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Choosing an Empirical Production Function: Theory, Nonnested Hypotheses, Costs of Specifications

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CHOOSING AN EMPIRICAL PRODUCTION FUNCTION:

THEORY, NONNESTED HYPOTHESES, COSTS OF MISSPECIFICATION

by

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Abstract: This paper considers the specification of a production function, using crop-yield response to agricultural lime as an example. The paper first reviews neoclassical production theory, observing that this theory specifies neither the form nor the arguments of a production function. Several functions that are consistent with the theory are then evaluated by goodness-of-fit tests, by nonnested-hypothesis tests, and by "costs of misspecification." Neither goodness-of-fit tests nor nonnested-hypothesis tests provide a clear choice among the candidate functions; costs of misspecification provide some choice.

Keywords: functional forms, neoclassical production theory, production functions, response functions, specification.

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CHOOSING AN EMPIRICAL PRODUCTION FUNCTION:

THEORY, NONNESTED HYPOTHESES, COSTS OF MISSPECIFICATION

A production function is a purely technical relationship, void of economic content (e.g., Chambers, p.7). According to Dorfman, Samuelson, and Solow (p. 131):

> The production function is a description of the technological conditions of production, and the economist takes no direct responsibility for ascertaining it. Instead he regards it as falling within the purview of the technologist or engineer. But there seems to have been a misunderstanding somewhere because the technologists do not take responsibility for production functions either. They regard the production function as an economist's concept, and, as a matter of history, nearly all the production functions that have actually been derived are the work of economists rather than of engineers.

Any attempt to fit a production function immediately confronts the specification problem--choosing arguments and algebraic form of the function. Economic theory provides mainly generic conditions of specification and provides little guidance for specifying a function to describe a particular production process. Satisfactory specification must consider the technological conditions governing that production process.

This paper considers a simple production process: crop-yield response to agricultural limestone by alfalfa, corn, and soybeans. It first considers several functional forms that satisfy the generic requirements of neoclassical production theory. It then considers the requirements of agronomic technology, which eliminate one function permitted by neoclassical theory but leave several remaining candidates. These remaining candidates are evaluated by goodness-of-fit measures, by nonnested-hypothesis tests, and by costs of misspecification. The principal purpose of the paper is to examine the process of choosing a production function, especially the assumptions used. As Hildreth emphasizes (p. 62), realistic assumptions may increase the accuracy of the inferences, unrealistic assumptions generally beget biased inferences.

Neoclassical Production Theory

The neoclassical production function for a single output and two variable inputs can be written

$$y = f(x_1,x_2) \tag{1}$$

where y is the quantity of output and x_i is the quantity of the ith variable input (e.g., Silberberg, p. 69). Equation (1) can include any finite number of variable inputs; $n = 2$ merely allows geometric representation. The properties of (1) are specified by assumptions (e.g., Chambers, pp. 7-9):

(a) $x_i \geq 0$ and finite (nonnegative, real inputs);

(b) $f(x_1,x_2)$ is finite, nonnegative, real valued, and single valued for all possible combinations of x_1 and x_2;

(c) $f(x_1,x_2)$ is everywhere continuous and everywhere twice continuously differentiable;

(d) $f(x_1,x_2)$ is subject to the "law" of diminishing returns.

Chambers reports other assumptions that are sometimes made. He emphasizes that all these assumptions are indeed assumptions, not universally maintained hypotheses.

Assumption (a) implies that any finite, nonnegative combination of x_1 and x_2, i.e., any point in the nonnegative quadrant of the Cartesian plane of x_1 and x_2 (nonnegative orthant if $n > 2$) is possible. Assumption (b) implies that each combination of x_1 and x_2 has only one associated y-value, assumed to be the maximum possible output for the given combination of inputs. Although assumptions (a) - (c) do obvious violence to "lumpy" inputs, they provide reasonable approximations for many inputs.

The law of diminishing returns has a variety of interpretations. Cassels asserts that "law of variable proportions" is a more accurate term and argues that, if successive units of one variable factor are added to a fixed quantity of another factor (or fixed quantities of a combination of factors), total output varies through three distinct phases (called stages in more recent literature). The first phase of the total-output curve begins at some point on the *X*-axis to the right of the origin where the ratio of the excessive factor (fixed input) to the deficient one (variable input) approaches infinity. (This requirement is closely akin to Chambers' "strict essentiality" assumption, p. 9.) Cassels also argues that the law is symmetrical: total output again approaches zero in phase three when the variable-input rate gets large enough.

Some recent writers reject the existence of a third phase: "...no marginal physical productivities can be negative, else output would not be maximal since it could be improved by the same set of factors by leaving some idle" (Samuelson, p. 58); "all marginal productivities are positive" (Chambers, p. 9). Chambers acknowledges that, when an entrepreneur operates in uncertainty, marginal productivities may sometimes be negative. These views about the possibility of production in phase III are consistent with Cassels' argument that only the second phase is economically relevant. (No one contends that phase III production is economically relevant.) If the assumption of nonnegative marginal productivities is imposed by allowing only functions that never decrease, and if such functions are fitted to data in which output decreases as input increases, parameter estimates will be biased.

No matter how phase III output is viewed, neoclassical theory permits a variety of production functions. This paper considers a one-input specialization of equation (1), $y = f(x_1, x_2^0)$, in which y is crop yield, x_1 is the limestone rate, and x_2^0 is a bundle of fixed inputs, either applied or intrinsic to the unamended soil. By itself, a limestone application would produce nothing, of course. There must be seed, other plant nutrients (e.g., nitrogen, phosphate, potash), cultural practices (cultivation or herbicides), and water (rainfall or irrigation), for example. If the principal interest in an experiment is the effect of limestone on yield, other applied inputs would be added at fixed rates to every experimental plot. Variability due to known intrinsic inputs would be controlled by arranging experimental plots in homogenous groups (e.g., blocks). Bias due to unknown, or unsuspected, variability in intrinsic inputs would be minimized by randomizing treatment assignments within each homogenous group. If there is no treatment-by-block interaction, yield response to lime will have the same general configuration in each block although, for a given input rate, yields may differ from one block to another.

A production function with a single variable input is a special case of a function with two or more variable inputs. It permits no examination of the possibilities for substitution among inputs, which interests economists. But many agronomic experiments are designed to investigate how output varies with variations in a single input. If production-function analysis is useful, it should be applicable to such special cases as well as to more general cases. Special cases may provide evidence on the discriminating power of the criteria for choosing an empirical production function.

Alternative Functional Forms

"In principle the variety of equations that may validly represent a production function is virtually limitless" (Ferguson, p. 61). This paper compares four:

Power: $$y = \beta_o + \beta_1 x^{\gamma},\ 0 < \gamma < 1 \tag{2}$$

Quadratic Spline: $$y = \beta_o + \beta_1 x + \beta_2 x^2 + \beta_3 z^2 \tag{3}$$

Square Root: $$y = \beta_o + \beta_1 x^{0.5} + \beta_2 x \tag{4}$$

Translog: $$y = \beta_o + \beta_1 x^{\beta_2 + \beta_3 \ln x} \tag{5}$$

where y is crop yield in common units per acre, x is the lime (agricultural limestone) rate in tons per acre, and ln x is the natural logarithm of x.

In (3), $z = x - x_o$ if $x > x_o$, $z = 0$ otherwise, and x_o is some fixed value of x, here taken as a design point of the experiment (table 4). This function joins two quadratics at $x = x_o$: $\hat{Y} = b_o + b_1x + b_2x^2$ if $x \le x_o$, , $\hat{Y} = a_o + a_1x + a_2 x^2$ if $x > x_o$, where $\hat{Y}$ is the predicted value of y. If the combined function and its first derivative are continuous at $x = x_o$, the estimates b_o, b_1, b_2, and b_3 of the four parameters in (3) provide estimates for all six parameters in the two separate functions (Fuller, p. 36):

$$a_o = b_o + b_3 x_o^2 \tag{6}$$

$$a_1 = b_1 - 2b_3 x_o \tag{7}$$

$$a_2 = b_2 + b_3 \tag{8}$$

In effect, z partitions the domain of (3) into two parts, and the estimated function gives correct predictions for both parts. If $b_3 = 0$, z has no effect and the first quadratic provides correct predictions over both parts of the domain ($a_k = b_k$, $k = 0, 1, 2$). Moreover, the t-statistic for b_3 measures the gain from including the fourth term in the estimated function, i.e., of representing response by two quadratics rather than only one. Thus, (3) can represent any relationship that a standard quadratic can, and it allows other possibilities as well. In that sense, (3) is more flexible than a standard quadratic; the added flexibility comes at the cost of estimating one additional parameter. If $a_1 = a_2 = 0$, the spline has a plateau at $\hat{Y} = a_0$ for $x > x_0$. For the alfalfa data, $a_1 = 0.0761$, $a_2 = -0.0038$, and the function approximates a plateau for $x > 3$ (fig. 1). (Fuller, pp. 36-40, discusses the use of splines to represent multiple inputs and higher-dimension surfaces. Gallant and Fuller discuss applications in which the join points must be estimated from the data.)

Because the empirical functions are fitted to observed yields rather than to increases above no-input yields, each function (2) - (5) includes an intercept (β_o). Every estimated intercept is positive, most significantly so (table 4), indicating that the unamended soil contains sufficient nutrients to produce some yield even when no variable input is applied. In some empirical applications (e.g., Heady and Dillon, p. 478), "yield" is defined as the increase above the no-input yield. Functions fitted to such increases have zero intercepts. With positive intercepts, functions (2) and (5) are not linear in the parameters, and they cannot be linearized (e.g., by a logarithmic transformation); they must be estimated by nonlinear procedures.

The power (Cobb-Douglas) function (2), without intercept, and the square-root function (4) have been widely used to represent crop response to applied nutrients (e.g., Heady and Dillon, p. 478). The quadratic-spline function (3) is a species of the widely used quadratic. The translog function (5), without intercept, is of more recent vintage and is a member of the class of so-called flexible functional forms. The list of candidates can be extended, of course. Griffin, Montgomery, and Rister, for example, describe twenty functional forms, including (2), (4), and (5) (their versions of (2) and (5) have zero intercepts). Relaxing one or more of the assumptions for (1) would extend the list further. If the requirement for continuous derivatives is relaxed, for example, linear splines (linear-plateau models) become admissible candidates (Anderson and Nelson; Perrin).

Among functions (2) - (5), only the translog can display all three stages of production; in the usual case ($\beta_2 > 0$, $\beta_3 < 0$), it has a short stage I as well as stages II and III. The power function (2) displays a single stage: stage II if $\beta_1 > 0$ and $0 < \gamma < 1$ (usual case), stage I if $\beta_1 > 0$ and $\gamma > 1$. Functions (3) and (4) display stages II and III in the usual cases ($\beta_1 > 0$, $\beta_2 < 0$, $\beta_3 > 0$, and $\beta_3 < |\beta_2|$ for (3); $\beta_1 > 0$, $\beta_2 < 0$ for (4)). If they display stage I at all, functions (3) and (4) display it throughout their domains.

According to Cassels's version of neoclassical production theory, which posits three stages of production, only the translog function (5) would be acceptable. Empirical data does not always display a stage I, however. "Under the usual conditions of crop and livestock production, there seems to be no strong empirical evidence for the existence of any but diminishing returns" (Dillon and Anderson, p. 7). The three sets of data used in this paper display only diminishing returns; if there is a stage I, either it occurs below the zero-input rate or it is masked by the wide spacing of the design points of the experiments. Even if the data do not display it, the translog imposes a stage I. When there is no stage I in the data, the spline and square-root functions may be reasonable alternatives to the translog.

According to Fuss, McFadden, and Mundlak (FMM), ". . . a wide variety of compatible functional forms will usually be available," and they list five criteria for choosing a single form (pp. 224-225):

1. *Parsimony in parameters*: Excess parameters exacerbate multicollinearity problems and, in small samples, seriously reduce error degrees of freedom.

2. *Ease of interpretation*: Prefer a form in which parameters have an intrinsic and intuitive economic interpretation and in which functional structure is clear.

3. *Computational ease*: Although nonlinear forms are feasible, linear-in-parameters systems have less expensive computations and more fully developed statistical theory.

4. *Interpolative robustness*: The chosen functional form should be consistent with maintained hypotheses in the range of the data.

5. *Extrapolative robustness*: The chosen functional form should be consistent with maintained hypotheses outside the range of the data.

Functions (3) and (5) violate criterion 1 (parsimony in parameters); both have more parameters than either of the other functions. Functions (2) and (5) violate criterion 3 (computational ease); neither can be transformed into a linear-in-parameters function. Widely available computer software makes nonlinear estimation less onerous than when FMM wrote, but much of this software uses iterative procedures that sometimes produce unstable estimates unless reasonable initial values for the parameters are provided. Function (2) violates criterion 5 (extrapolative robustness). If (2) displays stage II at all, it displays stage II throughout its domain and for any extrapolation beyond the domain. Although these are useful criteria, no single functional form will dominate in the sense that it satisfies every criterion at least as well as any other functional form. Compromise among the criteria is required.

Other Sources of Specification

Since economic theory specifies no response function, other sources of specification must be considered. For crop-yield response to lime, agronomic or horticultural sources are possible. (For animal-production problems, animal-nutrition information is possible.) According to agronomic technology, crops respond to lime because one or more of these conditions has been corrected: excessive hydrogen-ion activity, aluminum toxicity, manganese toxicity, calcium or magnesium deficiency, and molybdenum deficiency (Adams, 1978). Raising soil pH, by itself, mitigates the effects of all these conditions except calcium and magnesium deficiency. The usual means for raising soil pH is to apply lime, a mixture of the carbonates of calcium and magnesium. Such an application reduces calcium and magnesium deficiencies and mitigates the other effects of low pH. Soil pH is the principal criterion by which agronomists recommend lime applications

(Rowell, p. 883).

Like neoclassical production theory, agronomic technology provides no mathematical function to describe the relationship between crop yield and lime, but it indicates some characteristics of the relationship: (a) yield increases at a decreasing rate as pH increases (McLean and Brown, p. 287); (b) for most crop species, yield reaches a plateau somewhere below pH 7.0 and may even decrease as pH approaches 7.0, due perhaps to induced deficiencies of iron, manganese, or phosphate (Adams, 1969, p. 16; Rowell, pp. 892-96). Soil pH, itself, is affected by several factors, including soil texture, amounts and types of applied nitrogen, and season of the year. Within a growing season, for example, soil pH is usually highest at the end of winter, before fertilizer has been applied, and trends downward over the season (Adams, 1978). As soil pH rises above 6.0, lime becomes increasingly inefficient at raising pH (Adams, 1984, p. 222). Above pH 6.5, additional lime may increase pH little, but it never decreases pH. That is, large lime rates do not decrease soil pH but they may decrease crop yield.

The agronomic principle that crop yield reaches a plateau (zero marginal product) or may even decline (negative marginal product) for finite, perhaps relatively low, lime rates is more stringent than recent statements of the law of diminishing returns. Samuelson (p. 58) and Chambers (p. 9) insist that, although marginal product must eventually decline, it is everywhere nonnegative. By contrast, Cassel's interpretation requires that marginal physical product first goes to zero and then becomes negative as the variable-input rate increases sufficiently. Function (2) violates this more stringent requirement. Functions (4) and (5) satisfy the requirement if $\beta_2 < 0$ in (4) or $\beta_3 < 0$ in (5); both functions must decrease when x is large enough although x may have to be large indeed if $|\beta_2|/\beta_1$ ($|\beta_3|/\beta_2$) is small. Function (3) may decrease when x gets large, but the form of the function does not dictate that. If $|\beta_2| > \beta_3$, the function declines for large *x*, but if $|\beta_2| < \beta_3$ the function increases monotonically for sufficiently large *x*.

Hence, agronomic technology might be used to eliminate function (2). It does not eliminate function (3), (4), or (5).

The Data

Response functions were fitted to data from designed experiments for alfalfa, corn, and soybeans. Treatment means for the three experiments are reported in tables 1-3. Response functions were fitted to individual-plot data, however, not to the treatment means.

Alfalfa

Lime was applied in three forms: dolomite disked into plowed surface; dolomite, one-half plowed down, one-half disked into plowed surface; calcite disked into plowed surface. (Lime is a mixture of calcium carbonate and magnesium carbonate; the proportion of magnesium carbonate is higher in dolomite than in calcite.) Each form of lime was applied at five rates, ranging from none to twelve tons per acre. Thus, fifteen treatments (five lime rates in each of three forms) were applied, and each treatment was replicated four times in randomized complete blocks. Alfalfa yields are for three cuttings of hay. With form-of-lime represented by two dummy variables, an analysis-of-variance of the effects on alfalfa yield of form-of-lime and lime rate had a one-sided *P*-value of 0.63 for the null hypothesis that form-of-lime has no effect on yield. Subsequently, form-of-lime was ignored in fitting response functions, and the data were analyzed as though five lime rates were replicated twelve times (table 1).

Corn

Both conventional lime (ground limestone) and suspension lime (finely ground limestone suspended in water) were applied at five rates (not counting the control) shortly before corn was planted. Counting the control, eleven treatments were applied, each replicated four times in randomized complete blocks. Half the lime (both conventional and suspension) was disked in and then plowed down, and half was disked into the plowed surface.

Generally, the finer limestone is ground the faster it reacts with the soil. By the time the corn was harvested, roughly six months after lime was applied, there was no observable difference in yield between the two types of lime. An analysis-of-variance of the effects of type-of-lime and lime rate on corn yield had a one-sided *P*-value of 0.69 for the null hypothesis that type-of-lime has no effect. Subsequently, response functions were fitted to the pooled data, as though each nonzero lime rate was replicated eight times; the control was replicated only four times (table 2).

Soybeans

Seventeen lime-fertilizer treatments were replicated four times in randomized complete blocks. The treatments include six lime rates ranging from none to twelve tons per acre, combined with nitrogen rates of 20, 100, or 200 pounds per acre. Lime was applied only once, but nitrogen was applied annually four years in succession, during which time the plots were planted to cotton. In the fifth year, all plots were planted to soybeans with no additional fertilizer or lime. Soil pH and soil magnesium were measured on each plot before soybeans were planted (table 3).

Nitrogen fertilizer tends to reduce soil pH, the heavier the nitrogen rate the faster the reduction. Nonetheless, the data clearly show that lime affected soil pH even five years after it was applied. On plots fertilized annually with 100 pounds of nitrogen, for example, mean soil pH was 4.92 on unlimed plots, 7.60 on plots limed with twelve tons per acre. On plots fertilized annually with 200 pounds of nitrogen, mean soil pH was 4.40 on unlimed plots, 7.45 on plots limed with twelve tons per acre. Soil magnesium also increases as the lime rate increases, and it increases faster for dolomite than for calcite. The simple correlation between soil magnesium and lime rate was high ($r = 0.92$). Notwithstanding that result, an analysis-of-variance of the effects of dolomite on soybean yield had a one-sided *P*-value of 0.47 for the null hypothesis that there is no difference in yield between calcite and dolomite. Subsequently, both form-of-lime (dolomite vs. calcite) and soil magnesium were ignored in fitting response functions.

Goodness of Fit

Only the translog function (5) satisfies Cassel's version of neoclassical theory, which requires all three stages of production, and it also satisfies agronomic technology. Only the power function (2) satisfies versions of neoclassical theory that require marginal productivities to be nonnegative everywhere, but it violates the principles of agronomic technology. Nonnegative marginal productivities constitute an assumption. If that assumption is dropped, then functions (3) - (5) are acceptable to both neoclassical theory and agronomic technology. That is, neither agronomic technology nor neoclassical theory provides *a priori* grounds for excluding any of these three functions. If one of the three is the "true" function, it should have the smallest residual variance and the highest adjusted *R*-square (Johnston, p. 501). That is, statistical goodness-of-fit tests should choose among these three functions (provided, of course, that the relationships are not confounded with the effects of variables not controlled by the experimental designs).

Response functions were fitted to individual-plot data rather than to treatment means, and all four functions were fitted for comparison (table 4). With few exceptions (the coefficient of *LLIME* in the translog functions for alfalfa and corn, the intercept in the translog function for corn, and the exponent of *LIME* in the power function for soybeans), the *t*-statistic for each coefficient is approximately 2 or larger.

For alfalfa, the square-root function (4) has the highest adjusted R^2 and the smallest residual mean square among functions (3) - (5). For both corn and soybeans, the translog function (5) has the highest adjusted R^2 and the smallest residual mean square. Even the power function, which is inconsistent with agronomic technology, fits all three sets of data reasonably well (it fits the corn data better than either the square-root or the quadratic-spline). Figures 1, 2, and 3 graph the four functions and the treatment means.

In figure 1, mean alfalfa yield increases monotonically with lime (no stage III). The power function also increases monotonically with lime, as is its nature, but the spline, square-root, and translog functions increase to maxima and then decline. Figure 2 shows an anomaly in the treatment means for corn. The means increase rapidly to *LIME* = 2.0, decrease to *LIME* = 4.0, and increase again to *LIME* = 6.0. The decrease in mean yield at *LIME* = 4.0 is attributable entirely to suspension lime (table 2). In the pooled data, if $\bar{y}_k$ is mean yield for *LIME* = *k*, the linear contrast $(\bar{y}_2 + \bar{y}_6)/2 - \bar{y}_4$ has a *t*-value of 1.75 and a two-tailed *P*-value of 0.10, indicating a large difference. But there is no reliable explanation for this decrease, i.e., no *a priori* reason to expect it. The linear contrast $\bar{y}_2 - \bar{y}_6$ has a *t*-value of -0.82 and a two-tailed *P*-value of 0.40. Hence, $\bar{y}_2 = 138.6$ and $\bar{y}_6 = 144.2$ can be regarded as separate estimates of the same population mean. So regarded, there is a plateau from *LIME* = 2.0 to *LIME* = 6.0 if the low mean yield at *LIME* = 4.0 is ignored. The difference in soybean yield between the control plots and the maximum-yield lime rates is relatively small (table 3), and lime accounts for less than half of the variability in soybean yields. Figure 3 truncates the lower end of the yield scale to emphasize differences among the four functions.

All three figures illustrate differences among the functions, but these differences are most pronounced in figure 2 for corn. The power function increases monotonically throughout its domain, of course, and it would display this characteristic no matter how large the domain. The square-root function increases monotonically to a maximum at *LIME* = 3.7 and then declines. The quadratic spline reaches a local maximum at *LIME* = 1.8, declines to a local minimum at *LIME* = 4.0, and then increases to another local maximum at *LIME* = 6.0; the second local maximum is smaller than the first, so the global maximum in the domain *LIME* = 0 to *LIME* = 6.0 occurs at *LIME* = 1.8. The translog increases to a maximum at *LIME* = 5.0 and then declines.

According to the goodness-of-fit results, no single function fits all three sets of data best. For no set of data does the best-fitting function fit notably better than the next-best alternative. Moreover, it is unlikely that any of the four functions is the "true" function for any of the three sets of data. Choosing one function as "best," even for one set of data, requires additional criteria. The next two sections consider nonnested hypotheses and value-of-information as possible criteria.

Nonnested Hypothesis Tests

"Two models are said to be 'nested' when one is a special case of the other, obtained by parameter restrictions. Thus the Cobb-Douglas production function is nested within the constant elasticity of substitution (CES) production function. On the other hand, if one model cannot be expressed as a special case of the other, by parameter restrictions, the models are 'nonnested'" (Doran, p. 95). According to this definition, the power function (2) is nested within the translog function (5); the two functions are identical when $\beta_3 = 0$. Every other pair of functions (2) through (5) is nonnested.

Suppose Y is an n-vector of observed responses. Suppose further that H_0: $Y = f(X,\beta) + \epsilon_0$ and H_1: $Y = g(Z,\gamma) + \epsilon_1$ are two competing hypotheses to explain the variability in Y. X is an nxk matrix of exogenous observations and β is a k-vector of unknown parameters. Z is an nxl matrix of exogenous observations and γ is an l-vector of unknown parameters. Usually, ϵ_0 and ϵ_1 are assumed to be normal with zero means and variances $\sigma_0^2 I$ and $\sigma_1^2 I$. Several tests have been devised to choose between H_o and H_1 when they are nonnested.

Pesaran reports an N-test to test H_o against H_1 when both are linear in the parameters. Pesaran and Deaton modify the N-test for cases in which H_o is not linear in the parameters. Godfrey and Pesaran report an adjusted N-test for use when the sample is small or when the two competing hypotheses have unequal numbers of regressors and both hypotheses are linear in the parameters. Davidson and MacKinnon discuss three tests: a J-test, a C-test, and a P-test. They recommend the J-test when H_o is linear in the parameters, the P-test when H_o is not linear in the parameters, and the C-test as a simple preliminary test when H_o is not linear in the parameters and the derivatives of f with respect to the parameters are not easy to evaluate (Davidson and MacKinnon, p. 783). If H_o is linear in the parameters, the J-test and the P-test are identical; if H_o is not linear in the parameters, the J-test and the P-test yield different results in small samples but identical asymptotic estimates when H_o is true (Davidson and MacKinnon, p. 782). Doran (p. 100) discusses a JA-test and a J-test (same as Davidson and MacKinnon's J-test). He argues that the JA-test is exact when both hypotheses are linear and that the J-test is not exact but may be more powerful than the JA-test. All of these tests are conditional on the truth of H_o, not the truth of H_1. To test H_1, the simplest procedure is to reverse the roles of H_o and H_1 and do the test again (e.g., Doran, p. 99).

Table 5 reports *JA*-test statistics, *N*-test statistics, and *t*-values associated with the *P*-statistics for all three sets of data: alfalfa, corn, and soybeans. According to Davidson and MacKinnon (p. 792), the *N*-test statistic and the *P*-test statistic usually have opposite signs, and that holds in 26 of the 36 cases (72%) in table 5. For each set of data, test statistics are reported for all possible pairs of the power, the spline, the square-root, and the translog functions. (The power function is included for comparison even though it violates agronomic technology and is nested within the translog.)

The *JA*-test statistic is a *t*-value. Both the *N*-test and the *P*-test statistics are asymptotic standard normal variates. Asymptotic tendencies sometimes emerge for relatively small samples, but they sometimes emerge only as sample size gets large. All three samples here are relatively small, and standard errors of the *P*-statistics, included in the regression results, sometimes differ substantially from one. (Variances for the *N*-statistics are not byproducts of calculating the *N*-statistics, and they would be difficult to calculate.) This state of affairs suggests that *t*-values for the *P*-statistics will be easier to interpret than the *P*-statistics themselves and may be more reliable than either the *P*-statistics or the *N*-statistics. (Frank, Beattie, and Embleton, p. 600, contend that the *N*-statistic is more reliable than the *P*-statistic.) The *t*-values for alfalfa have either 53 or 54 degrees of freedom, those for corn have either 36 or 37 degrees of freedom, and those for soybeans have either 60 or 61 degrees of freedom. In all three cases, approximate critical values are 2.0 (5% significance) or 2.7 (1% significance).

"An hypothesis, which one would not wish to consider seriously in its own right, can be a perfectly effective tool for disproving an alternative . . " (Pesaran and Deaton, p. 678). Hence, although the power function violates agronomic technology, it may provide a useful alternative hypothesis for evaluating other hypotheses. In the alfalfa data, no null hypothesis is rejected in favor of any alternative hypothesis for either 1% or 5% significance; thus, all three candidates (spline, square-root, and translog) are acceptable. In the corn data, by contrast, every null hypothesis is rejected in favor of at least one alternative by at least one test statistic, suggesting that none of the three candidates is acceptable. In the soybean data, the power function and the translog function are rejected in favor of at least one alternative by at least one test statistic. The spline and square-root functions are rejected by no test statistic in favor of an alternative function. So, neither the power function nor the translog function is acceptable, but both the spline and square-root functions are acceptable.

Cost of Misspecification

Neither goodness-of-fit nor nonnested-hypotheses provide an unambiguous choice among the spline, the square-root, and the translog functions for any of the three sets of data. "Costs of misspecification" provide a third set of evidence. Suppose that one of the three functions is the "correct" one, and determine the optimal input rate and the associated net revenue for that function. Then determine the optimal input rate for an alternate function and estimate the net revenue that input rate would yield according to the "correct" function. This alternate net-revenue estimate for the "correct" function will be no larger than the optimal net revenue (it will usually be smaller). The difference between the two net-revenue estimates can be regarded as a cost, the net revenue sacrificed by applying a nonoptimal input rate. This difference has been variously called "cost of the wrong decision" (Havlicek and Seagraves), "value of information" (Perrin), and "cost of misspecification" (Frank, Beattie, and Embleton).

Table 6 reports "costs of misspecification" for the three sets of data, alternately regarding the spline, the square-root, and the translog as the "correct" function. Costs of misspecification are also reported for the power function. Net returns are the returns above variable cost for a single year, assuming that the lime must be paid for by one year's crop. As the soybean data show, however, lime may affect soil pH (and thus crop yield) for more than one year. Hence, the reported costs of misspecification underestimate the true costs. If crop-yield data were available for more than one year, all optimal lime rates would be larger (unless the discount rate for future returns were excessive); costs of misspecification would probably also be larger.

Alfalfa

Among the three "correct" functions, the spline has the lowest optimal lime rate, the square- root the highest. If the optimal lime rate is determined by the translog, the largest cost of misspecification is $2.66 per acre (when the spline is the correct function). Similarly, if the optimal lime rate is determined by the spline, the largest cost of misspecification is $1.87 per acre (when the square-root is the correct function). If the optimal lime rate is determined by the power function, the cost of misspecification is larger, reaching $28.44 per acre when the spline is the correct function. As noted, alfalfa yield approaches a plateau between three and six tons of lime per acre. The power function seriously underestimates the marginal physical product of lime in the neighborhood of three tons per acre, seriously overestimates in the neighborhood of twelve tons per acre, and recommends too little lime at the assumed price of $16 per ton, spread. (For sufficiently low lime prices, it would recommend too much lime, since its marginal physical products are positive indefinitely.) Hence, if the correct function is either the spline, the square-root, or the translog, the maximum cost of misspecification is smallest when the spline determines the optimal lime rate.

Corn

Among the three "correct" functions, the spline has the lowest optimal lime rate, the square-root the highest. If the optimal lime rate is determined by the translog, the largest cost of misspecification is $6.76 per acre (when the spline is the correct function). If the optimal lime rate is determined by the spline, the largest cost of misspecification is $6.86 per acre (when the square-root is the correct function). If the optimal lime rate is determined by the power function, however, the largest cost of misspecification is $8.77 per acre (when the spline is the correct function). For the three "correct" functions, the maximum cost of misspecification is smallest when the optimal lime rate is determined by the translog, although costs of misspecification are almost as low for the spline.

Soybeans

Among the three "correct" functions, the square-root has the lowest optimal lime rate, the spline the highest. If the optimal lime rate is determined by the translog, the largest cost of misspecification is $1.40 per acre (when the square-root is the correct function). If the optimal lime rate is determined by the spline, the largest cost of misspecification is $2.62 per acre (again, when the square-root is the correct function). If the optimal lime rate is determined by the square-root, the maximum cost of misspecification is larger; for the power function, the maximum cost of misspecification is larger still, reaching $11.96 per acre when the spline is correct. According to the treatment means (figure 3), soybean yield reaches a maximum at about four tons of lime and then declines. The power function cannot adequately represent such a response pattern since it never declines, no matter how large the lime rate. It underestimates yield (and marginal physical product) in the neighborhood of two to four tons of lime per acre and overestimates yield (and marginal physical product) in the neighborhood of eight to twelve tons of lime per acre. Thus, determining optimal lime rates with the translog minimizes the maximum cost of misspecification, although costs of misspecification for the spline are only slightly larger.

Costs of misspecification are based on local analysis that is sensitive to variation in prices and has no recognized statistical foundation. Nonetheless, any function with a high cost of misspecification, even for a given set of prices, should be examined carefully. A function that recommends more than 6 tons of lime for the alfalfa data would be suspect, no matter how appealing its statistical properties.

Conclusions

To represent crop-yield response to agricultural limestone, this paper considers four functions: a power (Cobb-Douglas) function, a quadratic spline, a square-root, and a translog. Only the translog displays all three stages of production (Cassel's version of neoclassical production theory). Only the power function displays marginal productivities that are everywhere nonnegative. Agronomic technology indicates that, if the limestone rate is large enough, crop yield will decline absolutely (stage III production), contrary to predictions by the power function. Three other kinds of evidence for choosing among the three remaining candidates are then considered: statistical goodness-of-fit tests, nonnested hypotheses, and cost of misspecification.

If a production function is a purely technical relationship, as both Chambers and Dorfman, Samuelson, and Solow contend, then specification of the function is not an exclusively economic exercise. Specification for a particular production process must incorporate the technological conditions governing that process. For crop production, the power function is inconsistent with agronomic technology, thus reducing the acceptable set of functions from four to three. But the question remains: “Which of the three candidates best describes crop response to lime?”

Goodness-of-fit statistics or nonnested-hypothesis tests are intended to resolve such ambiguities. For the crop-production problem, however, goodness-of-fit statistics provide no clear choice among the three remaining candidates. Adjusted *R*-squares are similar for all three functions, and except for the coefficient of *LLIME* in the translog functions for alfalfa and corn and the translog intercept for corn, most individual coefficients differ significantly from zero. Nonnested hypotheses are no more discriminating: in the alfalfa data, no null hypothesis is rejected in favor of an alternative hypothesis; in the corn data, every null hypothesis is rejected in favor of at least one alternative by at least one test statistic; in the soybean data, curiously, the translog is rejected in favor of the power function by one test statistic. Costs of misspecification favor either the spline or the translog in all three sets of data; the square-root function is somewhat inferior, the power distinctly inferior. Since costs of misspecification are sensitive to prices, these results may be peculiar to the prices used in this study. Nonetheless, any function with a high cost of misspecification merits careful examination. For the alfalfa data, for example, many farmers would be skeptical of a recommendation of more than four tons of lime per acre, regardless of the alfalfa and lime prices.

For the crop-production problem examined in this paper, agronomic technology provides some, though not definitive, guidance for specifying a production function. Neither goodness-of-fit statistics nor nonnested hypothesis tests provide any additional guidance for specifying a function. Either technology or statistical tests may provide more definitive guidance for other production processes, but making that judgment requires examining additional data. For the data examined here, only the cost of misspecification provides any discrimination among the three candidate functions. That evidence favors either the spline or the translog in all three sets of data, with the spline slightly better in the alfalfa data, the translog slightly better in the soybean data. The power and square-root functions are distinctly inferior to the other two in at least one set of data.

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Table 1. Alfalfa Treatment Means

Lime (ton/acre)	pH[a] (Apr. 62)	Yield[a] (ton/ac, 3 cuts.)
0	5.28	0.89
1.5	5.90	3.01
3.0	6.28	3.60
6.0	6.79	3.82
12.0	7.13	3.84

[a] Mean of twelve observations--four replications of three forms of lime at each lime rate.

Experiment at Mayfield, Kentucky, on Grenada silt loam. Lime was applied in May 1958, alfalfa was seeded in 1961, yields were measured in 1962.

Table 2. Corn Treatment Means

	Yield (bu/ac)			
Lime (ton/ac)	Conv. Lime[a]		Susp. Lime[a]	Pooled
0	--	24.1[a]	--	24.1
0.5	107.2		99.6	103.4
1.0	119.8		127.0	123.4
2.0	133.9[b]		142.1	138.6[c]
4.0	135.6		126.8	131.2
6.0	148.0		140.5	144.2

[a] Mean of four replications at each lime rate; yield is corn at 15.5% moisture.

[b] Mean of three replications; corn drowned out on one replication.

[c] Weighted mean with number of plots as weights.

Experiment on Goldsboro sandy loam of the southern coastal plain of Virginia. Lime was applied in April 1978, before corn was planted.

Table 3. Soybean Treatment Means

Lime (ton/ac)	Annual N, 69-72 (lb/ac)	pH[a] (Mar. 73)	Mg[a] (lb/ac, Mar 73)	Yield[a] (bu/ac)
0	100	4.92	25.0	26.6
0	200	4.40[b]	6.7[b]	22.4[b]
1.0	20	5.90	87.0	28.7
1.0	100	5.48	57.5	30.5
1.0	200	4.98	31.8	29.5
2.0	100	6.12	118.5	34.0
2.0	200	5.45	68.5	31.4
2.0[c]	100	5.88	42.0	31.0
2.0[c]	200	5.38	22.5	31.1
4.0[c]	100	6.62	65.5	33.2
4.0[c]	200	6.18	45.8	31.4
4.0	100	6.50	169.0	32.9
4.0	200	6.22	131.0	32.4
8.0	100	7.20	309.5	30.7
8.0	200	7.38	312.0	31.2
12.0	100	7.60	342.0	31.2
12.0	200	7.45	335.2	31.9

[a] Mean of four replications for each lime-N combination. Experiment at Sand Mountain Substation, Crossville, Alabama, on Hartsells fine sandy loam.

[b] Mean of three replications; one replication eliminated as outlier.

[c] Calcite; all others dolomite.

Table 4. Estimated Lime-Yield Response Functions[a]

	Prediction Equation (Standard Errors in Parentheses)	Adj. R^2	Resid. Mean Sq.	Resid. df.
ALFALFA (ton/ac)				
ANOVA		0.712	0.507	52
Power[b]	$Y^* = 0.8830 + 2.1796\, LIME^{0.1404}$ (0.2007) (0.2739) (0.0498)	0.710	0.510	54
Quad. Spline[c]	$Y^* = 0.9022 + 1.7939\, LIME - 0.2901\, LIME^2 + 0.2863\, Z3^2$ (0.2792) (0.2325) (0.0522) (0.0615)	0.716	0.499	53
Sq. Root	$Y^* = 0.9000 + 2.1897\, LIME^{0.5} - 0.3896\, LIME$ (0.2759) (0.2440) (0.0669)	0.721	0.492	54
Translog[b,d]	$Y^* = 0.8721 + 1.7885\, LIME^{\,0.5044 - 0.1216\, LLIME}$ (0.2088) (0.3789) (0.2514)(0.0803)	0.717	0.498	53
CORN (bu/ac)				
ANOVA		0.866	173.73	34
Power[b]	$Y^* = 23.9881 + 94.8384\, LIME^{0.1330}$ (6.8631) (7.4257) (0.0255)	0.842	203.74	37
Quad. Spline[c]	$Y^* = 37.1863 + 124.6851\, LIME - 35.4213\, LIME^2 + 39.6923\, Z2^2$ (8.1727) (11.6172) (3.9024) (4.9657)	0.820	233.03	36
Sq. Root	$Y^* = 30.7000 + 119.3912\, LIME^{0.5} - 31.1058\, LIME$ (8.4696) (11.3578) (4.0606)	0.825	226.38	37
Translog[b,d]	$Y^* = 11.0677 + 110.6851\, LIME^{0.1908 - 0.0595\, LLIME}$ (20.1869) (19.6821) (0.0744) (0.0405)	0.854	189.23	36
SOYBEANS(bu/ac)				
ANOVA		0.442	9.289	58
Power[b]	$Y^* = 24.7157 + 6.1001\, LIME^{0.0787}$ (1.1485) (1.3177) (0.0715)	0.418	9.688	61
Quad. Spline[c]	$Y^* = 24.5355 + 7.2849\, LIME - 1.8071\, LIME^2 + 1.7956\, Z2^2$ (1.3481) (1.5298) (0.4460) (0.4726)	0.446	9.211	60
Sq. Root	$Y^* = 24.8157 + 6.5537\, LIME^{0.5} - 1.3855\, LIME$ (1.3159) (1.2368) (0.3248)	0.444	9.247	61
Translog[b,d]	$Y^* = 24.7130 + 5.0887\, LIME^{0.5820 - 0.2020\, LLIME}$ (1.1238) (1.3655) (0.2789)(0.1037)	0.449	9.172	60

[a] Y^* is predicted yield.

[b] Fitted by SAS PROC NLIN; standard errors are asymptotic estimates.

[c] $ZK = (LIME - K)$ if $LIME > K$, $ZK = 0$ otherwise; two polynomial pieces are joined at $LIME = K$, a design point of the experiment.

[d] *LLIME* is the natural logarithm of *LIME* with $LIME = 0$ replaced by $LIME = 0.01$.

Table 5. Nonnested-Hypothesis Test Statistics[a]

		Alternative Hypothesis			
	Null Hypothesis	Power	Q. Spline	Sq. Root	Translog
Alfalfa	Power		0.217	0.167	0.048
			-0.845	-0.349	-1.659
			1.591	1.505	1.589
	Q. Spline	0.502		0.502	0.502
		-1.192		0.038	-0.047
		0.504		0.501	0.506
	Sq. Root	0.398	0.373		0.402
		-1.579	-1.081		-1.261
		0.352	0.626		0.655
	Translog	0.006	0.055	-0.124	
		-1.535	-0.465	0.384	
		-0.013	0.303	-0.287	
Corn	Power		0.384	0.038	0.115
			-5.498**	-3.393**	-0.225
			3.082**	1.340	2.066*
	Q. Spline	3.972**		3.955**	3.940**
		-3.939**		-1.936	-1.889
		3.884**		3.794**	3.942**
	Sq. Root	2.562*	-0.208		3.191**
		-1.672	-4.576**		0.813
		2.554*	2.528*		3.192**
	Translog	0.000	0.511	-0.298	
		-2.241*	-4.225**	-1.487	
		-0.024	1.858	-0.894	
Soybeans	Power		0.474	0.593	0.173
			0.147	-0.212	-6.382**
			2.179*	1.760	2.176*
	Q. Spline	-0.428		0.035	0.146
		-1.822		-0.342	0.001
		-0.605		0.029	1.050
	Sq. Root	0.421	0.504		0.357
		-1.805	-0.865		0.020
		0.229	1.149		1.659
	Translog	0.013	-0.006	-0.007	
		-6.249**	-0.480	-0.772	
		0.510	0.443	-1.092	

[a] For each null hypothesis at the left, the top number in the row is the *JA*-statistic (a *t*-value) for the alternate hypothesis at the head of the column; the middle number is the *N*-statistic, the bottom number is the *t*-value for the *P*-statistic. For nonlinear null hypotheses, the *JA*-statistic is an asymptotic value.

*Significant at the 5% level; ** significant at the 1% level.

Table 6. Costs of Misspecification

"Correct" Function	Optimal[b] Values	Cost of Applying Optimal Levels Based on[a] Power	Spline	Sq. Root	Translog
Alfalfa					
Spline	2.75 T/ac 247.32 $/ac	28.44	0.00	6.82	2.66
Sq. Root	3.45 T/ac 234.65 $/ac	15.67	1.87	0.00	0.44
Translog	3.10 T/ac 236.85 $/ac	15.08	0.64	0.53	0.00
Corn					
Spline	1.66 T/ac 310.52 $/ac	8.77	0.00	32.58	6.76
Sq. Root	2.46 T/ac 285.95 $/ac	2.18	6.86	0.00	2.59
Translog	1.95 T/ac 275.85 $/ac	0.01	0.78	1.70	0.00
Soybeans					
Spline	1.23 T/ac 154.13 $/ac	11.96	0.00	4.08	0.38
Sq. Root	0.60 T/ac 154.59 $/ac	3.61	2.62	0.00	1.40
Translog	1.04 T/ac 152.39 $/ac	10.55	0.29	2.10	0.00

[a] Decrease in net returns (return above lime costs) by applying the optimal lime rate for the "incorrect" function when response to lime is described by the "correct" function.

[b] Optimal lime rates(tons/ac) and net returns ($/ac). Assumed prices: lime, $16/ton, spread; alfalfa hay, $80/ton; corn $2.30/bu; soybeans, $5.65/bu. Hay and grain prices are net of harvesting and drying costs.

Figure 1. Observed and Predicted Alfalfa Yields

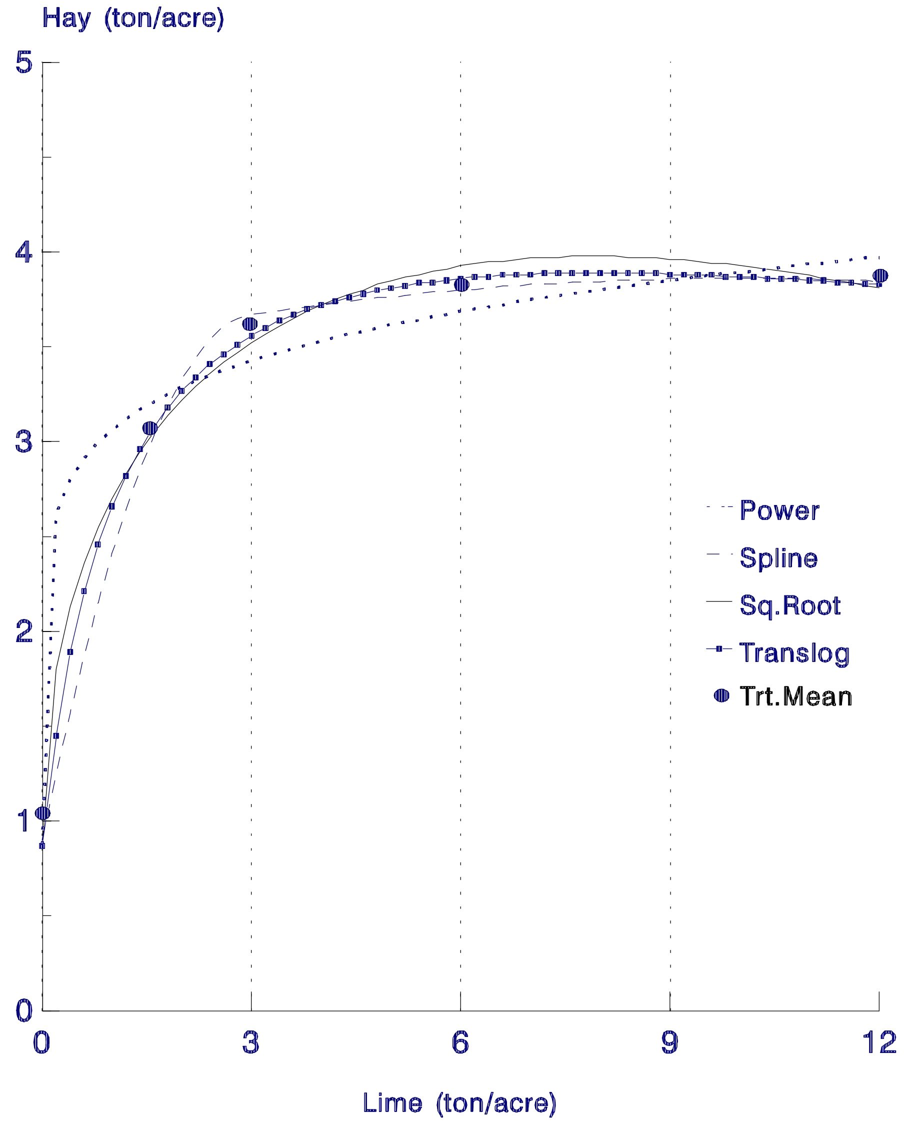

Figure 2. Observed and Predicted Corn Yields

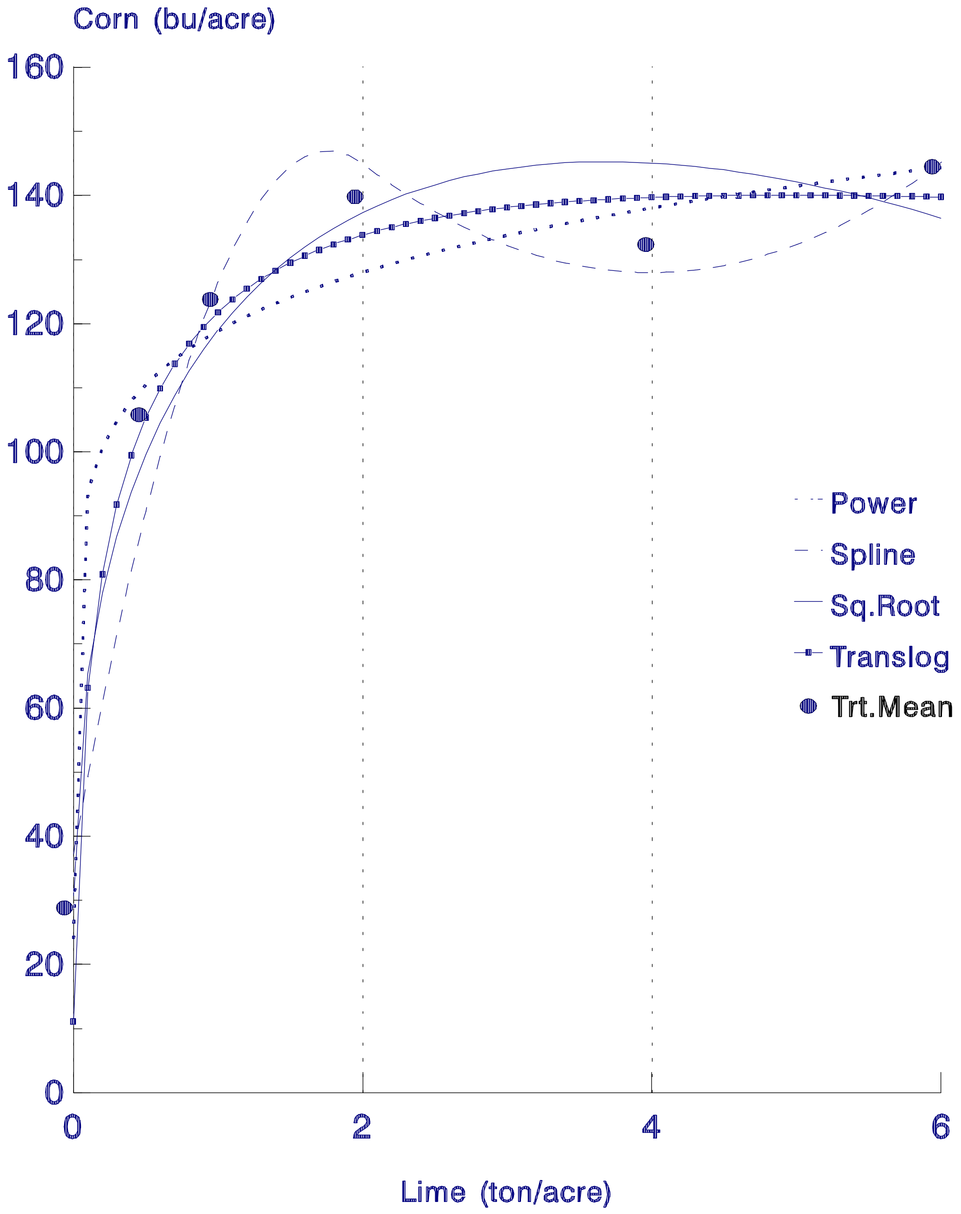

Figure 3. Observed and Predicted Soybean Yields