Comment on "Sigma: Issues, Insights, and Challenges" by F. O. Strasser, N. A. Abrahamson, and J. J. Bommer

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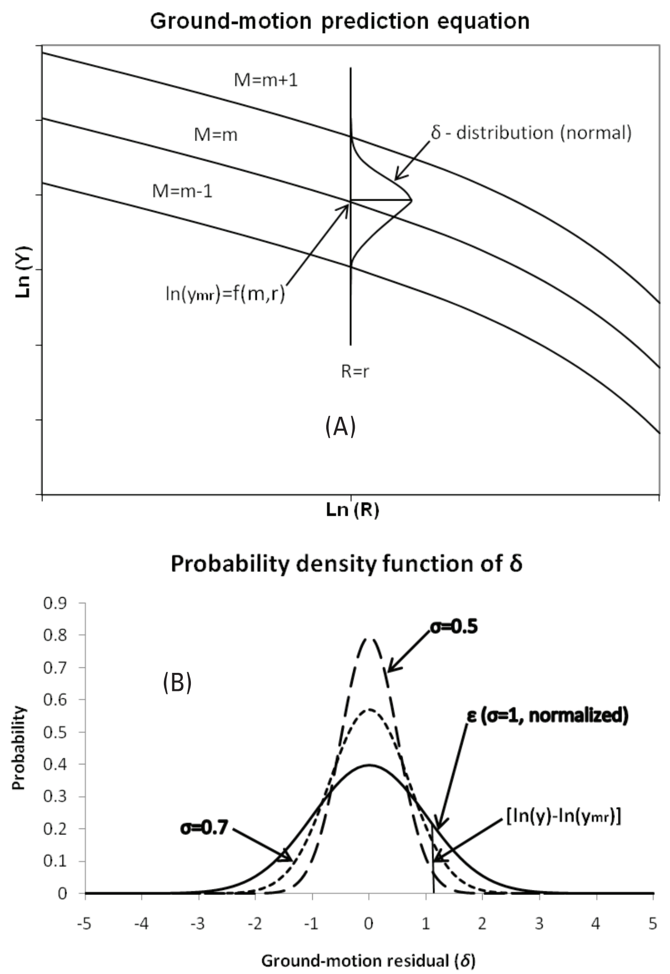
The recent paper "Sigma: Issues, Insights, and Challenges" by Strasser *et al.* (2009) provided an excellent review of the complexity and dependency of the standard deviation σ of ground motion, as well as the difficulty in reducing it. Strasser *et al.* (2009) concluded that "the value of σ has remained fairly stable over the past 40 years," although there have been systematic efforts to reduce it (Atkinson and Boore 2003; Abrahamson and Silva 2008; Strasser *et al.* 2008). In other words, Strasser *et al.* (2009) showed that the prospects of reducing σ are hopeless, even though the authors may still believe that "there is some hope to achieve reductions in σ, although the process may be labor-intensive." As stated by Strasser *et al.* (2009), their "attempt is made to identify the more promising approaches for reduction of σ." Their paper is not worth a single comment if σ is simply a statistical issue or personal belief. As stated by Strasser *et al.* (2009), however, "the effort involved [reducing σ] is likely to be worthwhile in the context of seismic hazard analysis." Their paper surely deserves a comment in the context of seismic hazard analysis, modern probabilistic seismic hazard analysis (PSHA) in particular, because the implications for society are far reaching, from seismic design of nuclear power plants, buildings, and bridges to earthquake insurance premiums.

Statistics is a common tool used by all disciplines for collection, analysis, interpretation, and presentation of data, and in particular for prediction and forecasting based on data. As shown by Strasser *et al.* (2009), ground motion Y at a site can be predicted by a statistical relationship, the so-called ground-motion prediction equation (GMPE), as

$$\ln(Y) = f(M,R) + \delta = f(M,R) + \varepsilon\sigma \,, \qquad (1)$$

where M is the earthquake magnitude, R is the source-to-site distance, and δ is the ground-motion residual, which can also be expressed as a product of the normalized residual ε and σ. The schematic relationships of GMPE are shown in Figures 1A and B. In modern GMPE, δ is assumed to be a normal distribution with a zero mean and the standard deviation σ (Figure 1A). As shown in Figure 1B, ε is a standardized normal distribution with a zero mean and standard deviation of 1. In other words, δ distribution depends on σ, but not on ε because ε is a standardized normal distribution with constant standard deviation of 1. As shown by Strasser *et al.* (2009), σ depends on M and R. Therefore, δ genuinely depends on M and R, and ε does not (standardized normal distribution). Bommer and Abrahamson (2007) also showed the dependency of δ and independency of ε (Figure 1 in Bommer and Abrahamson 2007).



▲ **Figure 1.** Schematic relationships for the ground-motion prediction equation (GMPE) (A) and the probability density function (PDF) of ground-motion residual (B).

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Now let's examine how GMPE and the dependencies are used in modern PSHA. As pointed out by Wang and Zhou (2007), modern PSHA is referred to as the Cornell-McGuire method and was based on the approximation of an earthquake as a single point source (*i.e.*, a point-source model for earthquakes; Cornell 1968, 1971; McGuire 2004). In modern seismology, however, an earthquake is considered a finite fault, particularly in the development of GMPE (Atkinson and Boore 2003; Bommer and Abrahamson 2006; Abrahamson and Silva 2008). In other words, the physical model (point source) that modern PSHA is based on is not consistent with modern earthquake science (finite fault). As shown by Cornell (1968, 1971), McGuire (2004), and Wang and Zhou (2007), the critical step in modern PSHA is to calculate the exceedance probability $P[Y \geq y]$ from GMPE, Equation 1. According to Cornell (1968, 1971; McGuire 2004), the core equation for hazard calculation in modern PSHA is

$$\gamma(y) = \sum \nu \iint \left[1 - \Phi\left(\frac{\ln y - \ln y_{mr}}{\sigma} \right) \right] f_M(m) f_R(r) dmdr \,, \qquad (2)$$

where γ is the annual probability of exceedance for a given ground motion y; ν is the activity rate; $f_M(m)$ and $f_R(r)$ are the probability density function (PDF) for earthquake magnitude (M) and source-to-site distance (R), respectively; $\ln(y_{mr}) = f(m, r)$, and $\Phi(t)$ is the cumulative probability function for δ and equal to the area under the probability distribution curve from $-\infty$ to $[\ln(y) - \ln(y_{mr})]$ (Figure 1B). The exceedance probability for δ is $1 - \Phi(t)$ and equal to the area under the probability distribution curve from $[\ln(y) - \ln(y_{mr})]$ to ∞ (Figure 1B). As shown in Equation 2, σ is a key parameter that influences hazard calculation. This can be seen in Figure 1B: the larger σ, the larger the exceedance probability is. Strasser *et al.* (2009) also showed that seismic hazard increases with σ, at low annual frequency of exceedance (10^{-4} or less) in particular (Figure 1 in Strasser *et al.* 2009). This is why σ becomes the controlling factor in modern PSHA. As also shown in Equation 2, the exceedance probability, $1 - \Phi(t)$, is calculated for δ (normal distribution with σ), but not for ε (standardized normal distribution with $\sigma = 1$) (Figure 1B). Bommer and Abrahamson (2007) confused δ with ε and erroneously stated that "the random variable considered in PSHA is ε not δ" (where δ is E in Bommer and Abrahamson 2007).

As pointed out by Wang and Zhou (2007), according to mathematical statistics (Benjamin and Cornell 1970; Mendenhall *et al.* 1986), Equation 2 is valid *if and only if* M, R, and δ are independent random variables. δ is not an independent random variable, however. Therefore, Equation 2 is not mathematically valid (Wang and Zhou 2007). Strasser *et al.* (2009) proved that the mathematical formulation of modern PSHA is not valid because neither δ nor σ is an independent random variable. In other words, σ is not used correctly in modern PSHA (Wang and Zhou 2007). This incorrect use of σ was also demonstrated in a recent study by Purvance *et al.* (2008), who found that PSHA calculations without consideration of σ (*i.e.*, zero aleatory variability) produce more consistent results compared with results from precariously balanced rocks.

In summary, the standard deviation σ is a statistical parameter that is used to quantify the variability of ground motions. Strasser *et al.* (2009) had shown that "the value of σ has remained fairly stable over the past 40 years" in spite of tremendous efforts and more additional data in recent years (Atkinson and Boore 2003; Bommer and Abrahamson 2006; Abrahamson and Silva 2008). This can be easily understood and explained in the context of statistics or of a normal statistical application. The reason σ becomes so critical in seismic hazard analysis is its role in hazard calculation of modern PSHA. In addition to the invalid physical model, modern PSHA is mathematically invalid because of the dependency of σ on M and R. In other words, σ has been incorrectly used in modern PSHA, leading to the so-called ergodic assumption (Anderson and Brune 1999). This incorrect use has led to so much effort to study σ. The paper by Strasser *et al.* (2009) was the latest such effort. It is not a concern if such effort is a purely statistical issue. But it is of great concern that the authors' effort is in the context of modern PSHA. Modern PSHA is not consistent with modern earthquake science and is mathematically invalid. Use of modern PSHA could lead to either unsafe or overly conservative engineering design.

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