

Chapter 6: Using the NPV Rule

In this chapter we will learn:

- 1) How to construct a pro-forma cash flow statement for a project. (The project NPV is calculated by discounting these cash flows back to time 0.)
- 2) How to account for project interactions.

Steps in the capital investment (capital budgeting) process

1. **Generation of investment proposals** (research/marketing department).

Types of projects (investments associated with real assets)

- A. New products or expansion/modification of existing products
- B. Replacement of equipment or building
- C. Comparison of alternative manufacturing processes
- D. Safety and environmental projects
- E. New advertising expenditures
- F. Abandonment of an existing project
- G. Other

2. Estimation of the **project's (expected) cash flows** (Chapter 6).

In this chapter, we assume that future project cash flows are known with certainty (i.e., risk free). In reality, future cash flows are almost never known with certainty. Therefore, the proper method is to use **expected** future cash flows in the NPV analysis.

Quick review from Chapter 2 notes: Assume a project requires a \$10,000 initial investment that will produce a time one cash flow of either \$11,000 (90% probability) or \$6000 (10% probability). Based on its risk, the appropriate discount rate is 8%. What is the NPV of the project?

Time 0 cash flow =
 Time 1 expected cash flow =
 NPV =

3. **Evaluation of cash flows.** Determine the riskiness of the project's expected cash flows and the project's opportunity cost of capital (Chapters 7 - 9). Higher risk cash flows have a higher opportunity cost of capital.

Each component of the project's cash flows could have a different risk level (and therefore a different opportunity cost of capital). This could have a big impact on project NPV. Example:

	0	1	2
Initial investment	-\$1050		
Risk-free cash flow (5%)			
High-risk cash flow (15%)			
Project cash flow	-\$1050	\$600	\$600

Project NPV (case #1, low risk cash flow at time one, high risk at time 2) =
 Project NPV (case #2, high risk cash flow at time one, low risk at time 2) =

4. **Project acceptance or rejection** based on the NPV method (Chapter 5). To calculate the project NPV, discount the expected future project cash flows at the opportunity cost of capital and subtract the initial investment.

Projects (investments in real assets) versus **investment in financial assets**. A project can have a positive or a negative NPV (although projects with a positive NPV are difficult to find). What about investments in financial assets?

Assuming the financial markets are perfect, efficient, and in equilibrium, the present value of the future expected cash flows from a financial asset is equal to the financial asset's current price.

Based on this, what is the NPV from an investment in the financial markets?

How does competition explain this?

- Current stock price = \$9
- Expected dividend in one year = \$1
Growth rate in dividends (in perpetuity) = 2%
Opportunity cost of capital (discount rate) = 12%

What is the present value of the stock's future dividends?

What is the expected return for the stock?

How should competition affect the stock price?

- Change the current stock price to \$11

What is the present value of the stock's future dividends?

What is the expected return for the stock?

How should competition affect the stock price?

Based on this discussion, what is the equilibrium stock price?

What is the expected return for the stock using its equilibrium stock price?

What implications does this have for allocating firm resources to evaluating investments in real versus financial assets?

Estimation of cash flows is probably the most important task in capital budgeting. Examine cash flows (rather than income)!

Need cash to pay for operating expenses, capital investment, interest, dividends. Remember, income taxes are an additional cash flow (a negative cash flow).

Example: Assume a project requires a \$10,000 initial investment (for the purchase of inventory). The inventory will be sold at the end of the period for \$16,000 (sold on credit, collected at time two). Cash wages paid to employees at time one = \$4000. Income tax rate = 34%. Corporation is an accrual-basis taxpayer.

Income Statement for period one

Revenue
CGS
Salary expense
Earnings before taxes (or taxable income)
Income Tax
Net Income

Cash flows by period

Time zero
Time one
Time two

A Note about Corporate Income Taxes - Taxes on project income should be computed based on the **corporation's marginal** income tax rate. Simple example (no state income tax)

	Project accepted	Project rejected
Taxable income	\$71,000	\$70,000
Income tax		

Corporate Tax Rates (Federal)

Bracket	Tax Rate
\$0 - \$50,000	15%
\$50,000 - \$75,000	25%
\$75,000 - \$100,000	34%
\$100,000 - \$335,000	39%
\$335,000 - \$10,000,000	34%
\$10,000,000 - \$15,000,000	35%
\$15,000,000 - \$18,333,333	38%
Above \$18,333,333	35%

What is the additional (marginal) income tax the firm has to pay if the project is accepted?

What is the marginal income tax rate for the project?

Make sure you include the marginal income taxes as an additional project cash flow

A corporation's marginal tax rate might change from year to year. Why would this rate change?

- 1)
- 2)

The marginal tax rate should be the combination of federal and state marginal income tax rates.

State corporate tax rates vary from state to state (and possibly from city to city within the state). The Kentucky corporate income tax rate schedule is:

Bracket	Tax Rate
\$0 - \$25,000	4%
\$25,000 - \$50,000	5%
\$50,000 - \$100,000	6%
\$100,000 - \$250,000	7%
Above \$250,000	8.25%

The federal tax code allows you to deduct state income taxes in computing federal taxable income. However, the Commonwealth of Kentucky does not allow you to deduct state income taxes.

Therefore: Federal taxable income = State taxable income minus state income taxes

Example #1. The year is 2002. What is the combined federal and state tax for a project if the corporation has \$60,000 of state taxable income without the project and \$60,001 of state taxable income with the project? (In other words, the project produces \$1 of additional **state** taxable income.)

Long method: Calculate state and federal taxes at the two different amounts of taxable income

A. What is the state and federal tax with \$60,000 of state taxable income?

2002 state taxable income = \$60,000
2002 state income tax equals

Assume that the corporation accurately estimated the 2002 state income tax and paid this amount to Kentucky during 2002. This means that there is no balance due, or refund owing, on the 2002 Kentucky income tax return filed in 2003.

- What are the consequences of having a balance due?
- What are the consequences of receiving a refund?

This 2002 state income tax payment is now a deduction in computing Federal taxable income for 2002.

2002 federal taxable income = \$60,000 less state income tax payment =
2002 federal income tax equals

Total 2002 federal and state income taxes with \$60,000 of state taxable income equals

B) Add one dollar of state taxable income and recalculate state and federal tax.

2002 state taxable income = \$60,001
2002 state income tax equals

2002 Federal taxable income = \$60,001 less state income tax =
2002 Federal income tax equals

Total 2002 federal and state income taxes with \$60,001 of state taxable income equals

C) How much additional income tax is paid when the firm earns one more dollar?

Easy formula for the computation of the combined federal and state marginal income tax rate. (This formula only works if the corporation's federal and state tax bracket is the same both with and without the project. If either the federal or state tax bracket changes with the addition of the project's income, then the **long method** must be used!)

T = state tax bracket + federal tax bracket (1 - state tax bracket)

Applying this formula to the above facts, the marginal income tax rate for the project is _____.

How much additional income tax is paid when the firm earns one more dollar?

Example #2. In 2003, state taxable income will be \$5,000,000 without the project and \$5,100,000 with the project. Does the project cause a change in tax brackets? How much additional income taxes must be paid?

Example #3. In 2004, state taxable income will be \$10,400,000 without the project, and \$11,400,000 with the project. Does the project cause a change in tax brackets?

For purpose of the examples given in class, we will assume that the corporation's marginal tax rate (**T**) is 34%.

Determination of Project Cash Flows

When evaluating a project, the relevant cash flows are the **incremental (or marginal) cash flows**. **Incremental cash flows** equals the difference in the firm's cash flows **with and without** the project.

We calculated the **incremental** corporate income taxes in the above examples.

1. An example of incremental cash flows

	0	1	2	3
Firm cash flows with project	\$900,000	\$1,050,000	\$1,050,000	\$1,050,000
Firm cash flows without project	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000
Project cash flows				

2. Average profitability versus incremental cash flows – Always evaluate projects based on the incremental cash flows (without regard to average profitability)

- **Example** – Assume that (on average) Factory X is barely profitable. Factory Y is much more profitable.

Expected yearly Factory X cash revenues = \$1,010,000
Expected yearly Factory X cash expenses = \$1,000,000
Expected yearly Factory X cash flow = \$10,000

Expected yearly Factory Y cash revenues = \$1,100,000
Expected yearly Factory Y cash expenses = \$1,000,000
Expected yearly Factory Y cash flow = \$100,000

A machine (critical to operating Factory X) breaks down. Without the machine, the factory cannot operate. With the machine, which costs \$30,000, Factory X should operate for 5 more years (producing expected cash flows of \$10,000 a year. The new machine does not affect Factory Y. So, Factory Y is also expected to continue operating (for 5 more years) producing expected cash flows of \$100,000 year.)

Should the machine be replaced?

	0	1	2	3	4	5
Firm cash flows with project						
Firm cash flows without project						
Project cash flows						

3. Incidental effects. When evaluating a new project, you must consider the incidental effects of the project's acceptance on the firm's cash flows from other projects.

- **Example:** Hallmark Cards Inc. is contemplating a new greeting card. Unlike the traditional greeting cards that retail for \$2, these cards will sell for 50 cents.
- **Example:** Compare the relative value of hosting the Super Bowl for Minneapolis and Miami.

In the previous discussion, we have emphasized analyzing *incremental* cash flows for a project

Firm cash flows with the project - Firm cash flows without the project = Project cash flows

The following discussion emphasizes other issues in determining cash flows. To simplify these tough subjects, I will only show the incremental cash flows from the acceptance of the project (rather than showing the detailed analysis of firm cash flows with the project versus firm cash flows without the project).

4. **Working capital adjustments.** Increases and decreases in working capital need to be considered in calculating cash flow.

Working capital = short-term assets (e.g., cash, A/R, inventory) minus short-term liabilities (e.g., A/P, taxes payable, wages payable)

An increase in working capital is treated as a negative cash flow

A decrease in working capital is treated as a positive cash flow

Analysis (for calculating yearly cash flows for a project)

Step 1 – Calculate yearly taxable income

Step 2 – Calculate the corporate income tax

Step 3 – Subtract income tax from taxable income

Step 4 – Calculate the amount of working capital as of the end of each year

Step 5 – Calculate the change in working capital from year to year

Step 6 – If there was an increase in working capital from the previous year, this increase in working capital is a negative adjustment in calculating cash flow. If there was a decrease in working capital from the previous year, the decrease in working capital is a positive adjustment in calculating cash flow.

Step 7 – Make the working capital adjustment to calculate the cash flow for the year (taxable income – income tax +/- working capital adjustment = cash flow)

An example

- An accrual-basis corporation is considering a project. In the project, the firm buys inventory and resells the inventory to the public. Assume the firm wants to maintain \$100,000 of inventory at all times throughout the 5-year project.
- The project will produce revenue of \$12,000,000, expenses (cost of goods sold) of \$9,000,000, and taxable income of \$3,000,000 for year one. (No depreciation – that is addressed later in the notes.)
- Revenues, expenses, and taxable income are expected to grow at 10% a year for five years (year 5 is the last year of the project). The growth is from an increase in sales (i.e., assume the inflation rate equals 0).
- Cash of \$50,000 is needed to start the project and is maintained throughout the life of the project.
- End of period accounts receivable balance equal to 1/12 of the year's revenue, and account payable equal to 1/12 of the year's expenses. Income taxes are paid in the current year

Questions

- Fill in the missing cells in the table to determine yearly project cash flow

	0	1	2	3	4	5	6
Revenue	12,000,000	13,200,000	14,520,000	15,972,000	17,569,200		
Cost of Goods Sold	-9,000,000	-9,900,000	-10,890,000	-11,979,000	-13,176,900		
Taxable Income	3,000,000	3,300,000	3,630,000	3,993,000	4,392,300		
Income Tax	-1,020,000	-1,122,000	-1,234,200	-1,357,620	-1,493,382		
Subtotal	1,980,000	2,178,000	2,395,800	2,635,380	2,898,918		
Adjustments							
Working Capital							
Cash Flow							
Balances							
Cash	50,000	50,000	50,000	50,000	50,000	0	0
Inventory	100,000	100,000	100,000	100,000	100,000	0	0
Accounts Receivable	0	1,000,000	1,100,000	1,210,000	1,331,000	1,464,100	0
Accounts Payable	0	750,000	825,000	907,500	998,250	1,098,075	0
Working Capital							

- What is cash revenue for year one?
What are cash expenses for year one?
Use these to verify year one cash flow
- Why is it reasonable to assume that accounts receivable and accounts payable increase as a percentage of revenue and expenses?

5. **Ignore sunk costs.** Project cash flows should not consider *past* expenditures for equipment, land, or other assets that will be used on the project. These are *sunk costs* and are irrelevant to the project selection decision. Only future (incremental) cash flows should be considered in your analysis of the project.

Example – Equipment purchased several years ago for \$100,000 will be used on a new project. The \$100,000 is a sunk cost and should be ignored in calculating the NPV for the project.

Example – Assume that the company purchased the equipment several years ago for \$100,000, but instead of paying cash, the firm agreed to make ten payments of \$10,000 (once a year). It currently has one remaining payment of \$10,000. The first \$90,000 of payments are clearly a sunk cost. What about the \$10,000?

6. **Take into account opportunity costs.** The *current* market value of equipment, land, and any other assets that will be used on the project should be considered as a time zero cash outflow. **Why?** If the assets are not used on the project, they can be sold. Therefore, by using the assets on the project, the firm is forgoing the opportunity to receive the positive cash flow from the sale of those assets. (Make sure you take into account the tax effects of sale.)

Example. A project will use equipment purchased several years ago for \$100,000. This asset has been fully depreciated, and therefore has a tax basis (tax book value) of \$0. The current market value for the asset is \$20,000. If the asset is not used on the project, it will be sold (it has no other use to the corporation). If sold, the corporation will pay tax at the rate of 34% on the \$20,000 gain. How should the use of this asset be considered in evaluation of the project?

Sunk cost. The \$100,000 purchase price is a sunk cost and is irrelevant to the project selection decision.

Opportunity cost. The following amount will be treated as a time zero negative cash flow (in addition to any other time zero cash flows).

Current market value =
Tax basis =
Tax gain (or loss) on sale =
Tax effect of sale =
Time zero “opportunity cost” cash outflow =

What if the accumulated tax depreciation was \$40,000 (leaving a tax basis of \$60,000)?

Current market value =
Tax basis =
Tax gain (or loss) on sale =
Tax effect of sale =
Time zero “opportunity cost” cash outflow =

7. **Ignore allocated overhead.** Accountants often allocate indirect expenditures to the firm’s revenue sources. These allocated expenses should be ignored in the analysis. An exception would be if the acceptance of the project causes these indirect expenditures to change.

Example. Corporate headquarter expenses (totaling \$10 million per year) are allocated to the various projects of the firm based on the revenue of these projects. Project X’s allocation is \$100,000.

If corporate headquarter expenses are the same whether the project is accepted or rejected then _____. If acceptance of the project requires the firm's corporate headquarters to hire two clerical staff at \$20,000 per year, then _____.

8. **Use tax depreciation in the calculation of project cash flows.** Tax depreciation of an asset is not a cash flow and therefore it does not directly affect project cash flows. However, tax depreciation will affect the amount of income tax the firm will have to pay if the project is selected. Therefore, because of this indirect effect, tax depreciation must be considered in the analysis of the project. Make sure you used *tax* depreciation to calculate the tax effects of the asset's depreciation (rather than *accounting* depreciation). **Example:**
- A. Acceptance of a project will require the purchase of equipment on January 1. Cost = \$1500 (3-year tax depreciation life - see page 130 of the textbook). The equipment is expected to run for four more years with a no salvage value at the end of its useful life. The corporation is a calendar-year taxpayer.
 - B. Project sales = 1000 units sold per year, current price = \$1 per unit. Customers pay cash.
 - C. Ignore expenses other than depreciation and income taxes. Assume the marginal tax rate is 34%.
 - D. Expected inflation rate = 2.9412% per year.
 - E. Discount rate = 5%
 - F. The firm uses straight-line depreciation for financial statement reporting purposes. Therefore accounting depreciation = _____ per year. If the asset had a \$500 salvage value, then accounting depreciation would be _____ per year.

Analysis (fill in the missing numbers for year 2)

	0	1	2	3	4
Unit sales		1000	1000	1000	1000
Price per unit		1.02941		1.09086	1.12294
Revenue		1029.41		1090.86	1122.94
Depreciation		-499.95		-222.15	-111.15
Taxable Income		529.46		868.71	1011.79
Income tax (at 34%)		-180.02		-295.36	-344.01
Subtotal		349.44		573.35	667.78
Adjustments					
Depreciation		499.95		222.15	111.15
Initial Investment	-1500.00				
Cash Flow	-1500.00	849.39		795.50	778.93
NPV at 5% = 1476.95					

9. **Expected inflation** is an important component of the determination of the project NPV.
- Use the **nominal discount rate** to discount cash flow estimates that **include inflation**, or
 - Use the **real discount rate** to discount cash flow estimates that **do not account for inflation** (i.e., cash flows in **constant dollars**).

Equation. $(1 + r_{\text{nominal}}) = (1 + r_{\text{real}})(1 + \text{expected inflation})$
 What is the **real** discount rate in the previous problem?

Notice: Inflation affects the revenues in the above project. However, since tax depreciation is based on the historical cost of an asset, inflation does not affect tax depreciation. Because of this rule, a special adjustment must be used in the calculation of real cash flows

Using real cash flows, method #1. Use real (constant dollar) cash flows for everything except for depreciation. Discount real (constant dollar) cash flows at the real discount rate. Discount depreciation cash flows at the nominal rate.

METHOD ONE

	0	1	2	3	4
Unit sales		1000.00	1000.00	1000.00	1000.00
Price per unit		1.00	1.00	1.00	1.00
Revenue		1000.00	1000.00	1000.00	1000.00
Income tax (at 34%)		-340.00	-340.00	-340.00	-340.00
Cash Flow		660.00	660.00	660.00	660.00
PV at 2% = 2513.10					
Depreciation income tax deduction		-499.95	-666.75	-222.15	-111.15
Cash flow from tax deduction (at 34%)		169.98	226.70	75.53	37.79
PV at 5% = 463.84					

$NPV = -\$1500 + \$2513.10 + \$463.84 = \1476.95

Using real cash flows, method #2. Adjust depreciation deduction for inflation. Discount all cash flows at the real rate.

Step 1. Adjust yearly depreciation by anticipated inflation

Year 1: $(\$1500 * 0.3333) / 1.029412^1 = \485.67

Year 2: $(\$1500 * 0.4445) / 1.029412^2 = \629.19

Year 3: $(\$1500 * 0.1481) / 1.029412^3 = \203.65

Year 4: $(\$1500 * 0.0741) / 1.029412^4 = \98.98

METHOD TWO

	0	1	2	3	4
Unit sales		1000.00	1000.00	1000.00	1000.00
Price per unit		1.00	1.00	1.00	1.00
Revenue		1000.00	1000.00	1000.00	1000.00
Depreciation		-485.67	-629.19	-203.65	-98.98
Taxable Income		514.33	370.81	796.35	901.02
Income Tax (at 34%)		-174.87	-126.07	-270.76	-306.35
Subtotal		339.46	244.73	525.59	594.67
Adjustments					
Depreciation		485.67	629.19	203.65	98.98
Initial Investment	-1500.00				
Cash Flow	-1500.00	825.13	873.93	729.24	693.65
NPV at 2% = 1476.95					

Sale of an asset - nominal versus real discount rates. A similar correction can be made to account for the sale of an asset. Example - Assume that the asset described above is sold at the end of the 3rd year for:

\$500 (real sale's price)

$(\$500)(1.029412^3) = \545.43 (nominal sale's price).

PV of the after-tax cash flow using the nominal sale's price

Nom. Sale's Price	\$545.43
Less: Adjusted Basis	<u>\$222.23</u> = $\$1500 - \$499.95 - \$666.75 - \111.08
Gain on Sale	\$323.20

Tax at 34%	<u>\$109.89</u>
Net Cash Flow	\$435.54
PV (3-years at 5%)	\$376.23

PV of the after-tax cash flow using the real sale's price

Real Sale's Price	\$500.00
Less: Adjusted Basis	<u>\$203.72</u> = \$1375.07 - \$458.31 - \$611.22 - \$101.82
Gain on Sale	\$296.28
Tax at 34%	<u>\$100.74</u>
Net Cash Flow	\$399.26
PV (3-years at 2%)	\$376.23

Calculation Hints. (Only one-half of the regular tax depreciation is allowed in the year of sale.)

$$\begin{aligned}
 \$1500 / 1.029412^3 &= \$1375.07 \\
 \$1375.07 \times 0.3333 &= \$458.31 \\
 \$1375.07 \times 0.4445 &= \$611.22 \\
 \$1375.07 \times 0.1481 \times 0.5 &= \$101.82
 \end{aligned}$$

10. Financing cash flows – Notice that we have assumed that the project has been financed with available cash. This might not always be the case. Sometimes the company might borrow money or issue stock to raise the funds needed undertake the project.

The format for analysis is the same. However, an adjustment to the project NPV might need to be made for financing effects. The proper analysis (discussed in Chapter 19) looks like this:

$$\begin{aligned}
 \text{Base case NPV} &= \$A \\
 \text{Financing NPV} &= \$B \\
 \text{Project NPV} &= \$A + \$B
 \end{aligned}$$

11. Evaluation of projects by foreign corporations

- A. Use the foreign country's currency
- B. Use the foreign country's inflation rate
- C. Use the foreign country's tax laws (tax rates, depreciation schedules, etc.)
- D. Use the foreign country's opportunity cost of capital

PROJECT INTERACTIONS (Situations when the simple NPV rule needs modification.)

1. **Optimal Timing of Investment:** Should the project be undertaken **now**, or should it be delayed and undertaken **in the future**? **Solution:** Which alternative yields the highest NPV?

Example: Tree Harvests (single harvest):

Assumptions:

- A. Young trees grow faster than mature trees. Assumption: board feet = 80 bd. feet x square root (age). Trees are currently 10 years old.
- B. Price of lumber increasing at 2.9412% per year. Current price = \$300 per 1000 board feet (or \$0.30 per bd. foot).
- C. Cost of harvest included in sale's price of lumber.
- D. Use a 5% nominal discount rate.

Age	Board Feet	Lumber Price	Sale's Price	PV (5%)	Return
10	252.9822	\$ 300.0000	\$ 75.8947	\$ 75.8947	
11	265.3300	\$ 308.8235	\$ 81.9401	\$ 78.0382	7.97%
12					
13	288.4441	\$ 327.2568	\$ 94.3953	\$ 81.5422	7.14%
14	299.3326	\$ 336.8820	\$ 100.8398	\$ 82.9611	6.83%
15	309.8387	\$ 346.7903	\$ 107.4490	\$ 84.1891	6.55%
16	320.0000	\$ 356.9900	\$ 114.2368	\$ 85.2453	6.32%
17	329.8485	\$ 367.4897	\$ 121.2159	\$ 86.1459	6.11%
18	339.4113	\$ 378.2982	\$ 128.3987	\$ 86.9053	5.93%
19	348.7119	\$ 389.4246	\$ 135.7970	\$ 87.5360	5.76%
20	357.7709	\$ 400.8783	\$ 143.4226	\$ 88.0490	5.62%
21	366.6061	\$ 412.6688	\$ 151.2869	\$ 88.4543	5.48%
22	375.2333	\$ 424.8061	\$ 159.4014	\$ 88.7607	5.36%
23	383.6665	\$ 437.3004	\$ 167.7775	\$ 88.9760	5.25%
24	391.9184	\$ 450.1622	\$ 176.4268	\$ 89.1075	5.16%
25	400.0000	\$ 463.4023	\$ 185.3609	\$ 89.1618	5.06%
26	407.9216	\$ 477.0318	\$ 194.5915	\$ 89.1446	4.98%
27	415.6922	\$ 491.0621	\$ 204.1307	\$ 89.0615	4.90%
28	423.3202	\$ 505.5051	\$ 213.9905	\$ 88.9175	4.83%

Which year should the company harvest the trees?

We assumed away any uncertainty. However, what if estimates are wrong (i.e., prices only increase at 2% a year, trees grow slower than expected, what if there is a fire)? How should uncertainty be included in the analysis?

In addition, a **multiple harvest** problem is much more complicated. (Earlier harvest allows the next set of trees to be planted earlier. This is advantageous because young trees grow faster.)

2. **Evaluating two mutually exclusive projects with unequal lives.** - The problem discussed in this section only applies in a special case: when you are evaluating mutually exclusive projects with unequal lives and when each project will be replaced with a new identical project at the end of its useful life (in perpetuity).

If the two projects are not mutually exclusive, then _____. If the mutually exclusive projects will not be replaced at the end of their lives, then _____.

Example – a firm is considering purchasing one of two different machines. Using a 2% real discount rate, which is the best machine to purchase if these machines will be replaced at the end of their useful lives in perpetuity? **Note:** The book’s example uses only costs (or cash outflows). This example is more general (it uses both positive and negative cash flows).

	Cost	Life	Cash Flow
Asset X	\$6000	3 years	\$2600 per year
Asset Y	\$4000	2 years	\$2600 per year

First – let’s discuss the wrong method to analyze these two projects

	0	1	2	3
Asset X				
Asset Y				

NPV_X (2%) (one purchase) =
 NPV_Y (2%) (one purchase) = \$1048.06

To properly analyze - Make the two machines comparable by equalizing the lives. Two methods: **Replacement Chains** and **Equivalent Annual Cash Flows**.

A. Replacement Chains - Lives equal in year 6 (also equal in year 12, year 18, year 24, etc.)

Asset	0	1	2	3	4	5	6
X	-6000	2600	2600	-3400	2600	2600	2600
Y	-4000	2600	-1400	2600	-1400	2600	2600

$NPV_X(2\%) = \$2909.79$. $NPV_Y(2\%) = \$3023.66$. Pick Y, it has the higher NPV over 6 years (also over 12, years, 18 years, 24 years, etc.)

B. Equivalent annual cash flow (EAC)

1) Use the single purchase NPV in the following formula:

$NPV / PV \text{ Ordinary Annuity Factor for } N \text{ years}$

PV annuity factor, 2%, 3 years = $2.883883 = [1 - 1/(1.02)^3] / 0.02$

PV annuity factor, 2%, 2 years = $1.941561 = [1 - 1/(1.02)^2] / 0.02$

$EAC_X =$

$EAC_Y = \$1048.06 / 1.941561 = \539.80

Pick Y because it has the higher EAC.

The PV of a perpetual stream of asset X (or Y) purchases is:

X:

Y: $\$539.80 / 0.02 = \$26,990.10$

Therefore, over the long term, asset Y is _____ more valuable than asset X.

Note: The above analysis assumes no inflation. Therefore, you would want to use a **real** as opposed to a **nominal** discount rate.

- If a real discount rate is used, make sure that the depreciation deduction is adjusted for inflation (see "method two" above).
- If a nominal discount rate is used, project cash flows would need to be adjusted for inflation. See book for a discussion.

Note: The above analysis assumes that the real cost and cash flows associated with operating the two assets are constant. However, it is likely that the real cost will change in the future.

For example, assume that the *real* cost of the machine falls at a rate of 20% per year. How should this additional piece of information affect the analysis?

3. **Replacement Analysis** – Should an old machine be replaced with a new machine *today or in the future*?

- Selling the old machine this year will probably result in a higher sales price than if it is sold next year.
- A new machine may run more efficiently, require less maintenance, produce more goods, etc.

To analyze, compare cash flows assuming machine is replaced today with cash flows assuming the machine is replaced in the future (e.g., in one year). Use the new machine's equivalent annual annuity (EAC) in the calculations. **Example:**

- A. Today is January 1 (calendar-year taxpayer). You purchased the “old” machine nine years ago for \$100,000 (MACRS class life of 7 years). It will last five more years and will produce an after-tax cash flow of \$15,000 per year at the end of each year. (At the end of its useful life, the old machine will be worthless.)
- B. The old machine is fully depreciated and therefore has a tax book value of \$0. Its current market value = \$70,000. Market value in one year = \$57,000.
- C. New machine (MACRS class life of 7 years) will cost \$150,000 and has a remaining useful life of 10 years. It produces an after-tax cash flow of \$40,000 per year. (The \$40,000 per year includes the cash flow effects of the depreciation deductions and sale at the end of its useful life.) At the end of its useful life, an identical new machine will be purchased with identical cash flows. (Replacement every 10 years will continue in perpetuity.) EAC for new machine (using a 2% discount rate) = \$23,301.02.
- D. Marginal tax rate: 34%
- E. All numbers are in constant dollars (i.e., no adjustment for inflation). Use a 2% real discount rate to determine present values.

Analysis:

- A. What are the cash flows if you replace the old machine today (instead of in one year)?
- Cash flows for new machine. (Hint: Replace with EAC.)
 - Sell the old machine at current salvage value of \$70,000. Time 0 cash flow = $\$70,000 - (\$70,000)(0.34) = \$46,200$
- B. What are the cash flows if you replace the old machine in one year (instead of today)?
- Cash flows of the new machine (starting at time 1). (Hint: As before, replace with EAC.)
 - Operate the old machine for one more year and therefore receive the \$15,000 cash flow one more year. This is a time 1 cash flow.
 - Sell the old machine in one year (at \$57,000). Time 1 cash flow = $\$57,000 - (\$57,000)(0.34) = \$37,620$.
- C. Summary of cash flows

Initial presentation of cash flows

	0	1	2	3	4	5	→
Replace now	-150000	40000	40000	40000	40000	40000	40000
	46200						
Replace in 1 year		-150000	40000	40000	40000	40000	40000
		15000					
		37620					
Project cash flow	-103800	137380	0	0	0	0	0

Replace “new machine” cash flows with EAC

	0	1	2	3	4	5	→
Replace now							
	46,200						
Replace in 1 year							
		15000					
		37620					
Project cash flow							

D. Calculate NPV = \$46,200 + (\$23,301.02 - \$15000 - 37620) / 1.02 = \$17,455.90.

Since NPV is positive, then **replace machine**. (NPV of “replace now” cash flows > NPV of “replace in one year” cash flows.)

4. Cost of Using Excess Capacity

Basic Issue - Does the use of the "excess capacity" of a machine cause it to wear out earlier? If so, consider the **additional** cost of an early purchase of a new replacement machine.

Example:

A. **Preliminary calculation of the project NPV.** A project requires an initial investment of \$100,000 and produces real cash flows of \$2,700 per year in perpetuity. Using a 2% real discount rate, the **preliminary** project NPV is \$35,000.

In addition to the above cash flows, this project will cause an old machine that the firm owns to wear out two years early.

B. Old machine will last 4 years without the project (low usage), but only 2 years with the project (high usage). Its salvage value is \$0 once it wears out. After tax costs of operating the old machine over 2 years (high usage) or 4 years (low usage) are as follows:

<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
	-10198	-10198	-10198	-10198
	-20000	-20000		

PV of operating costs (at 2%) assuming low usage =
 PV of operating costs (at 2%) assuming high usage =

Notice that these numbers are the same, and therefore can be ignored in this problem. (This will not always be the case!)

C. The replacement machine has a life of 4 years and will be replaced every 4 years with the same costs. The new replacement machine has more than enough capacity to handle the new project and all existing projects. After-tax **costs** of purchasing and operating the new machine are:

<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
-50000	-10000	-10000	-10000	-10000

(Note: time 0 in the above cash flow statement is the date of the purchase of the replacement machine – i.e., either time 2 or time 4.)

PV of a single purchase (at the real discount rate of 2%) = -\$88,077.29, EAC = -\$23,131.19

Analysis – The preliminary calculation of the project NPV is \$35,000. But, the costs of wearing out the old machine needs to be determined and used to calculate the correct project NPV.

The new replacement machine will either be purchased at the end of year 2 (project accepted) or at the end of year 4 (project rejected).

- PV at 2% of -\$23,131.19 at the end of year 3 = -\$21,797.03
- PV at 2% of -\$23,131.19 at the end of year 4 = -\$21,369.64
- Total PV = -\$43,166.68

- Preliminary NPV = \$35,000
- Adjustment for operating old machine at “high usage” = \$0
- Adjustment for wearing out old machine = -\$43,166.68
- Corrected NPV = -\$8,166.68. **Reject Project**

Alternative: Accept project and abandon in year 2 (or consider scaling back project) to avoid early purchase.

5. **Fluctuating Load Factors** can be summarized as follows - consider all alternatives. Read this section on your own. Not on the test.

Chapter 6 Review Questions

1. Know how to calculate the NPV of a project based on its expected future cash flows.
2. Why is it important to calculate the NPV of a project but not the NPV of an investment in the capital/money markets (e.g., stock, bonds, Treasury Securities)? Note: In discussing this topic, I assumed that the financial markets were perfect, efficient and in equilibrium.
3. What is a corporation's **marginal** income tax rate? Be able to use a federal and state tax bracket schedule and a corporation's taxable income (with and without the project) to calculate a corporation's marginal income tax rate.
4. What are some of the reasons a project's yearly income could be different than its yearly cash flow? Which should be used (future income or future cash flows) in determining the NPV of a project?
5. What is meant by a project's **incremental** cash flows? How is this distinguished from **average** cash flows?
6. What is meant by **incidental effects**? How should incidental effects be considered in analyzing a project?
7. Know how to calculate working capital adjustments in order to determine project cash flows.
8. What is a sunk cost and why should it be ignored? What is an opportunity cost and how should it be considered in the analysis of a project? Know how to calculate the after-tax opportunity cost of an asset used in a project.
9. How should allocated overhead be considered in analyzing a project?
10. Know how to compute tax depreciation for depreciable assets used on a project.
11. Know the formula relating nominal interest rates (nominal discount rates), real interest rates (real discount rates), and the expected inflation rate. Be able to calculate one value given the other two values. Know what “real” cash flows and “nominal” cash flows mean. Remember that real discount rates should be used to discount real cash flows and nominal discount rates should be used to discount nominal cash flows.
12. Remember that unadjusted depreciation benefits must be discounted using nominal discount rates. Know how to adjust depreciation so that depreciation benefits can be discounted using a real discount rate.
13. Know how to calculate the present value of nominal after-tax salvage values. Know how to adjust salvage values so that they can also be discounted using a real discount rate.
14. Understand (in brief) the terms ‘base-case’ NPV and financing NPV.
15. Understand how a foreign company should evaluate a project.
16. Know when trees should be harvested.

17. Know how to use replacement chains and equivalent annual cash flows (EAC) to evaluate two mutually exclusive projects with uneven lives. Remember that this problem only exists if the two projects are mutually exclusive and the 'machines' will be replaced (in perpetuity) at the end of their useful lives. Remember that the analysis usually uses real (constant dollar) cash flows, so a real discount rate (with a depreciation / salvage value adjustment) must be used. Intuitively, how should a project with increasing (or decreasing) real cash flows be handled?
18. Know when a machine should be replaced.
19. Similar to the example given in class, know how to incorporate the cost of using excess capacity.

Chapter 6 Practice Problems

1. Textbook "quiz" questions 1 – 10. (*Answers are at the back of the textbook*)
2. Assume that a project requires an initial investment of \$5000. The time 1 cash flow is either \$12,000 (10% probability), \$6000 (50% probability), or \$2500 (40% probability).
 - A. What is the time one expected cash flow for the project? *\$5200.00*
 - B. What is the NPV of the project using a 10% discount rate? *-\$272.73*
3. Assume that a corporation's 2002 state taxable income is \$80,000 without the project and \$81,000 with the project. As in class, assume that the firm will pay its state income taxes that it owes for 2002 during 2002. This means that these state income taxes will be a deduction in computing federal taxable income for 2002.

There is no change in tax bracket in this problem!

- A. What is the corporation's state income tax if the project is rejected? *\$4050*
- B. What is the corporation's state income tax if the project is accepted? *\$4110*
- C. How much more state income tax must the corporation pay if the project is accepted? *\$60*
- D. What is the corporation's federal income tax if the project is rejected? *\$14,073*
- E. What is the corporation's federal income tax if the project is accepted? *\$14,392.6*
- F. How much more federal income tax must the corporation pay if the project is accepted? *\$319.60*
- G. How much more federal and state income tax must the corporation pay if the project is accepted? *\$379.60*
- H. Use the simple formula to calculate the additional federal and state taxes that the corporation must pay if the project is accepted.

$$T = (0.06) + (1 - 0.06)(0.34) = 0.37960. \text{ Multiple } T \text{ times the incremental state taxable income generated by the project. } 0.37960 * \$1000 = \$379.60.$$

4. Assume that a corporation's 2002 state taxable income is \$78,000 without the project and \$79,000 with the project. As in class, assume that the firm will pay its state income taxes that it owes for 2002 during 2002. This means that these state income taxes will be a deduction in computing federal taxable income for 2002. **There is a change in the federal tax bracket in this problem!**
 - A. What is the corporation's state income tax if the project is rejected? *\$3930*
 - B. What is the corporation's state income tax if the project is accepted? *\$3990*
 - C. How much more state income tax must the corporation pay if the project is accepted? *\$60*
 - D. What is the corporation's federal income tax if the project is rejected? *\$13,517.50*
 - E. What is the corporation's federal income tax if the project is accepted? *\$13,753.40*
 - F. How much more federal income tax must the corporation pay if the project is accepted? *\$235.90*
 - G. How much more federal and state income tax must the corporation pay if the project is accepted? *\$295.90*
 - H. You cannot use the simple formula to calculate the additional federal and state taxes because the federal tax bracket changes if the project is accepted (from the 25% bracket to the 34% bracket)!
5. Today is January 1, 2002. ABC Inc. (a calendar-year taxpayer) will have \$70,000 of state taxable income in 2002 if it does not take Project Z. If ABC takes Project Z, its state taxable income will be \$72,000. How much MORE

income tax (both federal and state) will the firm need to pay if it takes the project? (As in class, assume that the firm will pay its state income taxes that it owes for 2002 during 2002. This means that these state income taxes will be a deduction in computing federal taxable income for 2002.) \$590

6. How much **more** income tax (both federal income tax and Kentucky income tax) will XYZ Inc. need to pay for the year 2002 if it takes Project A.

2002 Kentucky State taxable income if the firm does not take Project A = \$1,988,000
 2002 Kentucky State taxable income if the firm takes Project A = \$2,000,000

As in class, assume that the firm accurately estimates and fully pays its state income tax during the year 2002 to the state of Kentucky. Hint: Since you do not change tax brackets in this problem, you can use the 'easy' formula.

Answer = \$4,733.40

7. Using the following, what are the project's cash flows?

	0	1	2	3
Firm cash flows with project	\$850,000	\$1,075,000	\$1,075,000	\$975,000
Firm cash flows without project	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000
Project cash flows	<i>-\$150,000</i>	<i>\$75,000</i>	<i>\$75,000</i>	<i>-\$25,000</i>

8. Similar to the example of working capital adjustments in the notes, calculate the working capital balance, working capital adjustment, and project cash flow. (Answers in *italics*.)

	0	1	2	3	4	5	6
Revenue	6,000,000	6,600,000	7,260,000	7,986,000	8,784,600		
Cost of Goods Sold	-4,500,000	-4,950,000	-5,445,000	-5,989,500	-6,588,450		
Taxable Income	1,500,000	1,650,000	1,815,000	1,996,500	2,196,150		
Income Tax at 34%	-510,000	-561,000	-617,100	-678,810	-746,691		
Subtotal	990,000	1,089,000	1,197,900	1,317,690	1,449,459		
Adjustments							
Working Capital	<i>-150,000</i>	<i>-125,000</i>	<i>-12,500</i>	<i>-13,750</i>	<i>-15,125</i>	<i>133,363</i>	<i>183,013</i>
Cash Flow	<i>-150,000</i>	<i>865,000</i>	<i>1,076,500</i>	<i>1,184,150</i>	<i>1,302,565</i>	<i>1,582,822</i>	<i>183,013</i>
Balances							
Cash	50,000	50,000	50,000	50,000	50,000	0	0
Inventory	100,000	100,000	100,000	100,000	100,000	0	0
Accounts Receivable	0	500,000	550,000	605,000	665,500	732,050	0
Accounts Payable	0	375,000	412,500	453,750	499,125	549,038	0
Working Capital	<i>150,000</i>	<i>275,000</i>	<i>287,500</i>	<i>301,250</i>	<i>316,375</i>	<i>183,013</i>	<i>0</i>

9. Today is January 1, 2002. AAA Inc. is a calendar-year, accrual-basis, corporation. The following are balances at the beginning and end of the year for various working capital accounts. The first set of balances is calculated under the assumption that the firm does not take Project Y. The second set of balances is calculated under the assumption that the firm does take Project Y. What is the working capital adjustment for 2002 that will be made in determining cash flows for Project Y?

Do not take project

	<u>Beg Balance</u>	<u>End Balance</u>
Accounts Receivable	\$1000	\$1200
Inventory	\$5000	\$6000
Accounts Payable	\$4000	\$7000

Take project

	<u>Beg Balance</u>	<u>End Balance</u>
Accounts Receivable	\$1000	\$2700
Inventory	\$5000	\$7000
Accounts Payable	\$4000	\$9000

Answer = -\$500.00

10. Today is January 1, 2002. BBB Inc. is a calendar-year, accrual-basis corporation. The following are balances at the beginning and end of the year for various working capital accounts. The first set of balances is calculated under the assumption that the firm takes Project Z. The second set of balances is calculated under the assumption that the firm does not take Project Z. What is the working capital adjustment that will be made in determining cash flows for Project Z for 2002? Answer = +950.

Account balances if the firm TAKES the project

	<u>Beg Balance</u>	<u>End Balance</u>
Accounts Receivable	\$1000	\$1000
Inventory	\$4000	\$5000
Accounts Payable	\$6000	\$7000

Account balances if the firm DOES NOT TAKE the project

	<u>Beg Balance</u>	<u>End Balance</u>
Accounts Receivable	\$1000	\$850
Inventory	\$4000	\$3100
Accounts Payable	\$6000	\$4000

11. CCC Inc. is considering Project XXX. Use the following information to calculate the cash flow for Project XXX for the month of May. Answer = \$40,200

	April	May
Revenue		\$250,000
CGS		(\$150,000)
Gross Margin		\$100,000
Cash Selling and Administrative		(\$60,000)
Taxable Income		\$40,000
Income Tax at 34%		(\$13,600)
Subtotal		\$26,400
Adjustments		
Working Capital		You need to calculate this
Project Cash Flow		Your Answer
Working Capital Account Balances		
Accounts Receivable	\$25,000	\$24,800
Taxes Payable	\$10,000	\$23,600

12. A building was purchased several years ago for \$500,000. To date, \$350,000 of tax depreciation has been taken on the building. Therefore, the tax book value is \$150,000. Its current market value is \$600,000.

If the project is rejected the building will be sold (it has no other use to the corporation). Therefore, if the corporation did not take the project, it will receive \$600,000 from the sale of the building (less any income taxes it will need to pay because of the sale). Use the corporation's marginal income tax rate of 34% to compute the amount of income tax it owes if the building is sold.

What is the time zero after-tax opportunity cost of using the building on the project? *\$447,000 (a negative cash flow at time 0).*

13. A project will use a building purchased several years ago for \$250,000. So far, the corporation has taken a total of \$190,000 of tax depreciation. Therefore, the tax book value is \$60,000. The current market value of the building is \$95,000.

If the project is rejected the building will be sold (it has no other use to the corporation). Therefore, if the corporation did not take the project, it will receive \$95,000 from the sale of the building (less any income taxes it would need to pay because of the sale). Use the corporation's marginal income tax rate of 34% to compute the amount of income tax it owes if the building is sold.

What is the time zero after-tax opportunity cost of using the building on the project? *The time zero after-tax opportunity cost = \$83,100 (treated as a -\$83,100 cash flow at time zero)*

14. A firm is considering two possible projects. The firm has a marginal income tax rate of 34%. Both projects are identical in all aspects except:

Project A will use a fully-depreciated asset purchased 10 years ago for \$10000 with a current market value of \$6000.

Project B will use a fully-depreciated asset purchased 10 years ago for \$10000 with a current market value of \$5000.

Which of the two projects has the higher NPV?

- A. Project A.
- B. Project B. *(Correct Answer)*
- C. Both projects have the same NPV.

15. A firm is considering two possible projects. The firm has a marginal income tax rate of 34%. Both projects are identical in all aspects except:

Project C will use a fully-depreciated asset purchased 10 years ago for \$12000 with a current market value of \$5000.

Project D will use a fully-depreciated asset purchased 10 years ago for \$10000 with a current market value of \$5000.

Which of the two projects has the higher NPV?

- A. Project C.
- B. Project D.
- C. Both projects have the same NPV. *(Correct Answer)*

16. Similar to the depreciation example given in the notes, assume

- Equipment purchased on January 1. Cost = \$2500 (3-year tax depreciation life). The equipment is expected to run for four more years with no salvage value at the end of its useful life. The corporation is a calendar-year taxpayer.
- 1800 units sold per year, current price = \$1.50 per unit.
- Ignore expenses other than depreciation and income taxes. Assume the marginal tax rate is 34%.
- Expected inflation rate = 5% per year.
- Discount rate = 3% (real), 8.15% (nominal)

- A. What are the yearly “nominal” cash flows? What is the NPV using the nominal discount rate?
- B. What are the yearly “real” cash flows using “method 2”?
- C. What is the NPV using a real discount rate?
- D. Be able to calculate the NPV using “method 1” described in the notes.

	<i>Analysis Using Nominal Figures</i>				
	0	1	2	3	4
<i>Unit Sales</i>		1800	1800	1800	1800
<i>Price per unit</i>		1.57500	1.65375	1.73644	1.82326
<i>Revenue</i>		2835.00	2976.75	3125.59	3281.87
<i>Depreciation</i>		-833.25	-1111.25	-370.25	-185.25
<i>Taxable Income</i>		2001.75	1865.50	2755.34	3096.62
<i>Income Tax at 34%</i>		-680.60	-634.27	-936.81	-1052.85
<i>Subtotal</i>		1321.16	1231.23	1818.52	2043.77
<i>Adjustments</i>					
<i>Depreciation</i>		833.25	1111.25	370.25	185.25
<i>Initial Investment</i>	-2500.00				
<i>Cash Flow</i>	-2500.00	2154.41	2342.48	2188.77	2229.02
<i>NPV at 8.15%</i>	4854.41				

	<i>Analysis Using Real Figures (Method One)</i>				
	0	1	2	3	4
<i>Unit Sales</i>		1800.00	1800.00	1800.00	1800.00
<i>Sale's Price</i>		1.50	1.50	1.50	1.50
<i>Revenue</i>		2700.00	2700.00	2700.00	2700.00
<i>Income Tax at 34%</i>		-918.00	-918.00	-918.00	-918.00
<i>Cash Flow</i>		1782.00	1782.00	1782.00	1782.00
<i>PV at 3%</i>	6623.87				
<i>Depreciation income tax deduction</i>		-833.25	-1111.25	-370.25	-185.25
<i>Cash Flow from tax deduction (at 34%)</i>		283.31	377.83	125.89	62.99
<i>PV at 8.15%</i>	730.54				
<i>Initial Investment</i>	-2500.00				
<i>Total NPV</i>	4854.41				

	<i>Analysis Using Real Figures (Method Two)</i>				
	0	1	2	3	4
<i>Unit Sales</i>		1800.00	1800.00	1800.00	1800.00
<i>Price per unit</i>		1.50	1.50	1.50	1.50
<i>Revenue</i>		2700.00	2700.00	2700.00	2700.00
<i>Depreciation</i>		-793.57	-1007.94	-319.84	-152.41
<i>Taxable Income</i>		1906.43	1692.06	2380.16	2547.59
<i>Income Tax at 34%</i>		-648.19	-575.30	-809.26	-866.18
<i>Subtotal</i>		1258.24	1116.76	1570.91	1681.41
<i>Adjustments</i>					
<i>Depreciation</i>		793.57	1007.94	319.84	152.41
<i>Initial Investment</i>	-2500.00				
<i>Cash Flow</i>	-2500.00	2051.81	2124.70	1890.74	1833.82
<i>NPV at 3%</i>	4854.41				

17. Similar to the example given in class, assume that the asset purchased in the previous example is sold at the end of the third year. The “real” sale’s price is expected to be \$1200. Compute the present value of the after-tax salvage value using both nominal and real figures.

	<i>Nominal</i>	<i>Real</i>
<i>Sale's Price</i>	\$ 1,389.15	\$ 1,200.00
<i>Purchase Price</i>	\$ 2,500.00	\$ 2,159.59
<i>Depr yr one</i>	\$ (833.25)	\$ (719.79)
<i>Depr yr two</i>	\$ (1,111.25)	\$ (959.94)
<i>Depr yr three</i>	\$ (185.13)	\$ (159.92)
<i>Adjusted Basis</i>	\$ 370.38	\$ 319.94
<i>Gain</i>	\$ 1,018.78	\$ 880.06
<i>Tax</i>	\$ 346.38	\$ 299.22
<i>After Tax CF</i>	\$ 1,042.77	\$ 900.78
<i>PV</i>	\$ 824.34	\$ 824.34

18. Similar to the example given in class, assume that

- Young trees grow faster than mature trees. Assumption: board feet = 100 bd. feet x square root (age). Trees are currently 10 years old.
- Price of lumber increasing at 5% per year. Current price = \$500 per 1000 board feet (or \$0.50 per bd. foot).
- Cost of harvest included in sale's price of lumber.
- Use a 8.15% nominal discount rate.

Fill out the following table

Age	Board Feet	Price	Sale's Price	NPV (8.15%)	Return
10	316.2278	500.00	158.11	158.11	
11	331.6625	525.00	174.12	161.00	10.12%
12	346.4102	551.25	190.96	163.26	9.67%
13	360.5551	578.81	208.69	164.98	9.29%
14	374.1657	607.75	227.40	166.22	8.96%
15	387.2983	638.14	247.15	167.04	8.69%
16	400.0000	670.05	268.02	167.50	8.44%
17	412.3106	703.55	290.08	167.62	8.23%
18	424.2641	738.73	313.42	167.46	8.04%
19	435.8899	775.66	338.10	167.04	7.88%
20	447.2136	814.45	364.23	166.38	7.73%
21	458.2576	855.17	391.89	165.53	7.59%
22	469.0416	897.93	421.17	164.49	7.47%
23	479.5832	942.82	452.16	163.29	7.36%
24	489.8979	989.97	484.98	161.94	7.26%

Harvest in year 17.

19. Similar to the "tree harvest" example discussed in class, which of the following years should the company plan to harvest the trees? (Note that the discount rate is 7%.)

Age	Board Feet	Price	Sale's Price	NPV (7%)	Return
10	395.2847	\$ 400.0000	\$ 158.1139	\$ 158.1139	
11	414.5781	\$ 419.2000	\$ 173.7911	\$ 162.4216	9.92%
12	433.0127	\$ 439.3216	\$ 190.2318	\$ 166.1559	9.46%
13	450.6939	\$ 460.4090	\$ 207.5035	\$ 169.3847	9.08%
14	467.7072	\$ 482.5087	\$ 225.6728	\$ 172.1647	8.76%
15	484.1229	\$ 505.6691	\$ 244.8060	\$ 174.5433	8.48%
16	500.0000	\$ 529.9412	\$ 264.9706	\$ 176.5611	8.24%
17	515.3882	\$ 555.3784	\$ 286.2355	\$ 178.2531	8.03%

18	530.3301	\$ 582.0365	\$ 308.6715	\$ 179.6496	7.84%
19	544.8624	\$ 609.9743	\$ 332.3520	\$ 180.7775	7.67%
20	559.0170	\$ 639.2531	\$ 357.3533	\$ 181.6603	7.52%
21	572.8220	\$ 669.9372	\$ 383.7547	\$ 182.3191	7.39%
22	586.3020	\$ 702.0942	\$ 411.6392	\$ 182.7727	7.27%
23	599.4789	\$ 735.7947	\$ 441.0934	\$ 183.0381	7.16%
24	612.3724	\$ 771.1129	\$ 472.2083	\$ 183.1305	7.05%
25	625.0000	\$ 808.1263	\$ 505.0789	\$ 183.0638	6.96%

Harvest when the tree is _____ years old. *Answer = harvest when the tree is 24 years old.*

20. Refer back to the previous problem. Which year should the company harvest the tree if the discount rate was 7.5% instead of 7%? *Answer = harvest when the tree is 20 years old.*
21. Similar to the "tree harvest" example, which of the following years should the company plan to harvest the trees? (Note that the discount rate is 6%.)

Age	Board Feet	Price	Sale's Price	NPV (6%)	Return
10	316.2278	\$ 500.0000	\$ 158.1139	\$ 158.1139	
11	331.6625	\$ 515.0000	\$ 170.8062	\$ 161.1379	8.03%
12	346.4102	\$ 530.4500	\$ 183.7533	\$ 163.5398	7.58%
13	360.5551	\$ 546.3635	\$ 196.9942	\$ 165.4001	7.21%
14	374.1657	\$ 562.7544	\$ 210.5634	\$ 166.7859	6.89%
15	387.2983	\$ 579.6370	\$ 224.4925	\$ 167.7538	6.62%
16	400.0000	\$ 597.0261	\$ 238.8105	\$ 168.3520	6.38%
17	412.3106	\$ 614.9369	\$ 253.5450	\$ 168.6219	6.17%
18	424.2641	\$ 633.3850	\$ 268.7225	\$ 168.5998	5.99%
19	435.8899	\$ 652.3866	\$ 284.3687	\$ 168.3174	5.82%
20	447.2136	\$ 671.9582	\$ 300.5088	\$ 167.8026	5.68%
21	458.2576	\$ 692.1169	\$ 317.1678	\$ 167.0801	5.54%
22	469.0416	\$ 712.8804	\$ 334.3706	\$ 166.1719	5.42%
23	479.5832	\$ 734.2669	\$ 352.1420	\$ 165.0979	5.31%
24	489.8979	\$ 756.2949	\$ 370.5073	\$ 163.8757	5.22%
25	500.0000	\$ 778.9837	\$ 389.4919	\$ 162.5213	5.12%

Harvest when the tree is _____ years old. *17 years old*

22. Refer back to the previous problem. Which year should the company harvest the tree if the discount rate was 5.5% instead of 6%? *21 years old*
23. Similar to the "evaluating two mutually exclusive projects with unequal lives" example given in class, assume that Asset A costs \$6000 and produces after-tax real cash flows of \$2600 per year in years 1 and 2 and \$2800 in year 3. Asset B costs \$4000 and produces after-tax real cash flows of \$2600 in year 1 and 2. Use a 3% real discount rate:
- What the EAC of Asset A (using the 3% discount rate)? *\$543.52*
 - What the EAC of Asset B (using the 3% discount rate)? *\$509.56*
 - What is the PV of a perpetual stream of Asset A purchases? *\$18,117.46*
 - Which of the two assets should the firm purchase? *Asset A*
24. A project has the following real (constant dollar) cash flows. Using a 3% real discount rate, what is the equivalent annual cash flow (EAC) for the project? *Answer = \$102.54*

0	1	2	3	4
-\$2000	\$0	\$0	\$1000	\$1650

25. Using a 6% discount rate, what is the equivalent annual cash flow (EAC) for the following cash flow stream?
 Answer = \$1075.62

0	1	2	3	4	5
-\$6000	\$2500	\$2500	\$2500	\$2500	\$2500

26. Using a 3% discount rate, what is the equivalent annual cash flow (EAC) for the following cash flow stream?
 Answer = \$549.87

0	1	2	3	4	5
-\$6000	\$1860	\$1860	\$1860	\$1860	\$1860

27. Refer back to the previous problem. If the time zero cash flow was -\$5500 instead of -\$6000, the EAC would be _____ than the correct answer to the previous problem.

- A. Higher (*Correct answer*)
 B. Lower

28. Using a 5% discount rate, what is the equivalent annual cash flow (EAC) for the following cash flow stream?
 \$294.15

0	1	2	3	4	5
-\$6000	\$1680	\$1680	\$1680	\$1680	\$1680

29. Refer back to the previous problem. If the time zero cash flow was -\$5500 instead of -\$6000, the EAC would be _____ than the correct answer to the previous problem. *Higher*

30. Similar to the "replacement analysis" example given in class, assume: (1) an old machine produces cash flows of \$15,000 per year for the next 5 years, at which time it will be worthless, (2) old machine's sale's price (today) = \$66,000, (3) old machine's sale's price (in one year) = \$58,000, (4) old machine is fully depreciated, (5) new machine will cost \$200,000, produce after-tax cash flows of \$28,000 for the next 12 years. At the end of this 12-year period, an identical machine with the same cash flows will be purchased. This will continue every 12 years forever. (6) Cash flows are in constant dollars – use the real discount rate of 3%. (7) The marginal tax rate is 34%.

- A. What is the (t = 0) after-tax salvage value of the old machine if it is sold today? \$43,560.00
 B. What is the (t = 1) after-tax salvage value of the old machine if it is sold in one year? \$38,280
 C. What is the EAC for the new machine? \$7,907.58
 D. What is the (t = 0) NPV of replacing the old machine with the new machine today? $(\$43,560 + \$7,907.58 / 1.03) - (\$15,000 / 1.03 + \$38,280 / 1.03) = -\$490.89$, do not replace today!

31. A project requires an initial investment of \$100,000 and produces real cash flows of \$3,600 per year in perpetuity. In addition to the above cash flows, this project will cause an old machine that the firm owns to wear out two years early. More specifically, if the project is accepted, this existing machine will last only four more years. The new replacement machine will handle the capacity of the new project and all existing projects, will cost \$30,000 to purchase (at t = 4), and will cost \$5,000 per year to operate for eight years (t = 5 to t = 12). At the end of this eight-year period, an identical machine with the same cash flows will be purchased. This will continue every eight years forever.

If the firm rejects the project, the old machine will need to be replaced in six years with the new replacement machine. Cash flows for the replacement machine are the same as above, but the \$30,000 purchase of the replacement machine would be in year 6 and the \$5,000 operating cash out flows would be for years 7 - 14. Again, this will continue every eight years forever.

As with the example given in class, ignore the costs of operating the old machine.

Assume that the figures presented are real cash flows, so use a 3% real discount rate.

- A. What is the preliminary NPV? (In other words, what is the NPV ignoring the costs of causing the old machine to wear out 2 years early?) $\$20,000$
- B. What is the EAC for the new replacement machine? $-\$9,273.69$
- C. What is the PV of the additional costs associated with having to buy the new replacement machine at the end of the fourth year instead of the end of the sixth year? $\$15,766.14$ (negative cash flow).
- D. What is the corrected NPV for the project? $\$20,000 - \$15,766.14 = \$4,233.86$

32. Big Blue Corporation (a retailer of University of Kentucky merchandise) has \$5 million of extra cash that it plans to invest. It is considering two possible investments: Stock A or Stock B. Use the following information to answer the next two questions.

- Both stocks pay annual dividends (next dividend in one year). See time one expected dividend payments below.
- The dividends for both stocks are expected to grow at a constant rate in perpetuity. See growth rates below.
- Assume you invest \$5 million into either Stock A or Stock B
- Assume financial markets are perfect, efficient, and in equilibrium

Information about each stock

	Stock A	Stock B
Expected dividend in one year	\$2	\$1.50
Opportunity cost of capital	14%	9%
Annual dividend growth rate	2%	3%
Current price	$\$16 \frac{2}{3}$	\$25

Compare the NPV of investment of \$5 million in Stock A or Stock B. Which of the two investments has the higher NPV?

- A. Investment of \$5 million in Stock A.
- B. Investment of \$5 million in Stock B.
- C. Both investments have the same NPV. *Correct Answer*

Compare the expected return associated with investment of \$5 million in Stock A or Stock B. Which of the two investments is expected to have the highest return?

- A. Investment of \$5 million in Stock A. *Correct Answer*
- B. Investment of \$5 million in Stock B.
- C. Both investments have the expected return.