with his questioning, this time asking, "Sir, why are we doing all this?" The teacher replies: "We are just rounding off the work you have done."
>
> After about 20 minutes of board work, the teacher asks the students to go through all of their examples of data collection that they have done over recent weeks and write down whether they are qualitative or quantitative. Peter asks, "Sir what's the point of this? Aren't we going to do any work today?" the teacher responds with, "You need to know what these words mean." Peter replies, "But we know what they mean, you've just written it on the board so we know." (Phoenix Park, student teacher, Year 9)

This series of interactions was particularly interesting to observe because it was the group of boys who had been most resistant to open-ended work when they started at Phoenix Park who objected to the closed nature of the work the student teacher gave them. The boys repeatedly asked "whether they were going to do any work today," indicating that they did not regard copying off the board as work probably because it did not present them with a problem to solve. When the student teacher told them to classify data as quantitative or qualitative so that they would learn what the words meant, Peter questioned the point of this because they had already been told what they meant. Yet the mathematics teaching offered in this example is fairly characteristic of more traditional high school mathematics pedagogy, in which the teacher explains what something means to students, they copy it down from the board, and then they practice some examples of their own. The degree of resistance the students provided seems important to consider partly because it gives an indication of the ways students adapted to their school approach. Teachers often talk of students' resistance to the use of open problems or the need to discuss methods, but this interaction showed that even the most reluctant students changed their expectations for ways of working with careful support. In one week that I was in school, Jim had been absent for one lesson and the class had been taught by a substitute teacher from outside the department. When he returned, one of the previously resistant boys complained about the substitute teacher, saying to Jim: "It was terrible—we had this teacher who acted like he knew all the answers and we just had to find them."

The Phoenix Park teachers paid explicit attention to the learning practices students needed, teaching students *how to learn* as well as teaching them mathematics. Many of the practices were those that are valued in other reform-oriented classrooms, but teachers do not always give them such explicit attention. They assume that students will understand the need for their use and the changed practices they need to employ.

Time on Task

Another striking aspect of school mathematics at Phoenix Park related to the number of students choosing not to work in lessons, which continued to be a source of surprise to me. In their Year 9 questionnaire, students were asked to describe their mathematics lessons to someone from another school. The most popular description from 23% of students was "noisy." In the 100 or so lessons I observed at Phoenix Park, I would typically see approximately one third of students wandering around the room chatting about nonwork issues and generally not attending to the project they had been given. In some lessons, and for some parts of lessons, the numbers off task would be greater than this. Some students remained off task for long periods of time, sometimes all of the lessons; other students drifted on and off task at various points in the lessons. In a small quantitative assessment of time on task, I stood at the back of lessons and counted the number of students who appeared to be working 10 minutes into the lesson, halfway through the lesson, and 10 minutes before the end of the lesson. Over 11 lessons, with approximately 28 students in each, 69%, 64%, and 58% of students were on task, respectively.

The freedom that the students experienced to stop working when they wanted to seemed to be created by a number of interrelated facets of the Phoenix Park approach. It was partly to do with the nature of the mathematical approach and the fact that students could be wandering around the room and chatting with other students as part of their work. It also related to the fact that the students could all have been working on something different, which made it difficult for teachers to monitor the amount of work that they completed:

T: It gives some people more of a chance to muck about *(misbehave)*.

JB: Why?

T: Because, for instance, at the end of a lesson if the teacher wanted to check how much work you'd done he couldn't, but if you started at number 1 he would know that you hadn't got to number 20 or whatever. (Trevor, PP, Year 10, RT)

More important than either of these factors, however, is that the freedom the students experienced seemed to relate directly to the relaxed and nondisciplinarian nature of the three teachers and the school as a whole. Most of the time, the teachers did not seem to notice when students stopped working unless they became very disruptive. All three teachers seemed concerned to help and support students and, consequently, spent almost all of their time helping students who wanted help, leaving the others to their own devices. The three teachers were not markedly different in this regard, although Jim Cresswell's lessons were noticeably more chaotic than those of the other two teachers.

> I think the weakness of my teaching style would be very much that I depend on willingness and co-operation and, you know, if somebody is motivated to do the stuff they will achieve well. (Jim Cresswell)

Jim often told me that he was "no good at discipline," and my lesson observations showed that students in his classes were less on task than the classes of other teachers. This was partly because he treated the students in an adult way and some of the students took advantage of this. For example, there was a small classroom attached to Jim's room that nobody used. Jim used this room as a talking room, and students were meant to work in there if they wanted to talk and work, leaving the other students to work in quieter conditions. Jim was not concerned about his inability to see the students in this room, and he rarely asked students to work when they were not doing so unless they became disruptive. When Jim did tell students to work, the result was often ineffective. Typically the students would say something back to Jim, which sparked a debate between Jim and the student. At the end of this, the student usually went back to not working, and Jim would usually be called away to help somebody. In a number of Jim's lessons I observed, so few of the students appeared to be working that I started to have serious doubts about my research study. At the end of my research, I found out that some of the newer, more middle-class parents at the school had complained about Jim's teaching, which resulted in the principal visiting his lessons and telling Jim that only about 30% of students were on task.

Both Rosie Thomas and Martin Collins showed more overt concern to keep students on task than Jim. But while both teachers were more likely to react to the extremes of behavior that Jim tolerated, they nevertheless seemed unconcerned about students who sat and chatted through parts of their mathematics lessons. When the two teachers asked students to work, this often had little effect: The students worked for a few minutes, then went back to chatting. The degree to which students were on task in lessons also varied between classes, year groups, and aspects of lessons. In later sections, I explore the impact of the independence that students experienced over their work rate, which produced some surprising results.

Independence and Choice

There were many overt and covert ways in which the students at Phoenix Park were encouraged to be independent. This meant that they needed to take on some responsibilities as part of their mathematics approach in order to succeed. For example, the students were not given exercise books for their work: They used pieces of paper. At the start of activities, they were given blank or lined pieces of paper as well as graph paper if they needed it. The students each had a box file in which they kept their work. Nobody took charge of this process for the students; papers were not collected at the end of lessons. Students were meant to either take them home and bring them back again or store them in their box file. Students often came to lessons having forgotten or lost their work from the previous lesson and so took a new piece of paper and continued on that. Some of the students were disorganized, and their box files were made up of odd collections of extracts from different activities. At the end of each project, students were meant to gather together their work, present it in a coherent fashion, and summarize it. The students were rarely encouraged to be careful or tidy, and many of the finished projects looked messy compared with a more typical mathematics exercise book.

At around Easter of Year 10, the school sent pieces of coursework to the official examination offices. At Amber Hill, the projects were the only ones worked on for 3 weeks of Years 9 and 10. At Phoenix Park, the teachers and students could select the best projects from all those they had worked on. But the teachers gave the choice to the students, who were told to choose their best two pieces of work and give them in. The teachers gave the students guidance if asked. Often the pieces of work that were sent to the examination officials were unfinished either because the students showed little concern for the task of choosing their coursework or because the students had no complete projects to send:

L: They left it to the last minute as well, like they kept saying you've got to have work for your GCSE and that, but if you didn't hand

your projects in, in years 8 and 9 they weren't really bothered were they?

H: No.

L: And at the end now they say we need them. (Louise and Hannah, PP, Year 10, JC)

Here the students related the incompleteness of their work to the lack of enforced discipline or control from their teachers. It seemed surprising to me that the teachers gave such an important responsibility to the students, but this was consistent with the general approach of the school:

P: The amount you do is always up to you isn't it? How much homework you do and especially course-work for GCSE, it's your work, it's your responsibility, I mean however much work you get in, that's always going to be reflected in your mark. (Nile, PP, Year 10, JC)

Another important responsibility that the students held emanated from the choice that students were given about the projects they could work on and the direction in which the students took their work. The students at Phoenix Park were given considerable and varying amounts of freedom in their choice of work, their approach to work, the way in which they behaved in lessons, the organization of their work, and even their work environment. This choice and the students' independence had an important impact on their responses to mathematics, which are considered next.

THE STUDENTS' RESPONSES

Enjoyment

In interviews, conversations, and lesson observations at Phoenix Park, the students gave a much more varied picture of their enjoyment than the students at Amber Hill. The Amber Hill approach prompted a fairly consistent reaction from the students, whereas the Phoenix Park approach seemed to divide the year group into those who loved it, those who liked it, and those who hated it.

Approximately half of the Phoenix Park cohort liked mathematics most of the time, but their enjoyment depended on the particular projects they were doing.

Approximately one third of the students was more positive than this, and they seemed to like everything about mathematics. Questionnaires and interviews in Years 8 to 10 showed that these students liked the approach because it was varied, they were given a choice about what they did, and they had the freedom to work in any direction.

V: I thought the activities were really interesting because you had to work out for yourself what was going on, you had to use your own ideas.

JB: How does that compare to the SMP work you used to do in middle school?

V: Boring, it was boring doing stuff out of books. (Vicky, PP, Year 10, JC)

S: You're able to explore more, there's not many limits and that's more interesting. (Simon, PP, Year 10, JC)

However, this freedom was also the reason the third group of students hated the approach. Approximately one fifth of the cohort thought that mathematics was too open, and they did not want to be left to make their own decisions about their work. They complained that they were often left on their own not knowing what to do, and they wanted more help and structure from their teachers. The students felt that the school's approach placed too great a demand on them—they did not want to use their own ideas or structure their own work, and they said that they would have preferred to work from books. What for some students meant freedom and opportunity, for others meant insecurity and hard work. There were approximately five students in each class who disliked and resisted the open nature of their work. These students were mainly boys and were often disruptive—not only in mathematics, but across the school.

In the Year 8 questionnaire item that asked students to describe the *most interesting piece of mathematics* they had ever done in a lesson the Phoenix Park, students responded in a different way than the Amber Hill students. Whereas 49% of Amber Hill students chose *Logo*, Phoenix Park students described a variety of different projects. Five different projects were nominated by at least 5% of students: 11% of students chose *Logo*, 10% an activity called frogs, 9% a probability project, 8% the maths day (when they worked on mathematics projects all day), and 6% an activity called *limping seagulls*. Another 36% of students chose other class projects encountered over the past year. The question asked students to describe the most interesting piece of mathematics they had ever done in a school lesson. Many of the Amber Hill students described a lesson from elementary school or from Years 6 and 7. At Phoenix Park, all of the students described one of the projects they had experienced since starting at Phoenix Park in Year 8, and all descriptions were positive. For example:

"Horse racing was good because the answers were unexpected."

"The best piece of maths I think I have done was boxes as I did quite a long project."

"Statistics, I thought this was the most interesting, I wrote a large amount about marriages and divorces using the book *Social Trends*."

The Phoenix Park students' replies gave the impression that they were genuinely interested in the projects they had chosen, and they did not report that mathematics lessons were monotonous or boring.

In Year 8, students from both schools were asked in a questionnaire to state how often they enjoyed the mathematics they did in school (*always, most of the time, sometimes, hardly ever,* or *never*). This closed question produced similar results from the two schools. Forty-three percent of Amber Hill students and 52% of Phoenix Park students reported enjoying mathematics *always* or *most of the time.* However, the students responded very differently to open questions on the same questionnaire. One question asked the students to describe what they disliked about mathematics at school. Forty-four percent of Amber Hill students strongly criticized the mathematics approach, and 64% of these students criticized the textbook system. At Phoenix Park, 14% of students criticized the school's approach, and the most common response—from 23% of students—was to list nothing they disliked about mathematics at school. This compared with 6% of Amber Hill students. Table 5.1 presents all of the responses the students gave to the three different open questions on the questionnaire, which asked students what they liked, disliked, and would like to change about mathematics lessons. These three questions prompted 382 comments from the 160 Amber Hill students and 202 comments from the 103 Phoenix Park

TABLE 5.1
Year 8 Open Questionnaire Responses

Nature of Comment	*Amber Hill* *% (n = 382)*	*Phoenix Park* *% (n = 202)*
Enjoy open-ended work	14	38
Dislike textbook work	22	0
Cannot understand work	20	6
Can understand work	3	5
Work is interesting	4	21
Want more interesting work	15	19
Want more group work	5	0
Enjoy working alone/with others	8	4
Pace is too fast	9	3
Pace is about right	0	3

students (there were no significant differences between the number of comments per student at the two schools). The responses have been combined to present an overview of the issues important to the students.

This table shows that students at the two schools chose to address different issues when they were invited to give their own opinions on mathematics lessons. The Phoenix Park students chose to comment on the interest of their lessons and their enjoyment of open-ended work. These two sentiments comprised 59% of all the Phoenix Park comments. The Amber Hill students were more concerned about lack of understanding and their dislike of textbooks; these two comments comprised 42% of the Amber Hill responses. Many more of the Amber Hill students probably would have talked about open-ended work if they had ever experienced any, but at that time they had not yet worked on their coursework projects. In response to the three questions, there were 88 comments (23%) from Amber Hill students that related to their perceived lack of understanding of mathematics, compared with 6 comments (3%) from the Phoenix Park students.

In their Year 8 questionnaire, the students from both schools were asked to write a sentence describing their mathematics lessons. The three most popular descriptions from the 75 respondents at Phoenix Park were *noisy* (23%), *a good atmosphere* (17%), and *interesting* (15%). This contrasted with the three most popular responses from the 163 Amber Hill respondents, which were *difficult* (40%), *something related to their teacher* (36%), and *boring* (28%). The students' sentences were also coded as either *very positive, positive, neutral, negative,* or *very negative*. Table 5.2 shows the distribution of results for the two schools.

The overall picture of enjoyment gained from Phoenix Park was therefore more varied and significantly more positive than that gained from Amber Hill. At Phoenix Park, the vast majority of disaffection was suggested by a small proportion of students who showed opposition to school in general. A consideration of the various forms of data, including questionnaires, interviews, and lesson observations, suggests that approximately one third of the Phoenix Park students positively liked mathematics particularly because of its variety and openness, approximately one half of students enjoyed some of the projects some of the time and disliked others at other times, and the remaining students disliked the approach,

TABLE 5.2
Describe Maths Lessons: Coded Responses

%	*very positive*	*positive*	*neutral*	*negative*	*very negative*	*n*
AH	0	23	38	33	6	154
PP	5	38	32	25	0	67

particularly the freedom and openness they experienced. I consider this last group of students in more depth in a later part of this chapter.

Engagement

The General Picture. The Phoenix Park students varied in the extent to which they engaged with their mathematics. They were often left to decide whether or not they worked in class. This meant that some students worked with enthusiasm on their mathematics projects, while others would find talking or disrupting the class more interesting than their work. It was difficult for the students to work in a procedural way at Phoenix Park because the students constantly needed to make decisions about their project work. This meant that students tended to be either interested and working or uninterested and not working. The following extract is taken from the third lesson on the theme *36 fences*, which was described earlier and taught by Martin Collins. Some of the students have considered the areas of different rectangles with a perimeter of 36; others have moved on from this and have started to investigate the areas of different shapes.

> Mickey has found that the biggest area for a rectangle with perimeter 36 is 9 × 9 and is moving on to find the area of equilateral triangles, compared with other triangles; he seems very interested by his work. He finds one area and is about to find another when he is distracted by Ahmed, who tells him to forget triangles, he has found that the shape with the largest area made of 36 fences has 36 sides. He tells Mickey to find the area of a 36-sided shape too and leans across the table explaining how to do this excitedly. He explains that you divide the 36-sided shape into triangles and all of the triangles must have a 1cm base. Mickey joins in saying, "yes and their angles must be 10 degrees!" Ahmed says, "yes but you have to find the height and to do that you need the tan button on your calculator, T-A-N, I'll show you how. Mr Collins has just shown me." Mickey and Ahmed move closer together to do this. At another table, I ask Clare what she is doing; she says that she is working out the area of a hexagon and she shows me her diagram. She explains that she is working out the area by dividing it into six triangles; she has drawn one of the triangles separately. She says that she knows that the angle at the top must be 60 so she can draw it exactly to scale using compasses and find the area by measuring the height. Clare seems to have made these decisions on her own and she is clearly interested in her work. At another table, six girls have not started work even though we are 20 minutes into the lesson; they are sitting coloring on their folders; another group of boys are working out the areas of rectangles, but they do not seem to be particularly interested in what they are doing. (Year 8, Martin Collins)

This extract demonstrates the different amounts of enthusiasm and interest that were commonly in evidence during Phoenix Park lessons. Clare was not a high-attaining student, but she was interested in what she was doing and the decisions she had made. The six girls who were drawing on their folders were clearly not interested, and the small group of boys working out the areas of rectangles were not working with enthusiasm. Mickey and Ahmed were two high-attaining boys who were extremely involved in their work and who seemed genuinely excited to be discovering some new mathematical methods and relationships. The interest they showed for trigonometry, because they could *use* a tangent ratio to help them find something out within their project, was vastly different from the interest the Amber Hill students showed toward trigonometry. Students at both schools learned trigonometry within contextualized questions regarding shapes, but the Amber Hill students were introduced to trigonometric rations, then required to practice them within contextualized questions. The Phoenix Park students met trigonometric ratios when they needed them to solve problems. The students at Phoenix Park seemed to regard trigonometric ratios as exciting, reminding me of this description that Margaret Wertheim (1997) offers of the way she was introduced to pi when she was a child:

> When I was ten years old I had what can only be described as a mystical experience. It came during a maths class. We were learning about circles, and to his eternal credit our teacher, Mr Marshall, let us discover for ourselves the secret of this unique shape: the number known as pi. Almost everything you want to say about circles can be said in terms of pi, and it seemed to me in my childhood innocence that a great treasure of the universe had been revealed. Everywhere I looked I saw circles, and at the heart of every one of them was this mysterious number . . . It was as if someone had lifted a veil and shown me a glimpse of a marvelous realm beyond the one I experienced with my senses. (p. 3)

The students at Phoenix Park responded in similar ways to a number of their mathematical discoveries. This means that they experienced moments of wonder and excitement for at least *some* of their mathematics careers, which contrasted strongly with the Amber Hill students.

Another important difference between Amber Hill and Phoenix Park was that Phoenix Park students were not made to work. In interviews, the students did not talk about work they had done because they had been forced to but had gained little from in the way that the Amber Hill students did. They talked instead about the choice they had between involvement and doing nothing:

H: It was definitely a lighter lesson—you'd be involved and if you didn't want to be involved you'd sort of sit back and watch it all happen I suppose. (Hannah, PP, Year 10, JC)

Here Hannah does not give working without involvement as an option, whereas this was something of which the Amber Hill students were acutely aware. Although the freedom that the students at Phoenix Park experienced over their work rate meant that some students did very little, it also meant that some students worked in a motivated way. When students were asked to say the amount of time they worked in lessons, the results were interesting. Figure 5.1 presents the Phoenix Park students' results alongside those from Amber Hill to demonstrate the difference in the distribution of the results. At Phoenix Park, the students' times produced a symmetrical distribution, indicating that when students were given the freedom to work (or not), some students did very little work, but as many chose to do a lot. Indeed a much greater proportion of Phoenix Park students reported working for 51 to 60 minutes than Amber Hill students, who were made to work (12% at Phoenix Park, 2% at Amber Hill). Despite these differences, the means of the times given by Amber Hill and Phoenix Park students were identical (37 minutes). In some senses, this is remarkable given the difference in the freedom experienced by the two sets of students. Earlier I described the relaxed nature of Jim Cresswell's lessons and said that these lessons appeared to be more chaotic than those of Martin and Rosie. However, the means of the times given by students of the three teachers at Phoenix Park were as follows: Rosie, 40 minutes; Jim, 39 minutes; and Martin, 32 minutes. Martin was reported by the students to be the strictest of the three teachers. The similarity between the times given by students of different teachers and the times of students at the different schools adds further weight to the idea that making students work is not a particularly effective way to get students to think about mathematics.

The Uninterested Students. In every mathematics lesson I observed at Phoenix Park, between three and six students would do little work and spend much of their time disrupting others. I now try to describe the motivation of these 20 or so students, who represented a small but interesting group. The students who did little work in class were mainly boys, and they related their lack of motivation to the openness of the mathematical approach and, more specifically, the fact that they were often left to work out what they had to do on their own.

S: I tend to doss about a lot in maths *(mis-behave)*, half the time I can't be bothered to call miss over or ask her what I want to know, but I do realize that maths GCSE is pretty important.

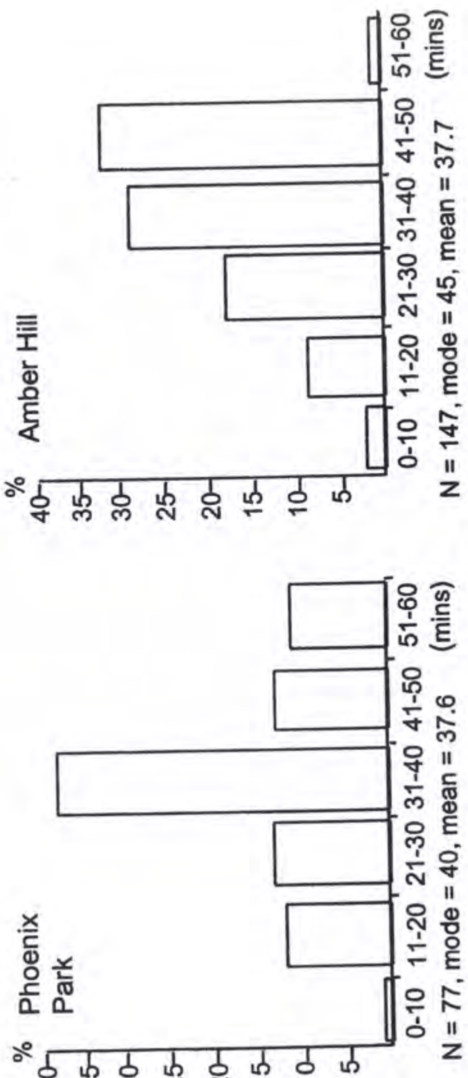


FIG. 5.1. Students' perceptions of time spent working in 60-minute mathematics lessons at Phoenix Park and Amber Hill.

JB: Why do you mess about in maths more than other subjects?

S: Because half the time if I ask for help I don't get it, or I don't get it until 20 minutes after I've asked. (Shaun, PP, Year 10, RT)

Many of the Phoenix Park students talked about the difficulty they experienced when they first started at the school working on open projects that required them to think for themselves. But most of the students gradually adapted to this demand, whereas the disruptive students continued to resist it. In Years 9 and 10, I interviewed six of the most disruptive and badly behaved students in the year group: five boys and one girl. They explained their misbehavior during lessons in terms of the lack of structure or direction they were given and, related to this, the need for more teacher help. These students had been given the same starting points as everybody else, but for some reason seemed unwilling to think of ways to work on the activities without the teacher telling them what to do. This was a necessary requirement with the Phoenix Park approach because it was impossible for all of the students to be supported by the teacher when they needed to make decisions.

The students who did not work in lessons were no less able than other students; they did not come from the same middle school and they were socioeconomically diverse. In questionnaires, the students did not respond differently from other students, even on questions designed to assess learning style preferences. The only aspect that seemed to unite the students was their behavior and the fact that most of them were boys. The reasons that some students acted in this way and others did not were obviously complex and due to a number of interrelated factors. Martin Collins believed that more of the boys experienced difficulty with the approach because they were less mature and less willing to take responsibility for their own learning than the girls. The idea that the boys were badly behaved because of immaturity was also partly validated by the improvement in the boys' behavior as they got older:

I: We have wasted a lot of time in the lessons, some of it, we have wasted time.

G: Yeah, we didn't used to do any work in lessons at all.

JB: But you take it more seriously now?

G: Yes.

JB: Why?

G: I'm not sure, in maths, then, we used to. . . .

I: Chuck *(throw)* stuff.

G: Yeah we always used to be chucking stuff and fighting, now we're a bit more serious. (Gary & Ian, PP, Year 10, JC)

The misbehaving students in each group were generally street-wise, confident students who seemed to enjoy being the center of attention. It was as if they had decided that school work was not for them, but they could gain satisfaction and self-esteem from being part of an antischool subculture. Other research studies have shown the presence of students with antischool values who gain pleasure from misbehaving (Ball, 1981; Willis, 1977), but the Phoenix Park students experienced more freedom than students generally do in schools. The result of this freedom seemed to be that they did little work. The students were also expected to do a lot in mathematics lessons. They were not asked to work through pages of a book following a rule. Instead they were asked to think for themselves, plan their work, and solve problems. They needed to make decisions and coordinate strategies. For many of the students, who were probably more inclined to "mess about" than work when they arrived at the school, this was too much, as one of the girls who shared lessons with them commented:

H: Well I don't think they were stupid or anything they just didn't want to do the work, they didn't want to find things out for themselves, they would have preferred it from the book, they needed to know straight away sort of thing. (Helen, PP, Year 10, MC)

Although the students at Phoenix Park who did little work in lessons were distinct from other students at the school, their behavior in lessons was only a more extreme version of a behavior displayed by most students at some times during lessons. The students worked when they wanted to work, which, for most students, meant intermittently.

S: But the tables that don't, even the tables that do get on with their work tend to jabber on a bit, like, Miss Thomas goes over to the table and she'll say "Oh did you see Neighbours *(TV Show)* last night?" to the other table and then they'll start talking and everyone will be talking. (Shaun, PP, Year 10, RT)

In summary, the students at Phoenix Park spent less time working than the students at Amber Hill, but they seemed to spend more time engaged with their work. This was not true for all of the students, but the widespread lack of interest evident at Amber Hill was rarely witnessed at Phoenix Park. This was replaced by a much more varied response to work, which, for most students, included both times of involvement and times of nonmathematical activity.

Students' Views About the Nature of Mathematics

The students at Phoenix Park were very different from the students at Amber Hill in the way they viewed mathematics. This was because most of the students believed mathematics to be an active, inquiry-based discipline. In the Year 9 questionnaire item that asked students to prioritize either thought or memory (see chap. 4), 65% of Phoenix Park students chose thought, compared with 36% of Amber Hill students. The majority of the Phoenix Park students did not regard mathematics to be a rule-bound subject involving set methods and procedures that they needed to learn; they saw it as a subject of explorations, negotiations, and inquiry:

A: You explore the different things and they help you in doing that. (Alex, PP, Year 10, JC)

P: You can do it at your own level, what suits you, and it's very sort of open. You can use it in different ways, you can do different things more than with set questions.

S: You're able to explore, there's not many limits and that's more interesting. (Philip & Simon, PP, Year 10, JC)

The students also had a sense of mathematics as a subject that allowed them to think deeply—to go beyond surface features of questions:

P: It's when you like learn new ways of doing things or you're like doing quite well on a problem . . . you're taking it really far, the investigation, you're getting really deep into it . . . you feel like you're learning quite a lot more. (Philip, PP, Year 10, JC)

There is evidence that many students regard mathematics to be a collection of procedures that allow them to answer questions in a short space of time (Schoenfeld, 1988). The Phoenix Park students did not seem to have this shallow view of mathematics; they were aware of the depth of the subject—the different layers that may be encountered. The students also demonstrated an unusual awareness of the diversity and breadth of mathematics. They did not regard mathematics as a vast collection of "sums"; they seemed to have a richer and more balanced view of the subject:

A: I used to think that maths was just sums and hard work.

JB: Don't you now?

A: No, not really, some of it is, but there's a lot more stuff involved in it as well.

JB: What other stuff?

A: Well, different sorts of—well there's loads of different things, theories and stuff like that, formulas, algebra, shapes and stuff. (Alex, PP, Year 10, JC)

JB: Has doing the projects changed the way you think in any way?

D: Yes 'cause like bookwork—say it's just all sums or whatever, but that's only like one really small part of maths isn't it?

JB: Mmm.

A: If you're doing all problems and that you can learn about all the different areas. All the really advanced maths is a lot more to do with theorems and theories and that sort of thing than just sums. (Danny & Alex, PP, Year 10, JC)

Neither Danny nor Alex particularly liked mathematics compared with their other school subjects, but this did not appear to affect the way in which they constructed their views about the *nature* of mathematics. Both students showed that they regarded mathematics as a diverse subject in which "sums" were "only one really small part." In their Year 8 questionnaire, students were asked to describe one or more situations when they had used mathematics outside school. Seventy-seven percent of the Amber Hill students' comments related to money or shopping, and no descriptions were given of situations requiring the use of data handling, shape, or space. At Phoenix Park, 53% of comments also related to money and shopping, but 14% of students described less typical activities such as: sorting out a magazine collection, classifying option choices at a club, laying slabs in the front garden, organizing a bank account, reading a map, and organizing a route for a paper round. These were not examples that the students had been told about in class or contexts they had encountered in lessons.

In many of my lesson observations, the students approached and talked about mathematics in ways that were qualitatively different from most students I have observed in mathematics classrooms over the last 15 years. They showed that they were not only interested in the answers to the investigations and problems, but were aware of the importance of the methods and processes they used along the way:

P: Sometimes I can't really think how things can be used, but it's the process and the method, I suppose, and the way you look at it. (Philip, PP, Year 10, JC)

The students' awareness of the methods and processes they used in their work can probably be related to the encouragement their teachers

gave them to think about methods and strategies and the careful attention teachers paid to ways of working. Students were often asked to think about what they had been doing in lessons and to plan the direction of the rest of their work for homework. These homeworks stand in direct contrast to the more typical "finish up to question 20" mathematics homework. They explicitly required students to think about strategies and methods. There were many indications that the teachers were successful in this regard, and that the unusually dynamic views the students held about mathematics were formed in response to their project-based work. All of the students contrasted this work with the SMP bookwork they encountered in middle school:

H: It's more interesting now, you're not just working through a book doing the same things. (Helen, PP, Year 9, RT)

S: You go right through the pages of a book until you've finished it and then it takes you to other pages, all pretty much the same stuff, you can't really experiment with work in books. (Shaun, PP, Year 9, RT)

L: It gives you more freedom here and it lets you find things out for yourself, where a book would just give you all the answers and stuff and you wouldn't have to find things out for yourself, you have to find things out for yourself and its more interesting and I think you tend to remember it more when you've found things out for yourself. (Louise, PP, Year 10, JC)

The Phoenix Park students had all experienced a book-work approach to mathematics prior to their project-based work, and the contrast they offered between the two approaches focused on the more dynamic nature of the mathematics they encountered in their project work. They talked about the way that books did not give them anything to *find out* or *explore;* they merely gave them *set work* that they had to *work through*. The students highlighted the *procedural* aspect of book work, which, they said, made mathematics less interesting and useful for them.

The significance of the students' project work to the active views of mathematics that they had developed was also demonstrated by the results of their Year 9 questionnaire. At Phoenix Park, the students worked in an entirely open way until Christmas of Year 10, when they started preparing for examinations. At this time, the mathematics approach became considerably more procedural as the students were required to work through short, procedural examination questions. When my case study year group was in Year 9, I gave a questionnaire to students in Years 8, 9, and 10. This asked students to prioritize either thought or memory. Sixty-six percent of students completing Year 8 and 65% of students completing Year 9 thought it was more important to think hard about questions than remember similar questions. This proportion fell to 48% of students completing Year 10. At another point in the questionnaire, the students were asked to rank different areas of mathematics in terms of importance. Five percent of Year 8 students and 8% of Year 9 students thought that "remembering rules and methods" was the most important part of mathematics; in Year 10, this increased to 17% of students. Responses to the same questions given to three year groups at Amber Hill remained constant between the three year groups (17%, 15%, 15%).

The Phoenix Park students' responses to their Year 10 examination preparation indicate that the change from project work to a more formal mathematics approach prompted a corresponding change in the students' views about mathematics. Cobb, Wood, Yackel, and Perlwitz (1992) also found this to be true of students who worked on projects and then reverted to a textbook approach. This caused many more of the students to think that success in mathematics involved following a teacher's set methods. At Phoenix Park, the project-based approach had expanded the students' views of mathematics and caused them to regard mathematics as an active, exploratory discipline; in contrast, the examination work caused many students to go back to some of their old views about the limited nature of mathematics, thus eradicating some of the school's positive achievements.

Independence and Creativity

The students at Phoenix Park were encouraged in many different ways to be independent in mathematics, mainly through the degree of choice they were given and the responsibility they needed to take for their work. In their Year 9 questionnaire, students were asked to describe mathematics lessons, and 11% of students *chose* to comment on the independence they experienced in their lessons. For example, "what you do is mostly up to the pupils." None of the Amber Hill students responded in this way. When teachers at Phoenix Park interacted with students, they treated them as if they were equals. If they asked students to do something and the students asked why, they would explain rather than say, "because I said so." The teachers did not seem to distance themselves from students, and the gap between teachers and students was not distinct. This seemed to have a direct effect on the students. When they interacted with adults, even strangers, they were confident and chatty; they never appeared to be nervous or intimidated as many of the Amber Hill students did.

When visitors walked into the classrooms at Phoenix Park, which was a common occurrence, the students were unconcerned whether they were school inspectors, visiting dignitaries, or parents. They would always chat to adults, run around, misbehave, or swear at each other in the same relaxed manner regardless of who was with them. When the principal walked into lessons, the students would not change their behavior in any way, and those who were not doing work would continue not to do work. In many of my conversations with students and observations of them around the school, I was often reminded of the students at Summerhill school, the famously progressive English school described in Neill's (the principal) book: *Summerhill* (1985). Neill attributed the confidence and ease with which his students treated adults to the progressive approach of Summerhill school, which, he claimed, took away their fear and oppression (Neill, 1985).

The independence and responsibility encouraged in the students at Phoenix Park seemed to have a direct effect on their approach to mathematics. In a general sense, the students seemed less oppressed and constrained than many students of mathematics, and they seemed to take a more creative approach to mathematics than was typical for school students. In a questionnaire given to the students in Year 10, 82% of Phoenix Park students agreed with the statement "It is important in maths to use your imagination," compared with 65% of Amber Hill students—a statistically significant difference. The students' creative approach to mathematics was also demonstrated by an applied activity I gave them in Year 9 called *Planning a Flat* (apartment). In this activity, the students were asked to design a flat in a given space and locate and draw the rooms and furniture. A major, but unexpected difference between the students at Amber Hill and Phoenix Park related to the designs students produced. The students were invited to design a flat to suit a person or people of their choice (e.g., a student, a couple, a family, or themselves). The choice of rooms they would have in the flat was left entirely up to the students. All the students in both schools included in their designs at least one bedroom, bathroom, living room, and kitchen. However, approximately one third of the Phoenix Park students also included more unusual rooms. In the 89 designs produced by the students at Phoenix Park, there were 35 examples of *unusual* rooms including 7 games rooms, 4 soccer rooms (generally including small 5-a-side pitches), 3 indoor swimming pools, 3 studies, 2 hi-fi rooms, 2 children's playrooms, 2 cocktail bars, and 1 each of: a bouncy castle room, a pool room, a Jacuzzi, a computer room, a gym, a garage, a bowling alley, a utility room, a piano room, and a disco room. At Amber Hill, there were 99 flat designs that included 2 pool rooms, 2 swimming pools, 1 playroom, and 1 store room. It appeared that the students at Phoenix Park included the rooms they wanted to have in their flats, whereas the Amber Hill students included the rooms they thought they *should* have – the rooms of which they felt a teacher or I would approve.

The lack of constraint the Phoenix Park students experienced in these different domains, and the lack of domination or control that was imposed by teachers, seemed to have contributed toward the confidence of the students at Phoenix Park, the creativity they demonstrated, and the relaxed way in which they appeared to make and take decisions:

> A: That's the way I am . . . I just kind of do things in my own way, if it pulls off, it pulls off, if it doesn't then that's down to me. (Andy, PP, Year 10, RT)

SUMMARY

The mathematics approach at Phoenix Park was unusual particularly because of its openness, the degree of choice the students were given, the independence students were encouraged to develop, and the freedom the students had over their work environment and work rate. These features of the mathematics approach should be located within the overall context of Phoenix Park School, which was an unusually progressive institution that aimed to develop students' independence and decision-making abilities. The Phoenix Park approach was mathematically different from the majority of schools because learning mathematics was not based around the learning of different mathematical procedures. Rather, the students were engaged in activities and projects in which the need for certain mathematical methods became apparent. This approach necessitated a relaxation of the control teachers had over the structure and order of the classroom. The Phoenix Park teachers were not concerned about this, in line with their general approach to mathematics teaching and learning. Their concern was to give students mathematically rich experiences and help them use mathematics, rather than maintain order and a high work rate. They were concerned with the quality rather than the quantity of the students' mathematical experiences and with understanding rather than coverage. This meant that the Phoenix Park classrooms looked different from those of Amber Hill, and the students' experiences were also markedly different.

An important feature of Phoenix Park's approach was the care and attention the teachers paid to teaching students how to learn in an open system. Teachers had high expectations for their students, and they regarded the gap between where students were and where they needed to be (Black & Wiliam, 1998) as their teaching challenge. This approach required a lot from the Phoenix Park teachers. They needed to know where each of the

projects may lead and which would be particularly interesting or important mathematical directions for students. They needed to know a lot about the students – what they knew and what would be most helpful for them to work on. They also needed to know the mathematics of the projects in some depth. Shulman and others have termed the combination of the knowledge of mathematics, students, and teaching ideas that the Phoenix Park teachers enacted *pedagogical content knowledge* (Ball & Bass, 2000; Shulman, 1986). The teachers used this knowledge in the pursuit of some important teaching practices. For example, they asked insightful questions that took students to the heart of the mathematical issues, they listened carefully to students' responses considering what they told them about the students' thinking, and they based their questions and guidance on what they learned in these interactions. As Ball and Bass have pointed out, such interactions require "pedagogically useful" knowledge (Ball & Bass, 2000, p. 89). It also seems important to note that the Phoenix Park teachers were extremely skillful in the ways in which they navigated students around the mathematical terrain. When students were stuck, teachers asked them to explain what they knew so far, they listened to students carefully, and they selected appropriate questions and interventions that helped students move forward. As a result of such careful teaching, many of the Phoenix Park students regarded mathematics to be a dynamic, flexible subject that involved exploration and thought. They valued the importance of mathematical processes, and the views they developed were, according to a wide range of literature (Doyle, 1983; Erlwanger, 1975; Schoenfeld, 1988), extremely unusual. Additionally, the students displayed a freedom, creativity, and lack of constraint in their interactions and behaviors, which appeared to derive directly from the approach of the school.

At Phoenix Park, the students did not believe lessons to be uniform and monotonous. Instead they regarded their lessons as varied, and their enjoyment of lessons depended on the particular activities they encountered. The students also displayed varied levels of engagement, which differed between students as well as between lessons and parts of lessons. A small but important proportion of the year group at Phoenix Park misbehaved in lessons and said they did not like the school's approach. However, it was difficult to know whether the students' lack of motivation caused their negative views about mathematics, whether it was the other way around, or whether neither one caused the other.

In the next chapter, I present the results of different assessments and consider the ways in which the difference between the two schools' approaches affected the students' understanding of mathematics.