

21

Time and Agricultural Production Processes

Chapters 2! 18 treated production processes in a comparative statics framework, and the time element was largely ignored. This chapter introduces time as an explicit component of marginal analysis. Goals for farmers other than profit maximization over a single production period are introduced. Time is introduced as an explicit variable within a single period planning model. Basic procedures for discounting or compounding revenues and costs over several production periods are outlined, and a profit maximization model in which revenues and costs occur over several production seasons is presented.

Key terms and definitions:

- Alternative Goals
- Long Run Profit Maximization
- Accumulation of Wealth
- Time as an Input
- Macroeconomic Policy
- Inflation
- Real Interest Rate
- Net Worth
- Discounting
- Compounding
- Present Value
- Multiple Period Production

21.1 Introduction

Throughout much of this text, the implicit assumption has been that farmers during a single production season are interested in maximizing profits, or in maximizing revenue subject to constraints. While these may be valid goals for a farmer within a single production season, farmers do not normally enter farming, produce for one season and then exit. Farmers look at the occupation over a time period of 20 years or more. They normally do not expect to exit from farming within a short time period. As a result, most farmers have goals and objectives that go beyond single season profit or constrained revenue maximization. These goals and objectives may not be totally inconsistent with short run profit or constrained revenue maximization, but they may not be entirely consistent either.

In addition to having an impact on the goals of the decision maker, time influences agricultural production in other ways. Time can be thought of as a scarce resource or input that must be allocated in a manner consistent with the goals of the farm manager. A dollar earned some time into the future cannot be treated the same way as a dollar earned today. Revenues and costs must be discounted or compounded. Time is an inherent part of virtually all agricultural finance issues. Marginal analysis can be used as a basis for making decisions within a time frame encompassing several production periods.

21.2 Alternative Goals of a Farm Manager Over Many Seasons

21.2.1 Long-Run Profit Maximization

Just as a farmer is interested in maximizing profits in a single production season, a farmer might also be interested in maximizing expected long run profit. The maximization of profit over a twenty year period may not entail making the same set of choices that would be made if the farmer were interested in maximizing profit in each of 20 successive single season production periods.

Long run profit maximization may require making larger expenditures on durable inputs such as land and machinery, for example, early within the 20 year period. A farmer interested in long run profit maximization over a number of years has a long run planning horizon and will make investment decisions with expected payoffs some years away consistent with the long run goal. The decisions made by farmers are not unrelated to the length of the planning horizon.

There are many examples of farm enterprises that must inherently involve a planning horizon of more than one year. The farmer who starts a pick-your-own strawberry enterprise is a year and a half or more away from revenue from the sale of the strawberries. An individual planning to grow apples or Christmas trees on land not previously used for that purpose must be interested in long run rather than short run profit maximization. If farmers all were only interested in single season profit or constrained revenue maximization, they would never go into such enterprises.

A goal of long run profit maximization will frequently require short run profit and income sacrifices during the early years of the planning horizon, with the hope or expectation of making greater profits during the latter years. However, a dollar today is worth more than a dollar obtained a year or more from now. A dollar today could earn interest in a bank. This is foregone income until the dollar is earned. Inflation makes a dollar earned today more valuable than the same dollar earned a year from now. These elements must enter if a farmer is interested in finding a decision path that maximizes profits over a long term planning horizon.

Moreover, season to season variability in profits or farm income is often greater if long run rather than short run profit maximization is the goal. The variability is of importance to the extent that it influences the survival of the farm firm at some point during the planning horizon. Short run survival is essential to long run profitability, and hence long run profits are subject to the constraint that the firm survive in the short run.

21.2.2 Accumulation of Wealth

Farmers sometimes seem less interested in their own incomes than in ensuring that their children inherit a lot of farmland. Some might argue that the accumulation of wealth or net worth, usually in the form of farmland, is more important to most farmers than the goal of maximizing long run profits. The maximization of long run profits and the accumulation of wealth in the form of farmland are not entirely inconsistent goals, but neither are the two goals entirely consistent.

Some non-farm businesses pride themselves in paying consistent annual dividends to their stockholders. However, aggressive companies are more likely to reinvest such profits in the firm. If these reinvestments are successful, the price of the stock in the company will increase. Farmers make similar choices. A farmer might spend profits from last season's crop on family living expenses. A farmer interested in maximizing single season profits might use part of last year's profits to purchase inputs such as fertilizer for this year's crop, with the hope of increased profits for the coming year. A farmer interested in the accumulation of wealth might more likely use part of last year's profits as a down payment on additional land, and the profitability of the crop for the coming season may be reduced to a degree. The balance between these three uses provides important clues with respect to the underlying goals of the farm manager.

21.2.3 Other Goals

Farmers, much like consumers, are subject to peer pressure. Nearly every farmer wants to be recognized by his or her neighbors as a good farmer. This peer pressure manifests itself in curious ways. Some farmers enter contests designed to see who can produce the greatest yields for a given crop. Yield maximization is almost always inconsistent with profit maximization. Livestock shows may be enjoyable for farmers. Often there is no clear link between the characteristics of livestock deemed important in the show ring and profit maximization. However, large profits may be made if there are others who believe such characteristics are desirable. Money can be made if a buyer desiring the particular traits can be found.

Farm machinery provides another example. Examples of farmers who appear to have as their primary goal the accumulation of the best set of farm machinery can easily be found. A self-respecting farmer would not want to be placed in a situation whereby his tractor was considered by his neighbors to be somehow inadequate, even though it may provide excellent service at low cost. Such peer pressure even extends to the color or brand of farm machinery, with certain brands being favored in certain areas.

The quality of life has a great deal to do with specific decisions made by the farmer. Some farmers produce cattle not to make a profit but rather to enjoy the associated lifestyle (to the consternation of ranchers who are attempting to produce cattle for a profit). The home on ten acres represents a desirable hobby farm for some workers in urban areas.

21.3 Time as an Input to the Production Process

Farmers normally do not think of time as an input to the production process, but time can be thought of as an input. Labor is an input only to the extent that it provides a stream of services over time that presumably increase the profitability of the firm. Farm machinery is not an input in the sense that fertilizer and seed are inputs to the production process. Rather the farm machinery also provides a stream of services over time. It is the stream of services, not the machinery itself, that contributes to the profitability of the firm. Time suitable for performing planting, tillage and harvest activities is limited by weather conditions, and field time used to perform one operation cannot be reused to perform another field operation.

Much of farm management involves the allocation of the scarce resource time in such a way that profits to the farm are maximized. Consider a farmer who has available during a single production season a limited number of hours (T^0) suitable for the performance of planting (T_p), tillage (T_t) and harvest (T_h) activities. Assume that the farmer produces two crops, corn and soybeans. The farmer has already determined the acreage of y_1 and y_2 to be grown, and is interested in allocating available time to each crop. Revenue is sum of the output of the two crops multiplied by their respective prices ($R = p_1 y_1 + p_2 y_2$). Output of each crop is a function of the time available in planting, tillage and harvest operations for that crop. That is $y_1 = y_1(t_{p1}, t_{t1}, t_{h1})$; $y_2 = y_2(t_{p2}, t_{t2}, t_{h2})$.

Form the Lagrangean:

$$\begin{aligned} \text{21.1} \quad L = & p_1 y_1(t_{p1}, t_{t1}, t_{h1}) + p_2 y_2(t_{p2}, t_{t2}, t_{h2}) \\ & + D[T_p - t_{p1} - t_{p2}] + ([T_t - t_{t1} - t_{t2}] + R[T_h - t_{h1} - t_{h2}]) \end{aligned}$$

where y_1 is corn, y_2 is soybeans, t_{p1} is the time used in the planting operation per unit of output y_1 , t_{p2} is the time used in the planting operation per unit of output y_2 , t_{t1} is the time used for tillage operations per unit of output y_1 , t_{t2} is time used for tillage operations per unit of output per unit of output y_2 , t_{h1} is the time used for harvest operations per unit of y_1 .

Then the first order conditions for single season maximization of revenue subject to the constraint imposed by the the time available for planting, tillage and harvest operations is:

$$\text{21.2} \quad p_1 M_1 / M_{p1} = D$$

$$\text{21.3} \quad p_2 M_2 / M_{p2} = D$$

$$\text{21.4} \quad p_1 M_1 / M_{t1} = ($$

$$\text{21.5} \quad p_2 M_2 / M_{t2} = ($$

$$\text{21.6} \quad p_1 M_1 / M_{h1} = R$$

$$\text{21.7} \quad p_2 M_2 / M_{h2} = R$$

$$\text{21.8} \quad T_p - t_{p1} - t_{p2} = 0$$

$$\text{21.9} \quad T_t - t_{t1} - t_{t2} = 0$$

$$\text{21.10} \quad T - t_{h1} - t_{h2} = 0$$

By Rearranging 21.2 - 21.7 :

$$\text{21.11} \quad p_1(M_1/M_{p1}) = p_2(M_2/M_{p2}) = D$$

$$\text{21.12} \quad p_1(M_1/M_1) = p_2(M_2/M_2) = C$$

$$\text{21.13} \quad p_1(M_1/M_{h1}) = p_2(M_2/M_{h2}) = R$$

The Lagrangean multipliers in this problem can be readily interpreted as the imputed values for an additional unit of available time in the planting, tillage or harvest seasons for each crop, in terms of revenue to the farm. For each operation, the farmer should allocate time in such a way that the last unit of time results in the same marginal increase in revenue for both outputs y_1 and y_2 . In general, the Lagrangean multipliers D , C , and R will not necessarily be the same value, but will depend on both on how much time is required for the operation per unit of output, and how much time is available.

The Lagrangean multipliers indicate the imputed value of time in each period in the production of various commodities. If the magnitudes of the shadow prices, or Lagrangean multipliers are large for a particular period, the available time is posing a severe restriction on production of the various products. A Lagrangean multiplier near zero for a particular period suggests that although time in that particular period is of some positive value in the production process, it does not pose a severe restriction.

The example presented here is only illustrative. The farmer will likely break time down into weekly or perhaps even daily increments. In a daily model, each day would have its own Lagrangean multiplier, but the same marginal rules for allocating time between enterprises would apply. These rules suggest that the farmer should do first the operation that contributes the most to the revenue for the entire farm.

A similar approach could be used determining the sequencing of events for for allocating farm machinery use, as well as determining the sequencing of chores to be done in allocating hired labor each day between enterprises. Mathematical programming models have been used in Kentucky, Indiana and in other states in extension work with farmers. A primary objective of these models is to determine how available field time, labor and machinery should be allocated between corn and soybean production. The sequence of events taking place during the production season is divided into a calendar of weekly events. Farmers specify the available field time, labor and machinery during each week of the production season, and the model allocates each time related input according to the rules suggested here. Thus, results provide an indication of how time related inputs should be allocated during each week of the growing season.

21.4 Time, Inflation, Interest Rates and Net Worth

A dollar earned a year from now is not the same thing as a dollar earned today. A dollar spent today is not the same thing as a dollar spent a year from now. There are two reasons for this. First, a dollar earned today could have been placed in the bank and interest would have accrued. That interest is foregone if the dollar is earned a year from now. Moreover, the opportunity cost of a dollar spent today is the interest that could have been earned if the dollar

had not been spent today. Government policy at the federal level over time plays a significant role in determining the profitability of agriculture over time.

Inflation or deflation in asset values clearly affects agricultural investments, particularly those in land, over time. Because of inflation, a dollar earned a year from now is less valuable than a dollar earned today. A dollar borrowed today can be paid back with cheaper dollars earned a year from now. Interest rates paid by banks on savings and charged by banks on loans should reflect these intertemporal differences in the value of a dollar. The interest rate charged or paid by banks less the inflation rate is sometimes called the real interest rate. Thus the interest rates charged or paid by banks reflect not only the real interest rate, but also the general rate of inflation in the economy. Interest rates on low risk investments, such as savings accounts insured by the federal government, also are of concern. The imputed value of dollars not invested in the farm can be represented by the return in a risk free investment.

Macroeconomic policies pursued over time by the federal government have a significant impact on decisions made by farmers. Time affects the opportunity cost, or imputed value for dollars that could be invested in farming or as an alternative, in a low risk savings account. Moreover, over time, inflation and deflation can increase or decrease the value of a farmer's real estate holdings and other assets, thus affecting net worth.

21.5 Discounting Revenues and Costs

Discounting is used in order to determine what a specific amount of revenue obtained at some future point in time would be worth today, or to determine the current amount of a cost incurred at some future point in time. The examples presented here illustrate how to calculate discounted present values for a stream of revenues that occur over a period of several years, but the same approach could be applied to costs that do not all occur at the start of the production period.

21.5.1 The Present Value of a Dollar

The present value of a dollar earned one period from now can be determined by dividing the dollar by 1 plus the market rate of interest. If the interest rate is 8 percent and the period of time is one year, then the present value of a dollar is $1/1.08 = \$0.926$. Suppose that the dollar is instead earned five years from now. The present value of that dollar is $1/(1.08)^5 = \$0.681$. Suppose that an enterprise generates a dollar in revenue at the end of each of 5 years. The present value of the 5 dollars thus generated is:

$$\text{¶1.14} \quad 1/(1.08) + 1/(1.08)^2 + 1/(1.08)^3 + 1/(1.08)^4 + 1/(1.08)^5 = \$3.99$$

A general rule for determining the present value (PV) of a dollar earned at the end of each of n years is:

$$\text{¶1.15} \quad PV = E \sum_{n=1}^N 1/(1+i)^n$$

where $n = 1, \dots, N$

N is the number of years
 i is the market interest rate.

21.5.2 Discounting Revenues with the Present Value Formula

The amount of money returned at the end of the year will probably vary from year to year. For example, if at the end of year 1, 40 dollars is returned, year 2, 50 dollars; year 3, 20 dollars; year 4, 10 dollars and year 5, 100 dollars, the present value formula provides the value of the revenues at the start of year 1:

$$\text{¶1.16} \quad PV = 40/(1+i) + 50/(1+i)^2 + 20/(1+i)^3 + 10/(1+i)^4 + 100/(1+i)^5$$

A general present value formula is:

$$PV = \sum (R_j/(1+i)^j)$$

For all $j = 1, \dots, n$ where n is the number of years.

R_j is the revenue from the j th year.

A farmer may consider the purchase of a machine that will return 100 dollars per year in increased revenue above any variable costs to run the machine and keep it in repair. At the end of five years, the machine is worn out, but has a salvage value of 150 dollars. The present value formula can be used to determine what the farmer could afford to pay for the machine for an interest rate of 8 percent:

$$\text{¶1.17} \quad PV = 100/(1.08) + 100/(1.08)^2 + 100/(1.08)^3 + 100/(1.08)^4 + \\ 100/(1.08)^5 + 150/(1.08)^5 = \$501.36$$

The discounted revenue from the machine is \$501.36. The farmer can afford to pay up to \$501.36. The current price of the machine could be subtracted from its present value (PV) to obtain the discounted net present value (NPV). If the NPV is positive and the assumptions are correct, the farmer will make money on the investment.

Such an approach could easily be applied to a large durable goods investment such as a farm tractor. The major problem is in obtaining the needed revenue data for the machine. Ideally, returns should represent the marginal revenue attributed to the machine with costs other than the purchase price of the machine subtracted. It is sometimes difficult to determine the revenues that should properly occur as a result of owning the new machine. For example, a larger tractor may result in increased revenues because of improved timing of planting, tillage and harvest operations, but these revenues are sometimes difficult to measure.

Another issue involves the determination of the expected life of the tractor, and its salvage value at the end of the expected life. The true expected life is normally very different from the assumed life that is allowed for federal tax purposes. The proper interest rate to be used is another problem. For example, the interest rate could be one of the interest rates charged by the local bank on borrowed funds, or it could be one of the rates paid on a savings account.

The present value formula has been modified to determine the present value of an asset with an infinite life span, such as a piece of land. The present value formula for an asset with an infinite life span is:

$$21.18 \quad PV = R/i$$

where R is the annual return attributable to the asset over all costs other than the asset itself, and i is the assumed interest rate. For example, corn land with a return of 300 dollars per acre over all costs other than the cost of the land, is worth $300/i$ dollars per acre. If the interest rate is ten percent, then the land is worth 3000 dollars per acre. If the land is selling for 2500 dollars per acre, its net present value is $3000 - 2500 = 500$ dollars per acre.

21.5.3 Compounding Revenues and Costs

The discounting process presented in section 21.5.2 makes it possible to determine what revenues and costs occurring over a period of years would be if all were measured at the start of the production period. Compounding is the process used to determine revenues and costs at the end, not the beginning of the planning horizon. The examples used here apply to costs, but the same approach could be used to determine revenues at the end of the planning horizon.

The process of discounting revenues and costs may be an unfamiliar one, but the process of compounding revenues and costs should be familiar to anyone who has purchased an item with borrowed money. The process of compounding costs involves nothing more than the accumulation of the loan amount (or principal) and interest charges over the time span that the item is owned.

Suppose that a farmer purchases a truck for 10,000 dollars. The farmer intends to sell the truck at the end of three years for \$6000. The farmer could have instead used the money to buy a certificate of deposit to mature in three years with a ten percent interest rate compounded annually. At the end of the first year the truck cost the farmer $10,000(1.10) = 11,000$ dollars. At the end of the second year, the truck cost $11,000(1.10) = 10,000(1.10)^2 = 12,100$. At the end of the third year, the truck cost $12,100(1.10) = 10,000(1.10)^3 = 13,310$. If, at the end of the third year, the farmer sells the truck for 6,000 dollars, then the cost of the truck was $13,310 - 6,000 = 7,310$ over the assumed three years of ownership.

If the farmer borrows the money for the purchase of the truck from the bank, then the interest charge on the loan could be used. However, the example becomes more complicated in that the farmer will likely pay back the loan in monthly or annual installments. This means that over the three years, the farmer will have, on the average, owed the bank far less than the full 10,000, and the interest payments would be based on what was actually owed. However, if the truck had not been purchased, the payments the farmer would have made to the bank on the loan over the three years would have instead been put in a savings account which would have earned interest. The interest on the savings account represents an opportunity cost that should also be charged to the truck.

A general formula for compounding costs is :

$$21.19 \quad C = C_{n-1}(1+i) + C_{n-2}(1+i)^2 + \dots + C_0(1+i)^n$$

where C is the total costs incurred to the end of the n th year, C_0 is the cost at the start of the first year, C_{n-1} is the cost incurred at the start of the n th year, C_{n-2} is the cost incurred at the start of the $(n-1)$ th year, n is the number of years, and i is the assumed interest rate.

A similar formula could be used to discount revenues, except that revenues usually occur at the end, rather than the start of the year.

21.6 Polyperiod Production and Marginal Analysis

Marginal analysis can be applied to problems in which there is time between the occurrence of costs and the return of revenue from the sale of the product. In order to do this, compounding or discounting is used in order to compare revenues obtained at the end of a year with costs occurring at the start of the year. The approach can then be expanded to take into consideration production decisions involving enterprises where both costs and returns occur over a period of several years.

Suppose that a farmer incurs production costs for output y_1 at the start of the year, but revenues do not result until the end of the year. Total revenue is:

$$\text{21.20} \quad TR = p_1 y_1$$

where y_1 is output, and p_1 is the price of the output.

Total cost (TC) is a function of output:

$$\text{21.21} \quad TC = c(y_1)$$

Revenues occur at the end of the year, so they are discounted to the start of the year:

$$\text{21.22} \quad PV = TR/(1+i) = p_1 y_1 / (1+i)$$

Costs occur at the start of the year, and need not be discounted.

The profit equation discounted to the start of the year is:

$$\text{21.23} \quad A = TR/(1+i) - TC$$

First order conditions require that the slope of the production function be equal to zero:

$$\text{21.24} \quad dA/dy_1 = [dTR/dy_1][1/(1+i)] - dTC/dy_1 = 0$$

$$p_1/(1+i) = c'(y_1)$$

In a purely competitive model, marginal revenue and price are the same thing. The discounted marginal revenue or price $[p_1/(1+i)]$ must equal the marginal cost $[c'(y_1)]$.

The problem could also be solved for first order conditions at the end of the production season. Costs would be compounded, but revenues occurring at the end of the production season would not be compounded. Total compounded cost would be:

$$\text{21.25} \quad TC(1+i) = c(y_1)(1+i)$$

Profit is:

$$\begin{aligned} \text{¶1.26} \quad A &= TR - TC(1+i) \\ &= p_1 y_1 - c(y_1)(1+i) \end{aligned}$$

The first order conditions again require the slope of the profit function to be zero:

$$\text{¶1.27} \quad dA/dy_1 = p_1 - c'(y_1)(1+i) = 0$$

$$\text{¶1.28} \quad p_1 = c'(y_1)(1+i)$$

$$\text{¶1.29} \quad p_1/(1+i) = c'(y_1)$$

In general, the marginal conditions remain the same regardless of whether the problem is solved at the start or the end of the year.

Now suppose that a farmer produces two commodities in which revenues occur at the end of the year, but costs occur at the start of the year. The farmer has the choice of producing commodity y_1 , which returns revenues in each of the years in a three year planning horizon; or producing commodity y_2 , which does not bring any returns in years one and two, but does bring a large return at the end of year three. Both commodities incur costs for each year that are assumed to be paid at the start of each year. The market rate of interest is i , and the farmer wishes to choose the combination of y_1 and y_2 that will result in maximum discounted profits over the three year period.

The discounted revenue from the sale of y_1 at the end of year 1 is:

$$\text{¶1.30} \quad R_{11} = p_{11} y_1 / (1+i)$$

where R_{11} is the revenue obtained from y_1 at the end of year 1 discounted to the start of year 1, and p_{11} is the price of y_1 at the end of year 1.

The revenue from the sale of y_1 produced during year 2, discounted to the start of year 1 is:

$$\text{¶1.31} \quad R_{12} = p_{12} y_1 / (1+i)^2$$

where R_{12} is the revenue obtained from y_1 at the end of year 2 discounted to the start of year 1, and p_{12} is the price of y_1 at the end of year 2.

The revenue from the sale of y_1 produced during year 3, discounted to the start of year 1 is:

$$\text{¶1.32} \quad R_{13} = p_{13} y_1 / (1+i)^3$$

where R_{13} is the revenue obtained from y_1 at the end of year 3 discounted to the start of year 1, and p_{13} is the price of y_1 at the end of year 3.

Since y_2 does not generate any revenues at the end of years 1 and 2, the revenue from y_2 can be calculated from the formula:

$$\text{¶1.33} \quad R_{23} = p_{23}y_2/(1+i)^3$$

where R_{23} is the revenue obtained from y_2 at the end of year 3 discounted to the start of year 1, and p_{23} is the price of y_2 at the end of year 3.

The present value of all revenues in all periods from the sale of y_1 is:

$$\text{¶1.34} \quad R_1 = R_{11} + R_{12} + R_{13}$$

The present value of all revenues in all periods from the sale of y_2 is:

$$\text{¶1.35} \quad R_2 = 0 + 0 + R_{23}$$

Since revenues from y_2 occur only at the end of period three.

Costs for the production of y_1 and y_2 accrue in each year, despite the fact that y_2 only produces returns in year 3. Costs in each year are assumed to be a function of the output level. Since the costs for year one occur at the start of year one, they need not be discounted. Costs occurring at the start of year two are discounted by the factor $1+i$, and costs occurring at the start of year 3 are discounted by the factor $(1+i)^2$.

Total discounted costs for the production of y_1 are:

$$\text{¶1.36} \quad C_1 = C_{11} + C_{12} + C_{13}$$

where C_1 = total costs over the three year period;

$C_{11} = c(y_{11})$ or the costs of producing y_1 in year one paid at the start of year one;

$C_{12} = c(y_{12})/(1+i)$, or the costs of producing y_1 in year two paid at the start of year two;

and $C_{13} = c(y_{13})/(1+i)^2$, or the costs of producing y_1 in year three paid at the start of year three.

Total discounted costs for the production of y_2 are:

$$\text{¶1.37} \quad C_2 = C_{21} + C_{22} + C_{23}$$

where C_2 = total costs over the three year period;

$C_{21} = c(y_{21})$ or the costs of producing y_2 in year one paid at the start of year 1;

$C_{22} = c(y_{22})/(1+i)$, or the costs of producing y_2 in year two paid at the start of year two;

and $C_{23} = c(y_{23})/(1+i)^2$, or the costs of producing y_2 in year three paid at the start of year three.

Profit is discounted revenue less discounted costs for both products:

$$\text{21.38 } A = R_1 + R_2 - C_1 - C_2$$

The necessary conditions for profit maximization require that the slope of the profit function be equal to zero with respect to both outputs:

$$\text{21.39 } \frac{\partial A}{\partial M_1} = \frac{\partial R_1}{\partial M_1} - \frac{\partial C_1}{\partial M_1} = 0$$

$$\text{21.40 } \frac{\partial A}{\partial M_2} = \frac{\partial R_2}{\partial M_2} - \frac{\partial C_2}{\partial M_2} = 0$$

$$\text{21.41 } \frac{\partial R_1}{\partial M_1} = \frac{\partial C_1}{\partial M_1}$$

$$\text{21.42 } \frac{\partial R_2}{\partial M_2} = \frac{\partial C_2}{\partial M_2}$$

Discounted marginal revenue must equal discounted marginal cost for both outputs. For outputs y_1 and y_2 :

$$\text{21.43 } \frac{\partial R_1}{\partial M_1} = p_{11}/(1+i) + p_{12}/(1+i)^2 + p_{13}/(1+i)^3$$

$$\text{21.44 } \frac{\partial R_2}{\partial M_2} = p_{23}/(1+i)^3$$

$$\text{21.45 } \frac{\partial C_1}{\partial M_1} = c_{1y_{11}} + c_{1y_{12}}/(1+i) + c_{1y_{13}}/(1+i)^2$$

$$\text{21.46 } \frac{\partial C_2}{\partial M_2} = c_{2y_{21}} + c_{2y_{22}}/(1+i) + c_{2y_{23}}/(1+i)^2$$

If second order conditions are met, these conditions would determine the allocation between the production of y_1 and y_2 that would globally maximize discounted profits under the assumed interest rate. These relationships can be rearranged to show that the ratio of discounted marginal revenues to discounted marginal costs should be equal to one in the production of both outputs.

If the farmer were constrained by the availability of inputs required to produce y_1 and y_2 , then the ratio of discounted marginal revenues to discounted marginal costs should be the same for the production of both outputs. However, in this case, the ratio would be equal to a number greater than 1.

21.7 Concluding Comments

This chapter has illustrated a number of ways in which time can be incorporated into economic analysis. The labor, machinery and field time available to a farmer during each period within the calendar of events occurring for a production season are limited, and available time within each period must be allocated consistent with the equal marginal return rule. Time is of even greater concern within a multi-period production framework. Goals and objective of farmers may change as the length of the planning horizon is altered. Application of the equal marginal return rules within a planning horizon encompassing many production periods involves either the compounding or discounting of revenues and costs. However, even within a multiple period framework, the equal marginal return rules still apply.

Problems and Exercises

1. Outline alternative goals a farmer might pursue other than short run profit maximization. Are there possible goals in addition to those listed in this book?
2. Why does compounding and discounting become an inherent part of marginal analysis in a multiperiod framework?
3. Suppose that an enterprise generates \$1000 in revenue in each of 5 years. The interest rate is 9 percent. What is the present value of the stream of revenue generated over the 5 year period?
4. Assuming an interest rate of 10 percent, how much is an acre of land worth that generates \$200 in returns over costs (other than interest and principal payments on the land)? Why are farmers frequently willing to pay more than this value for an acre of land?

5. Suppose the following schedule of revenues and costs

Year	Revenue at end of year	Costs at start of year
1	\$2000	\$1000
2	\$3000	\$2000
3	\$2000	\$4000
4	\$5000	\$1000
5	\$2000	\$1000

Calculate the present value of revenues over costs at the start of year 1.

Calculate the future value of revenue over costs at the end of year 5.