

SKEWNESS

All about Skewness:

- Aim
- Definition
- Types of Skewness
- Measure of Skewness
- Example

A fundamental task in many statistical analyses is to characterize the *location* and *variability* of a data set. A further characterization of the data includes skewness and kurtosis.

Measure of Dispersion tells us about the variation of the data set.

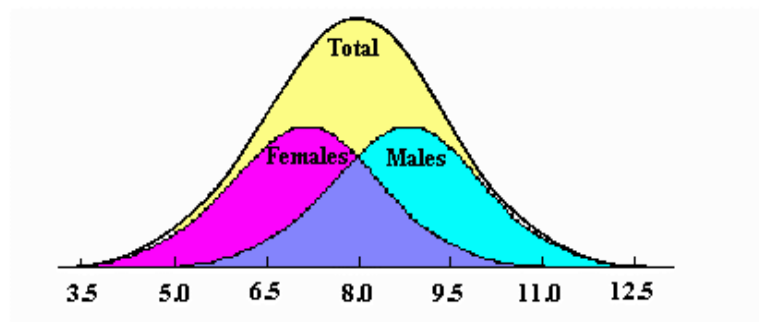
Skewness tells us about the *direction of variation* of the data set.

Definition:

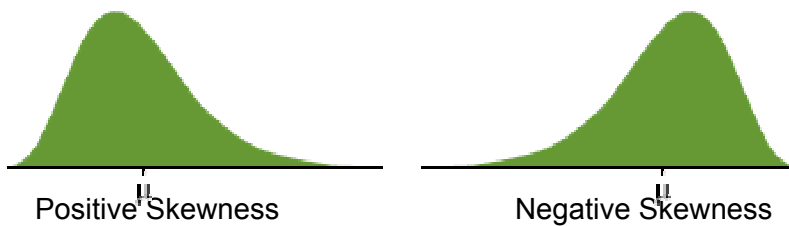
Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point.

Types of Skewness:

Teacher expects most of the students get good marks. If it happens, then the curve looks like the normal curve bellow:



But for some reasons (e. g., lazy students, not understanding the lectures, not attentive etc.) it is not happening. So we get another two curves.



The first one is known as positively skewed and the second one is known as negatively skewed curve.

Positive vs. Negative Skewness

These graphs illustrate the notion of skewness. The one on the left is positively skewed. The one on the right is negatively skewed.

Measure of Skewness:

1. Karl Pearson coefficient of Skewness

$$Sk = 3(\text{mean} - \text{median}) / \text{Standard Deviation} = 3(X - Me) / S$$

2. The skewness of a random variable X is denoted or $skew(X)$. It is defined as:

$$skew(X) = \frac{E[(X - \mu)^3]}{\sigma^3}$$

where μ and σ are the mean and standard deviation of X .

Interpretation:

1. If $Sk = 0$, then the frequency distribution is normal and symmetrical.
2. If $Sk > 0$, then the frequency distribution is positively skewed.
3. If $Sk < 0$, then the frequency distribution is negatively skewed.

Example:

The length of stay on the cancer floor of Apolo Hospital were organized into a frequency distribution. The mean length of stay was 28 days, the medial 25 days and modal length is 23 days. The standard deviation was computed to be 4.2 days.

Is the distribution symmetrical, or skewed?

What is the coefficient of skewness? Interpret.

Solution: Solve yours by using the formula.

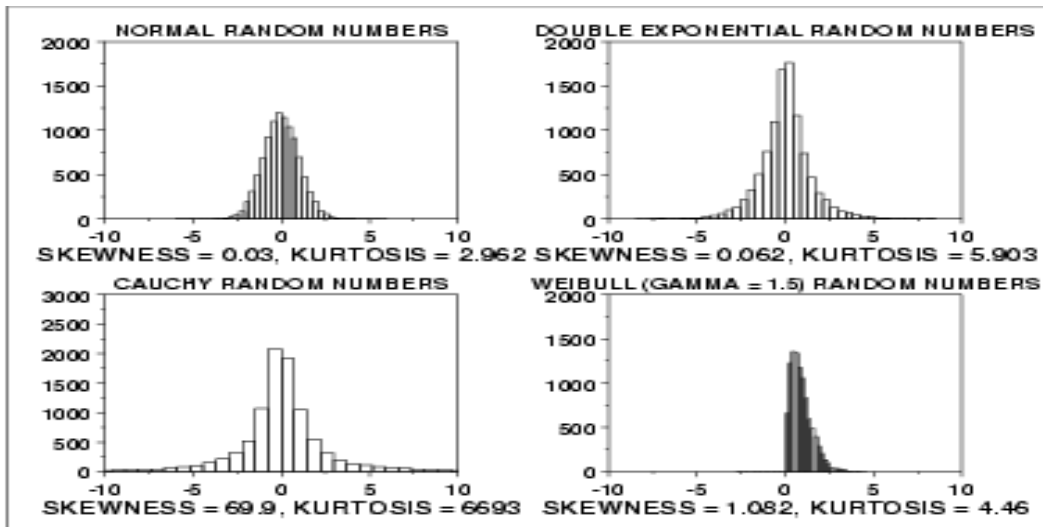
KURTOSIS

Kurtosis is a parameter that describes the shape of a random variable's probability distribution. Consider the two probability density functions (PDFs) in Exhibit 1:

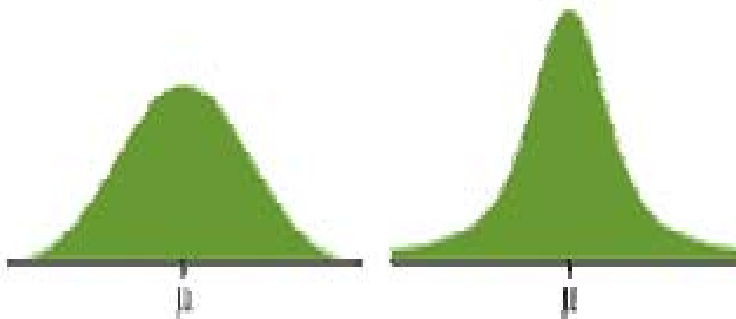
Low vs. High Kurtosis

Exhibit 1

These graphs illustrate the notion of kurtosis. The PDF on the right has higher kurtosis than the PDF on the left. It is more peaked at the center, and it has fatter tails.



Copyright (c) Longevity Analysis, LLC



These graphs illustrate the notion of kurtosis. The PDF on the right

Which would you say has the greater standard deviation? It is impossible to say. The distribution on the right is more peaked at the center, which might lead us to believe that it has a lower standard deviation. It has fatter tails, which might lead us to believe that it has a higher standard deviation. If the effect of the peakedness exactly offsets that of the fat tails, the two distributions will have the same standard deviation. The different shapes of the two distributions illustrate kurtosis. The distribution on the right has a greater kurtosis than the distribution on the left.

The kurtosis of a random variable X is denoted or $kurt(X)$. It is defined as

$$kurt(X) = \frac{E[(X - \mu)^4]}{\sigma^4}$$

where μ and σ are the mean and standard deviation of X .

About Kurtosis

The *Excel*TM help screens tell us that "kurtosis characterizes the relative peakedness or flatness of a distribution compared to the normal distribution. Positive kurtosis indicates a relatively peaked distribution. Negative kurtosis indicates a relatively flat distribution" (Microsoft, 1996). And, once again, that definition doesn't really help us understand the meaning of the numbers resulting from this statistic.

Normal distributions produce a kurtosis statistic of about zero (again, I say "about" because small variations can occur by chance alone). So a kurtosis statistic of 0.09581 would be an acceptable kurtosis value for a mesokurtic (that is, normally high) distribution because it is close to zero. As the kurtosis statistic departs further from zero, a positive value indicates the possibility of a leptokurtic distribution (that is, too tall) or a negative value indicates the possibility of a platykurtic distribution (that is, too flat, or even concave if the value is large enough). Values of 2 standard errors of kurtosis (*sek*) or more (regardless of sign) probably differ from mesokurtic to a significant degree.

The *sek* can be estimated roughly using the following formula (after Tabachnick & Fidell, 1996): For example, let's say you are using *Excel*TM and calculate a kurtosis statistic of + 1.9142 for a particular test administered to 30 students. An approximate estimate of the *sek* for this example would be: Since two times the standard error of the kurtosis is .7888 and the absolute value of the kurtosis statistic was 1.9142, which is greater than .7888, you can assume that the distribution has a significant kurtosis problem. Since the sign of the kurtosis statistic is positive, you know that the distribution is leptokurtic (too tall). Alternatively, if the kurtosis statistic had been negative, you would have known that the distribution was platykurtic (too flat). Yet another alternative would be that the kurtosis statistic might fall within the range between - 1.7888 and + 1.7888, in which case, you would have to assume that the kurtosis was within the expected range of chance fluctuations in that statistic. The existence of flat or peaked distributions as indicated by the kurtosis statistic is important to you as a language tester insofar as it indicates violations of the assumption of normality that underlies many of the other statistics like correlation coefficients, t-tests, etc. used to study the validity of a test. Another practical implication should also be noted. If a distribution of test scores is very leptokurtic, that is, very tall, it may indicate a problem with the validity of your decision making processes. For instance, at the University of Hawai'i at Manoa, we give a writing placement test for all incoming nativespeaker freshmen (or should that be freshpersons?) that produces scores on a scale of 0-20 (each student's score is based on four raters' scores, which each range from 0-5). Yearly, we test about 3400 students. You can imagine how tall the distribution must look when it is plotted out as a histogram: 20 points wide and hundreds of students high. The decision that we are making is a four way decision about the level of instruction that students should take: remedial writing; regular writing with an extra lab tutorial; regular writing; or honors writing. The problem that arises is that very few points separate these four classifications and that hundreds of students are on the borderline. So a wider distribution would help us to spread the students out and make more responsible decisions especially if the revisions resulted in a more reliable measure with fewer students near each cut point.

Conclusion

One last point I would like to make: the skewness and kurtosis statistics, like all the descriptive statistics, are designed to help us think about the distributions of scores that

our tests create. Unfortunately, I can give you no hard-and-fast rules about these or any other descriptive statistics because interpreting them depends heavily on the type and purpose of the test being analyzed. Nonetheless, I have tried to provide some basic guidelines here that I hope will serve you well in interpreting the skewness and kurtosis statistics when you encounter them in analyzing your tests. But, please keep in mind that all statistics must be interpreted in terms of the types and purposes of yo