Empirical Evaluation of Voting Rules with Strictly Ordered Preference Data

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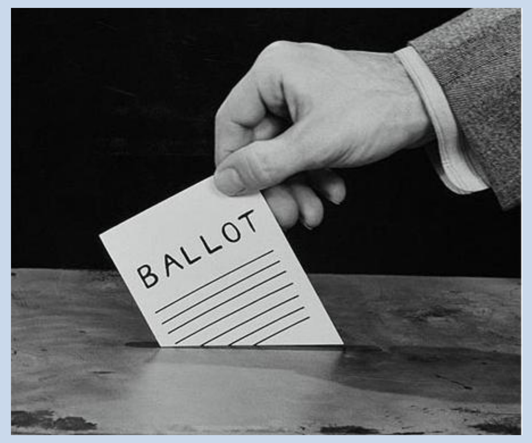
Motivation:

- Data collection and reasoning systems are moving towards distributed and multi-agent design paradigms.
- Knowledge and data aggregation method, such as voting, are used to combine these observations and preferences into a total, group ordering.
- Unfortunately, the study of voting systems often takes place in the theoretical domain due to a lack of large samples of sincere, strictly ordered voting data.



Objective:

- Identify, evaluate, and distribute a novel set of election data.
- An election is a strict, total ordering over a set of candidates (A > B > C).
- Characterize this novel set of data in terms of its:
  - ✓ Voting Rule Results
  - ✓ Occurrences of Condorcet's Paradox
  - ✓ Condorcet Efficiency
  - ✓ Preference Restrictions
  - ✓ Statistical Properties

The Data:

- We extract a subset of all possible 3 and 4 candidate elections from the Netflix Prize Dataset.
  - Over 100 million ratings of 17,770 unique movies.
  - Each set of 3 movies becomes a 3 candidate election
- Our resultant dataset contains over 5 orders of magnitude more data than all other election studies combined (10^12 elections).
- Users motivated by design of system to be more sincere.

	3 Candidate Sets			4 Candidate Sets		
	Set 3A	Set 3B	Set 3C	Set4A	Set 4B	Set 4C
Min.	350.0	350.0	350.0	350.0	350.0	350.0
1st Qu.	444.0	433.0	435.0	394.0	393.0	384.0
Median	617.0	579.0	581.0	461.0	461.0	438.0
Mean	963.8	881.8	813.4	530.9	530.5	494.6
3rd Qu.	1,041.0	931.0	901.0	588.0	591.0	539.0
Max.	22,079.0	18,041.0	20,678.0	3830.0	3396.0	3639.0
Elements	1,553,611.0	1,331,549.0	2,049,732.0	2,721,235.0	1,222,009.0	1,243,749.0

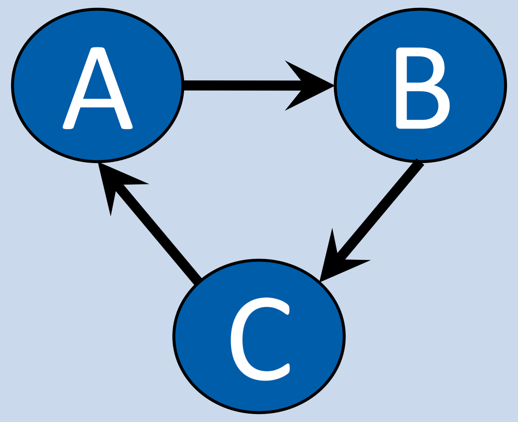
- Election Size Details

Condorcet's Paradox:

- Occurs when rational individual preferences combine to create irrational total ordering.
- Cyclic orderings over can be total (all candidates), partial (some candidates excluding the winner) and partial top (some candidates including the winner).

		Partial Cycle	Partial Top	Total
$m=3$	Set 3A	635 (0.041%)	635 (0.041%)	635 (0.041%)
	Set 3B	591 (0.044%)	591 (0.044%)	591 (0.044%)
	Set 3C	1,143 (0.056%)	1,143 (0.056%)	1,143 (0.056%)
$m=4$	Set 4A	3,837 (0.141%)	2,882 (0.106%)	731 (0.027%)
	Set 4B	1,864 (0.153%)	1,393 (0.114%)	462 (0.035%)
	Set 4C	3,233 (0.258%)	2,367 (0.189%)	573 (0.046%)

- Number of elections containing cycles.



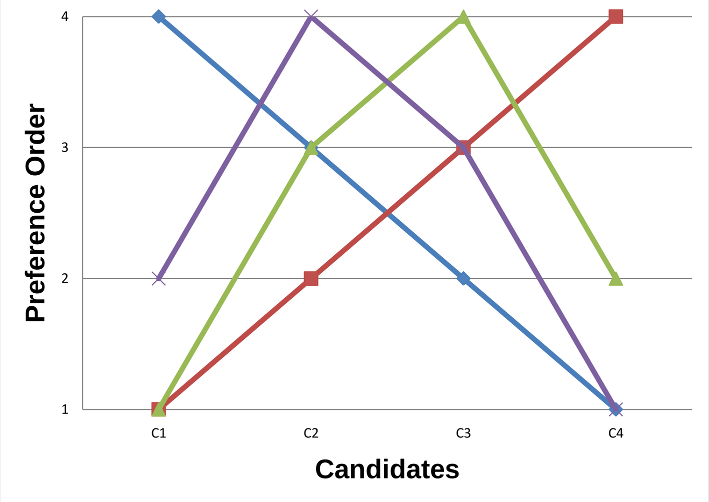
- A total voting cycle: A is preferred to B, which is preferred to C, which is preferred to A.

Single Peaked Preferences:

- Informally, it is the idea that some candidate, in a three candidate election, is never ranked last.
- Elections with single peaked preferences are more susceptible to manipulation.

		Single-Peak	Single-Mid	Single-Bottom
$m=3$	Set 3A	342 (0.022%)	0 (0.0%)	198 (0.013%)
	Set 3B	227 (0.017%)	0 (0.0%)	232 (0.017%)
	Set 3C	93 (0.005%)	0 (0.0%)	100 (0.005%)
$m=4$	Set 4A	1 (0.022%)	0 (0.000%)	1 (0.013%)
	Set 4B	0 (0.000%)	0 (0.000%)	0 (0.000%)
	Set 4C	0 (0.000%)	0 (0.000%)	0 (0.000s%)

- Elections demonstrating single peaked preference structures.



- Example of single peaked preference profiles.

Voting Rules:

- We evaluated our dataset under a variety of common voting rules.
- Results were compared using Spearman's Rank Order Correlation Coefficient
- Voting Rules Considered:
  - ✓ Plurality
  - ✓ 2-Approval
  - ✓ Copeland
  - ✓ Borda Count
  - ✓ Repeated Alternative Vote

		Plurality	2-Approval	Borda	RAV
Set 3A	Mean	0.9300	0.9149	0.9787	0.9985
	SD	0.1999	0.2150	0.1029	0.0336
Set 3B	Mean	0.9324	0.9215	0.9802	0.9985
	SD	0.1924	0.2061	0.0995	0.0341
Set 3C	Mean	0.9238	0.9177	0.9791	0.9980
	SD	0.208	0.2130	0.1024	0.0394
Set 4A	Mean	0.9053	0.9578	0.9787	0.9978
	SD	0.1691	0.0956	0.0673	0.0273
Set 4B	Mean	0.9033	0.9581	0.9798	0.9980
	SD	0.1627	0.0935	0.0651	0.0263
Set 4C	Mean	0.8708	0.9516	0.9767	0.9956
	SD	0.2060	0.1029	0.0706	0.0404

- Mean and Standard Deviation of Spearman's Rho between each voting rule and Copeland order.

Condorcet Efficiency:

- The frequency with which a voting rule selects the Condorcet Winner, when one exits.
- Condorcet Winners are generally accepted to be the "best choice" when one exists.
- A common metric to compare the quality of voting rules.

		Condorcet Winners	Plurality	2-Approval	Borda	RAV
$m=3$	Set 3A	1,548,553	0.9665	0.8714	0.9768	0.9977
	Set 3B	1,326,902	0.9705	0.8842	0.9801	0.9980
	Set 3C	2,041,756	0.9643	0.8814	0.9795	0.9971
$m=4$	Set 4A	2,701,464	0.9591	0.9213	0.9630	0.9966
	Set 4B	1,212,370	0.9626	0.9290	0.9693	0.9971
	Set 4C	1,241,762	0.9550	0.9253	0.9674	0.9940

•Condorcet Efficiency of the various voting rules understudy. Ratio of number of Condorcet Winners chosen by a rule to the total number of Condorcet Winners

Statistical Models of Elections:

- Due to a lack of large datasets many researchers generate election pseudo-data with statistical models.
- These models define distributions over all (#Candidates)! possible strict orderings.
- The distributions used to generate this data may or may not be grounded in reality.
- We evaluate 4 static and 3 generative distribution models:

Static Models:
- ✓ Impartial Culture
- ✓ Dual Culture
- ✓ Uniform Culture
- ✓ Unequal Unique Probabilities

Generative Models:
- ✓ Impartial Anonymous Culture
- ✓ Polya Eggenberger Urn Model
- ✓ Fitted Impartial Anonymous Culture

Findings:

- Almost all voting rules give the same winner.
- Most voting rules highly Condorcet Efficient.
- Almost no voting cycles or single peaked preferences.
- None of the statistical models are well supported..
- Results consistent with other empirical studies in the field.

•This work is supported by NSF EAGER grant CCF-1049360