

**Chapter 7 – Introduction to Risk, Return and the Opportunity Cost of Capital**  
**Chapter 8 – Risk and Return (section 8-2 and 8-3)**

These chapters describe how risk is measured and is part of a three-chapter sequence describing how the risk of a project's cash flows determines the discount rate (the opportunity cost of capital) for these cash flows. We then use that discount rate to calculate the present value of the project's future expected cash flows. Subtracting the initial investment gives us the project NPV.

You will notice a slight difference in notation between these notes and the text. Please take this into account in your studying.

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**Through most of our discussion of Chapter 6, we assumed that the future cash flows of a project are known with certainty.** However, in most circumstances future cash flows are uncertain. When a corporation's future cash flows are uncertain:

- **Step one** - Calculate **expected** future cash flows.

Calculation of project cash flows (under certainty) was discussed in Chapter 6. This involved determining project revenues, expenses, tax depreciation, corporate income taxes, working capital adjustments, inflation, etc.

To calculate expected cash flows for risky projects, the types of calculations done in Chapter 6 may have to be performed many times resulting in something like this:

Project A's time one cash flow in a booming economy = \$155  
Project A's time one cash flow in a normal economy = \$135  
Project A's time one cash flow in a recession economy = \$40

To calculate expected cash flows, you need to determine the probabilities:

Probability of a booming economy = 20%  
Probability of a normal economy = 60%  
Probability of a recession economy = 20%

What is the expected project cash flow for time one in the above example?

In the above example, I have assumed that project cash flows depend only on the type of economy. This is obviously a simplification. Several other factors can affect project cash flows, each with their own cash flows and probabilities:

Project A's time one cash flow in a booming economy and high demand from consumers = \$200  
Project A's time one cash flow in a booming economy and low demand from consumers = \$110

Probability of a high demand from consumers = 50%  
Probability of a low demand from consumers = 50%

- **Step two** - Discount expected cash flows at the opportunity cost of capital to determine their present value.

The opportunity cost of capital is based on the risk of the cash flows (higher risk, higher opportunity cost of capital)

Notice that the above example gives project cash flows and probabilities. We use these to calculate expected cash flows (and the present value of these cash flows).

Sometimes you might want to calculate present values of individual parts of the project

PV of cash flows from Part A of project  
+ PV of cash flows from Part B of project  
PV of project cash flows

Separate present value calculations (of the two parts) will give you the same answer as you would get by calculating the PV of the project cash flows.

However, separate calculations may allow you to focus on the risks of the different parts of the project. For example:

Part A has high risk (use a high opportunity cost of capital)  
Part B has low risk (use a low opportunity cost of capital)

Project has \_\_\_\_ risk (use a \_\_\_\_ opportunity cost of capital)

**The opportunity cost of capital for cash flows** is the expected return for financial assets with the exact same amount of risk as these cash flows.

- Risk-free cash flows should use a discount rate equal to the expected return for risk-free securities (e.g., the one-month Treasury-Bill interest rate, approximately 5%)
- Higher risk cash flows should be discounted at a higher discount rate to reflect their higher level of risk. For example, consider a project with cash flows that are just as risky as an investment in the S&P500 stocks.

Examine Table 7-1 (page 156 of the textbook). Then describe how the opportunity cost of capital would be determined for the risky cash flows of this particular project.

Relevant Table 7-1 information

Average annual return for the S&P 500 stocks (1926 – 1997) = 13.0%  
Average annual return for Treasury Bills (1926 – 1997) = 3.8%

Some questions and observations

1. What is the expected return (over the next year) for an investment in the S&P stocks?

To answer, add:

- Current one-month Treasury Bill rate
- Market risk premium for S&P 500 stocks (proxied by the average historical premium)

2. In the previous equation, we assumed two things:

- The *market risk premium* is constant across time (for evidence, see Figure 7.3, page 159.)
- The average historical premium is a good approximation of the unobservable *market risk premium*.

3. Based on this, what would happen if Treasury Bill interest rates increased from 5% to 10%?

- Expected S&P return with a 5% risk-free interest rate =
- Expected S&P return with a 10% risk-free interest rate =

4. Problems with using historical data (for the period 1926-1997) to approximate the market risk premium

- The S&P 500 had experienced very high returns over the last several years of that period pushing up our estimate of the market risk premium.
- Results from a recent survey of financial economists imply a market risk premium of a little over 6%.
- Brealey and Myers suggest the use of a market risk premium towards the upper end of a 6% - 8.5%.

**For examples used in class, we will use 5% as the risk-free interest rate and 8.4% as the market risk premium.**

### Computing the opportunity cost of capital for risky cash flows of a project

From the above discussion, we know how to calculate the opportunity cost of capital for risk free cash flows and for cash flows that are just as risky as the S&P 500 stocks. What about cash flows of different amounts of risk?

In general, to calculate the opportunity cost of capital for a risky cash flow, you must determine (A) its risk, and (B) the expected return for financial assets that have that exact same amount of risk. In this class, we will:

- Calculate risk as the beta for the project's cash flows
- Use the CAPM to calculate the expected return for financial assets with the same beta risk as the project's cash flows
- Use this expected return (calculated by using the CAPM equation) as the opportunity cost of capital for the risky cash flows.
- Use this opportunity cost of capital as the discount rate to determine the present value of the future expected cash flows. Subtract the initial investment to determine the project NPV.

### Statistical and other tools used in the calculation of the beta of a project's risky cash flows

- Calculation of the present value of the future expected cash flows
- Calculation of returns and expected returns
- Calculation of the variance of returns
- Calculation of the standard deviation of returns
- Calculation of the correlation of returns

**This next section of the notes is meant to build up your intuition of the determinants of beta. A detailed example follows. You are expected to review most of this detailed example on your own.**

ABC Corporation is considering an investment of \$100 (at  $t = 0$ ) in either project A or project B. Each project will last only one year. Time 1 cash flows from these projects depend on the state of the economy. Cash flows for a \$100 risk-free investment and for a \$100 investment in the "market" are provided for comparison purposes.

State	1	2	3
Economy	Boom	Normal	Recession
Probability	20%	60%	20%
Risk-Free	\$105	\$105	\$105
Market	\$143	\$116	\$76
Project A	\$155	\$135	\$40
Project B	\$15	\$105	\$136

Based on the initial investment of \$100, we can calculate a one-period rate of return.

State	1	2	3
Economy	Boom	Normal	Recession
Probability	20%	60%	20%
Risk-Free	5%	5%	5%
Market	43%	16%	-24%
Project A	55%	35%	-60%
Project B	-85%	5%	36%

We can also calculate the one-period rate of return in each economic state based on the present value of the future expected cash flow. (The calculation of the present values is discussed at the end of the notes.)

- PV of the market investment cash flows = \$100 (same as the initial investment) NPV = \$0
- PV of the risk-free investment cash flows = \$100 (same as the initial investment) NPV = \$0
- PV of project A cash flows = \$99.8382 (less than the initial investment) NPV = -\$0.1618
- PV of project B cash flows = \$102.0885 (more than the initial investment) NPV = \$2.0885

State	1	2	3
Economy	Boom	Normal	Recession
Probability	20%	60%	20%
Risk-Free	5%	5%	5%
Market	43%	16%	-24%
Project A	55.2512%	35.2188%	-59.9352%
Project B	-85.3069%	2.8519%	33.2177%

## Expected Values

### 1. The Expected Time 1 Cash Flow

- $E(CF_m) = (\$143)(0.2) + (\$116)(0.6) + (\$76)(0.2) = \$113.4$
- $E(CF_{rf}) = \$105$
- $E(CF_A) = \$120$
- $E(CF_B) = \$93.2$

### 2. The Expected Return – based on the initial investment

- $E(r_m) = (43\%)(0.2) + (16\%)(0.6) + (-24\%)(0.2) = 13.4\%$ , or  
 $E(r_m) = (\$113.4 / \$100) - 1 = 13.4\%$
- $E(r_{rf}) = (\$105 / \$100) - 1 = 5\%$
- $E(r_A) = (\$120 / \$100) - 1 = 20\%$
- $E(r_B) = (\$93.2 / \$100) - 1 = -6.8\%$

### 3. The Expected Return – based on the present value of the future expected cash flow

- $E(r_m) = (\$113.4 / \$100) - 1 = 13.4\%$
- $E(r_{rf}) = (\$105 / \$100) - 1 = 5\%$
- $E(r_A) = (\$120 / \$99.8382) - 1 = 20.1945\%$
- $E(r_B) = (\$93.2 / \$102.0885) - 1 = -8.7067\%$

## Measures of Risk – Variance and Beta

When the risk of the project (or investment) is high, the variance and standard deviation will be high. However, note that variance and standard deviation measure **total risk** both diversifiable ('unique' risk) and non-diversifiable risk (or 'market' risk).

The other measure of risk, beta, measures only the portion of the risk that is not **diversifiable**.

4. **The variance of returns** – use the returns and expected return that are calculated based on the present value of the future expected cash flow. (In the following formula,  $\pi_i$  is the probability of state  $i$  occurring.)

$$\sigma_a^2 = \sum_{i=1}^N (\tilde{r}_{ai} - E(r_a))^2 \pi_i$$

- $\sigma_m^2 = 0.045904 = [(0.43 - 0.134)^2 0.2 + (0.16 - 0.134)^2 0.6 + (-0.24 - 0.134)^2 0.2]$
- $\sigma_{rf}^2 = 0$
- $\sigma_A^2 = 0.166539 = [(0.552512 - 0.201945)^2 0.2 + (0.352188 - 0.201945)^2 0.6 + (-0.599352 - 0.201945)^2 0.2]$
- $\sigma_B^2 = 0.160521$

5. **Standard deviation** equals the square root of the variance.

- $\sigma_{rf} = 0$
- $\sigma_m = 0.214252$
- $\sigma_A = 0.408091$
- $\sigma_B = 0.400651$

6. The **correlation coefficient** must be calculated in order to calculate the beta. The correlation coefficient measures the tendency of two asset's returns to move in the same direction. (Cov (a, b) is the covariance between the returns of assets a and b.) Again, use the returns and expected return that are calculated based on the present value of the future expected cash flow.

$$\rho_{ab} = \frac{\sum_{i=1}^N (\tilde{r}_{ai} - E(r_a))(\tilde{r}_{bi} - E(r_b))\pi_i}{\sigma_a \sigma_b} \equiv \frac{\text{cov}(a, b)}{\sigma_a \sigma_b}$$

- $\rho_{mA} = \text{Cov}(m, A) / (\sigma_m \sigma_A) = [(0.43 - 0.134)(0.552512 - 0.201945)(0.2) + (0.16 - 0.134)(0.352188 - 0.201945)(0.6) + (-0.24 - 0.134)(-0.599352 - 0.201945)(0.2)] / [(0.214252)(0.408091)] = 0.949675$
- $\rho_{mrf} = 0.0$
- $\rho_{mB} = -0.872593$

The maximum correlation coefficient is 1.0 (perfect positive correlation) and the minimum is -1.0 (perfect negative correlation). A correlation coefficient of 0 means that the two asset's returns are uncorrelated.

The 'market' investment and project A have a very high positive correlation. This indicates that when the 'market' has a return that is higher than its expected return, project A should also have a return that is higher than its expected return (and vice versa). The opposite relationship (a negative correlation) is found between the 'market' investment and project B. The returns of the 'market' investment and the risk-free investment are uncorrelated.

## 7. Should ABC Corp. invest in Project A and / or Project B?

Investors in ABC Corp. want the corporation's managers to only invest in positive NPV projects. To calculate the NPV, discount the expected  $t = 1$  cash flow at the opportunity cost of capital for each of the projects.

$$\begin{aligned} \text{NPV}_A &= -\$100 + \$120 / (1 + \text{opportunity cost of capital for project A}) \\ \text{NPV}_B &= -\$100 + \$93.2 / (1 + \text{opportunity cost of capital for project B}) \end{aligned}$$

As stated above, the opportunity cost of capital depends on the riskiness of the projects. How risky are these two projects?

Well-diversified investors in ABC Corp. are concerned with how these projects will affect their investment portfolio's risk. They are not concerned with risk that will be diversified away.

If an investor is invested in the market (the market portfolio), then beta is the relevant risk measure (not variance or standard deviation). **Beta** measures the contribution of an asset to the riskiness of the 'market portfolio.' The market portfolio has a beta of one. Betas greater than one are more risky than the market (and vice versa).

### Calculation of beta

$$\beta_a = \frac{\rho_{ma} \sigma_m \sigma_a}{\sigma_m^2} = \frac{\text{cov}(m, a)}{\sigma_m^2} = \frac{\rho_{ma} \sigma_a}{\sigma_m}$$

- $\beta_A = [(0.949675) (0.408091)] / 0.214252 = 1.80887$
- $\beta_{rf} = 0.0$
- $\beta_M = 1.0$
- $\beta_B = -1.63175$
  
- Project A is **more** risky than the market portfolio.
- Project B is **less** risky than the market portfolio.

### 8. A quick look at the Capital Asset Pricing Model (CAPM)

Using certain semi-reasonable assumptions, it can be shown that all risk-averse investors will want to invest in two assets: i) the market portfolio, and ii) a risk-free asset.

The **market portfolio** is a portfolio of **all** risky assets (stocks, bonds, real estate, fine art, human capital, etc.) held in proportion to their market value. The **risk-free asset** could be Treasury Bills or some other risk-free asset (bank saving's account, bank CD's, money-market mutual funds, etc.). **Example:**

\$10000 total investment  
 \$2000 in a bank certificate of deposit - risk-free investment  
 \$8000 in a global mutual fund - market investment

Based on the discussion in the notes so far, we learned that **beta** is the appropriate measure of risk for investors that own a portion of the market portfolio. This also applies to an investment such as the one above (i.e., part investment in the market portfolio and part in a risk-free investment).

**Remember:** Beta measures the contribution of an asset to the riskiness of the market portfolio (after taking into account the reduction in risk from diversification).

Betas greater than one are more risky than the market portfolio.  
 Betas less than one are less risky than the market portfolio.

Risk-averse investors will not want to own high-risk assets unless they are compensated with a **low price** (or equivalently a **high expected return**).

**Example:**

	Price	Expected T = 1 Cash Flow	Risk
Asset I	\$100	\$110	High
Asset J	\$100	\$110	Low

Notice that both assets have the same expected return (10%), but asset I has more risk. This is not an equilibrium relationship. Investors will not want to own asset I if they can own asset J that has the same expected return, but lower risk. What will happen to asset I's **price** and expected **return**?

Based on this, if an asset has above-average risk, then investors will require an **above-average expected return** (and vice versa).

9. The **Capital Asset Pricing Model (CAPM)** is one finance model that takes into account this risk/return tradeoff. According to this model, the **required return** for asset **a** is:  $r_a = r_f + \beta_a (E(r_m) - r_f)$

Notice that the CAPM uses beta for a measure of risk.

In a CAPM setting, the **required return** means the expected return required by investors based on the asset's risk (beta) and market-wide expected returns [ $r_f$  and  $E(r_m)$ ]. Graphically, the CAPM equation gives the formula for the **Security Market Line (SML)**, see Figure 8-7 in the text, page 196. For example, assume that  $r_f = 5\%$  and  $E(r_m) - r_f = 8.4\%$ . The intercept of the SML is the risk free rate (5%). The slope is the market risk premium (8.4%).

What is the required return for the following investments / projects? Also, how are these required returns related to expected returns? **Assuming the CAPM is correct:**

$\beta_{rf} = 0$	$r_{rf} = 5\% + 0 (8.4\%) = 5\%$
$\beta_m = 1$	$r_m = 13.4\%$
$\beta_A = 1.80887$	$r_A = 20.1945\%$
$\beta_B = -1.63175$	$r_B = -8.7067\%$

**The Expected Return – based on the initial investment**

- $E(r_m) = (\$113.4 / \$100) - 1 = 13.4\%$
- $E(r_{rf}) = (\$105 / \$100) - 1 = 5\%$
- $E(r_A) = (\$120 / \$100) - 1 = 20\%$
- $E(r_B) = (\$93.2 / \$100) - 1 = -6.8\%$

**The Expected Return – based on the present value of the expected cash flows**

- $E(r_m) = (\$113.4 / \$100) - 1 = 13.4\%$
- $E(r_{rf}) = (\$105 / \$100) - 1 = 5\%$
- $E(r_A) = (\$120 / \$99.8382) - 1 = 20.1945\%$
- $E(r_B) = (\$93.2 / \$102.0885) - 1 = -8.7067\%$

**Notice that for financial assets:**

Required return (using CAPM) = expected return (based on the initial investment)  
 Required return (using CAPM) = expected return (based on the PV of the future expected cash flows)

**Notice that for the two projects:**

Required return (using CAPM)  $\neq$  expected return (based on the initial investment)  
 Required return (using CAPM) = expected return (based on the PV of the future expected cash flows)

**As discussed earlier, competition in the financial markets (plus market perfection and efficiency) causes:**

The cost of the financial asset (reflected in the initial investment) to equal the present value of the future expected cash flows (NPV = \$0)

Resulting in required returns = expected returns (based on initial investment)

**However, this should also happen for any asset that trades in an active and competitive market place**  
(land, fine art, coins)

#### 10. Graphical presentation of the CAPM (the SML)

#### 11. Expected rates of return can differ from required rates of return for:

- **Assets that do not trade on an active and competitive secondary market** (greater probability of mispricing).
- **Projects:** the expected return (based on the initial investment) for a project can be more or less than the required return implied by the CAPM.

#### 12. As a first estimate of the **opportunity cost of capital**, we will use the required return based on the CAPM. In other words, to determine the appropriate discount rate for a project:

- Calculate the project's beta.
- Use the CAPM equation to determine the expected return (and required return) for financial assets with the same amount of beta risk.
- Use this expected return (required return) as the discount rate for calculating the NPV of the project (or as the cutoff rate used with the IRR method).

For projects A and B

$$NPV_A = PV \text{ of expected (time 1) cash flow} - \text{initial investment} = \$120 / (1 + 0.201945) - \$100 = \$99.8382 - \$100 = -\$0.1618$$

$$NPV_B = PV \text{ of expected (time 1) cash flow} - \text{initial investment} = \$93.20 / (1 - 0.087067) - \$100 = \$102.0885 - \$100 = \$2.0885$$

Plot the SML and projects A and B on the same graph

### **Circular logic needed to calculate beta**

1. Notice that the present value of the future expected cash flow is needed to calculate the returns in each economic state (based on the present value) and expected return (based on the present value)
2. The returns in each state (based on the present value) and the expected return (based on the present value) are needed to calculate the standard deviation of returns
3. The returns in each state (based on the present value) and the expected return (based on the present value) are needed to calculate the correlation of returns
4. The standard deviation of returns and the correlation of returns is needed to calculate beta
5. Beta is needed to calculate the CAPM required rate of return
6. The CAPM required rate of return is needed to calculate the present value of the future expected cash flows
7. The present value of the expected cash flows is needed in step one

Your solution at step 6 will eventually converge to the correct number if you loop through process enough times.

However, there is another method for calculating the present value of the expected cash flows without going through the above circular loop. Interested students can come by my office for a discussion.

### **Chapter 7 and 8 Review Questions**

1. What is the opportunity cost of capital? What is typically used as the opportunity cost of capital for projects with risk free cash flows? What is meant by a constant market risk premium? Assuming a constant market risk premium, what happens to the expected return for the S&P 500 stocks if there is an increase in the risk free interest rate?
2. Historically, what has been the average difference between the returns on the S&P 500 stocks and Treasury Bills? Is this a good estimate for the market risk premium? According to Brealey and Myers, what is a good estimate for the market risk premium?
3. Know how to calculate:
  - A) Expected cash flows
  - B) Expected returns (based on the initial investment)
  - C) Expected returns (based on the present value)
  - D) Variance of returns (based on the present value)
  - E) Standard deviation of returns (based on the present value)
  - F) Correlation coefficient (based on the present value)
  - G) Beta
4. What is the maximum and minimum correlation coefficient? How is the maximum and minimum correlation coefficient related to the maximum and minimum possible beta?
5. What are "real life" examples of the risk-free security and the market portfolio?
6. What is the beta risk of the market portfolio? What is the beta of the risk free asset? What does it mean (in terms of risk) if the beta is (a) greater than one, (b) less than one, or (c) less than 0?
7. How is the beta riskiness of an asset related to its market value and expected return? Be able to use the CAPM equation to calculate required rates of return for assets based on their beta risk. (This graphs the security market line.) What are examples of assets that fall on the SML? How do market forces ensure that these assets locate on the SML?
8. What does it mean if a project locates above (or below) the SML with respect to the project's NPV?

**Chapter 7 and 8 Practice Problems** - Plenty of practice problems are given in the notes. Be sure to work through these practice problems to help your prepare for the exam. Here are a couple more practice problems.

1. If an asset's standard deviation of returns is 20% and the market's standard deviation of returns is 25%, what is the maximum and minimum possible beta for the asset? *Maximum = 0.80, minimum = -0.80.*
2. Assume that the risk-free interest rate is 5% and the market risk premium is 8.4%. What is the required rate of return for the following financial assets?

Beta for financial asset AAA = -1.5. *Answer = -7.6%*  
 Beta for financial asset BBB = 1.5. *Answer = 17.6%*

3. What is the expected return (based on the initial investment) for the two financial assets described above if the market is in equilibrium (and perfect and efficient)? *-7.6% and 17.6% respectively.*
4. Project Z requires an initial investment of \$100 and has a time one expected cash flow of \$95. The beta of Project Z's time one cash flow is -1.5. What is the NPV of Project Z? *\$2.81*
5. Both Project X and Y require an initial investment of \$1000 and both have an expected one-year return of 10%. Using this information and the other information provided below, which project has the higher NPV?

	0	1
Project X expected cash flows	-\$1000	\$1100
Project Y expected cash flows	-\$1000	\$1100

Risk-free interest rate = 5%  
 Market risk premium = 8.4%  
 Discount rates determined by the CAPM equation

Correlation between the market returns and Project X returns = -0.2  
 Correlation between the market returns and Project Y returns = +0.2

Annual standard deviation of returns for the market = 20%  
 Annual standard deviation of returns for the Project X = 40%  
 Annual standard deviation of returns for the Project Y = 20%

Hint: You have enough information to calculate the NPV of the two projects. But you can also answer the problem without calculations if you correctly use some finance intuition. *Solution:*

*The beta for Project X =  $[(-0.2)(0.40)] / 0.20 = -0.4$*

*The beta for Project Y =  $[(+0.2)(0.20)] / 0.20 = +0.2$*

*With a lower beta, Project X will have a lower discount rate and a higher NPV*

6. A project will produce one of two possible cash flows in one year (cash flows and probabilities given below). The beta for these cash flows is 1.2. The risk free rate is 5% and the market risk premium is 8.4%. Using the CAPM, what is the present value of the project's time one cash flow? *\$92.11*

Type of Economy	Good	Bad
Probability	60%	40%
Project Cash Flow	\$130	\$70

Hint: Calculate the expected time one cash flow. Then calculate the discount rate using the CAPM. Use the discount rate to calculate the present value.

7. Which of the following two assets has the highest beta? The difference between the two assets is **bolded**.

Asset A

Expected cash flow = \$130

Initial investment = \$100  
Standard deviation of Asset A's returns = **30%**  
Standard deviation of the Market's returns = 20%  
Correlation between Asset A's returns and the market's returns = -0.5

Asset B

Expected cash flow = \$130  
Initial investment = \$100  
Standard deviation of Asset B's returns = **40%**  
Standard deviation of the Market's returns = 20%  
Correlation between Asset B's returns and the market's returns = -0.5

- A. Asset A has the highest beta. (*Correct Answer*)
- B. Asset B has the highest beta.
- C. Both assets have the same beta.

8. Which of the following two assets has the highest beta? The difference between the two assets is **bolded**

Asset AA

Expected cash flow = \$130  
Initial investment = \$100  
Standard deviation of Asset AA's returns = **30%**  
Standard deviation of the Market's returns = 20%  
Correlation between Asset AA's returns and the market's returns = 0.0

Asset BB

Expected cash flow = \$130  
Initial investment = \$100  
Standard deviation of Asset BB's returns = **40%**  
Standard deviation of the Market's returns = 20%  
Correlation between Asset BB's returns and the market's returns = 0.0

- A. Asset AA has the highest beta.
- B. Asset BB has the highest beta.
- C. Both assets have the same beta. (*Correct Answer*)