GROWING STOCK AND STAND DENSITY

The growth rate achieved by any given forest stand is largely determined by two factors:

1. The initial productive capacity of the site
2. The amount and composition of the growing stock present in the stand.

Methods for evaluating the first factor, productive capacity, are discussed in Chapter 2. Procedures for expressing and quantifying the second factor constitute the subject matter of this chapter.

Specific growing stock characteristics of importance are:

1. The species present.
2. The numbers of trees by species and size class categories.
3. The spatial distribution of the trees.

All three of these characteristics are subject to control by the forest manager. This control is achieved through the manager's selection of silvicultural and
harvest strategies. The strategies selected usually represent attempts to utilize the productive capacity of the site as fully as possible for production of the commercial product mix that best furthers the management objectives of the forest owner.

The degree of crowding present in a stand and the level of site utilization attained are evaluated quantitatively through expressions of stand density and stocking. Measures of stand density are direct functions of such stand statistics as number of trees per acre or basal area per acre. Stocking measures, on the other hand, involve a comparison of the existing stand with some norm that has been established with a particular product objective in mind (Bleckford et al., 1957). The important distinction here is that two foresters evaluating the growing stock in a particular stand should obtain the same values for stand density statistics regardless of their intentions concerning subsequent management of the stand. However, if one intended to manage the stand for sawtimber production and the other envisioned pulpwood production, their assessments of stocking could differ considerably. It should be noted that, while most textbooks and scientific articles draw a distinction between stocking measures and stand density measures, common usage often fails to recognize this difference and many foresters frequently treat the terms stand density and stocking as synonymous.

Some measures of stand density, such as the number of trees per acre, are simply direct functions of measurable stand statistics, while others involve comparisons with previously established limiting relationships (e.g., the maximum number of trees that can be supported at a given age). Measures of stocking are usually expressed in relative terms, such as the basal area or volume per acre in a stand as a percentage of the same variable in a hypothetical stand considered to be the norm for the age and site index involved. Stocking is often expressed qualitatively as a judgment of the adequacy of the growing stock at the present stage of stand development in terms of the overall product goal and the potential of the site. For example, stands are often characterized as understocked, well-stocked, fully stocked, or overstocked. Regardless of the measure used to express density or stocking, the adequacy of the stocking or stand density must be evaluated through its effect on product yields.

Measures of stand density are more precise and generally more useful in analysis and estimation of growth and yield than stocking measures. In application, the utility of any given measure of stand density depends on the objective of the analysis. Certain measures may prove satisfactory when the objective is to estimate aggregate stand yield from measured stand variables. The same measures may be inadequate in an analysis of the subtle effects of various silvicultural treatments on the development of individual tree components in the stand.

It is important to remember that almost all measures of stand density and stocking are dynamic rather than static. Direct measures of stand density such as number of trees per acre or basal area per acre will obviously change as time passes. However, values for stand density measures that involve comparisons with limiting relationships and values for stocking measures will also generally change as the stand ages, with understocked stands becoming more adequately stocked as time passes. It is also possible for stocking levels to be drastically reduced at one or more points in the life of the stand as a result of catastrophic mortality. Typical causes of this type of mortality are storm damage, insect or disease attack, and fire damage.

3.1 STAND DENSITY AND GROWTH

The significance of stand density stems from the fact that, within limits, the greater the amount of growing space per tree, the faster the tree will grow. Therefore, for any given stand, the present average tree size has been determined by the growing space previously occupied by the tree. Control of density at stand establishment, and subsequently by thinning and other silvicultural means, is therefore an important aspect of timber management. Most empirical information concerning the effect of stand density on growth and yield comes from a large number of thinning and spacing studies that have been undertaken around the world with many different species. A small, but representative, subset of these studies is included in the References at the end of this chapter. From these and similar studies, some general conclusions can be drawn concerning the effect of stand density on tree and stand growth.

3.1.1 Height Growth

Empirical evidence from thinning experiments indicates that for many commercially important species height growth is not greatly affected by the manipulation of stand density. The average height of the stand may be changed by thinning, depending on the thinning method, but within wide limits of stand density, height growth seems to be unaffected, especially when the comparison is restricted to dominant and codominant trees.

Spacing experiments generally support this conclusion within moderately wide limits of spacing, but the extent to which the generalization applies depends on the species. The height growth of many hardwood species is significantly less at low densities than at moderate or high levels. Very dense natural stands of western hemlock and lodgepole pine display greatly reduced height growth, and some spacing studies with fast-growing conifers such as Douglas-fir and slash pine have shown significantly greater height growth for 10-by-10 feet spacings than for 6-by-6 feet plantings.
3.1.2 Diameter Growth

Spacing and thinning experiments have consistently shown increases in breast height diameter (dbh) growth with decreasing stand density. Stands with wider spacings or stands previously thinned, in time, have larger average diameters than similar stands with closer spacings or comparable unthinned stands.

Intercor competition affects diameter growth at surprisingly low stand densities, particularly in the case of fast-growing, shade-intolerant species. As a result, very low densities are required to produce maximum diameter growth throughout the life of an even-aged stand. At any given age, there is a lower limit of stand density below which no further increase in diameter growth will result from continued density reduction. At density levels below this lower limit, the trees are growing free of intertree competition and are usually referred to as open-grown trees. However, other vegetation present on the site may also affect diameter growth even though tree density is below this competition limit (Grusko, 1970). The general effect of spacing on average stand diameter is illustrated in Figure 3.1.

In stands where competition has caused a reduction in diameter growth compared to open-grown trees, the response to increases in growing space resulting from thinning varies with species, age, and quality of the site. Older trees with greatly reduced crowns do not respond as much as younger trees of comparable stem size, and dominant trees which have been relatively less affected by competition respond less to increased growing space in terms of relative diameter growth rate than smaller trees in the same stand. As in the case of average stand height, the average stand diameter may be affected by thinning, depending on the thinning method. This thinning effect should not be confused with the effect of density changes on subsequent stand diameter growth.

3.1.3 Stem Form

Differences in stem form resulting from variations in stand density are a matter of potential concern in timber management since their existence complicates the process of accurately estimating individual tree volumes. Spacing experiments have shown that denser stands often have average dominant and codominant trees of the same height but of smaller diameter than less dense stands, while thinning studies have indicated that stem diameter in the lower bole increases relatively faster in thinned stands than in comparable trees in unthinned stands. Such stems will have different taper curves, but not necessarily different stem volumes, as would be implied by a standard volume equation based on breast height diameter and total height. In fact, the effect of stand density on a common measure of stem form, the cylindrical form factor, is not readily deduced from its separate effects on height growth and breast height diameter growth. The volumetric effect of stand density on stem form can best be evaluated by comparing trees of the same diameter and total height (for example, a codominant tree in a less dense stand with a dominant tree of the same total height and diameter in a denser stand). In practice, differences in stem form that result from stand density variation seldom have an economically significant impact on stem volume.

3.1.4 Stand Basal Area and Volume Growth

In assessing the effect of density manipulation on stand basal area and volume growth, it is important to distinguish between the relationships that apply in all-aged stands and those that are appropriate to even-aged situations. An all-aged stand is distinguished by two primary characteristics. First, the stand has no definite beginning or end in time; second, trees of all ages are spatially intermixed within the stand. The general form of the relationship between stand density and net annual cubic volume growth rate for such stands is shown in Figure 3.2. Knowledge of the form of this relationship between growth and the amount of growing stock, or stand density, is important in the management of such a renewable resource. When continued harvests are imposed on the stand, an equilibrium is said to exist if, for each time period, net growth plus snag growth is harvested. This equilibrium condition produces a stationary reserve growing stock and a constant sustainable harvest. For each level of reserve growing stock,

\[ \text{The cylindrical form factor is defined as the ratio of stem volume to the volume of a cylinder with} \]
\[ \text{height equal to the height of the tree and diameter equal to tree dbh.} \]
there is an associated sustainable equilibrium harvest, and for some level of the growing stock, there exists a maximum sustainable equilibrium harvest.

In contrast, any even-aged stand has a definite beginning in time. For some time after stand establishment, the higher the stand density (within limits), the greater the net growth in basal area and total cubic volume. Typical results from spacing experiments in even-aged stands appear in Figures 3.5 and 3.4.

In thinning experiments involving even-aged stands, the effects of stand density variation on growth in terms of basal area or cubic volume have been inconsistent, partly as a result of the confounding effects of age and merchantability limits. A generalized result is shown in Figure 3.5, where gross yield in an unthinned stand is approximately equalled by the cumulative harvests plus the remaining stock in a thinned counterpart. The results of most thinning studies support the general conclusion that thinning does not significantly affect gross cubic volume yield per acre, except where severe overcrowding would greatly restrict root and crown development in an unthinned stand or where stand density is reduced so severely by thinning that the site is clearly underutilized for some time. Although there have been instances where thinning has actually increased total gross site cubic volume production, the rationale for thinning in timber management is generally based on economic considerations and a reduction of mortality loads rather than on the expectation of increased total production.

3.2 MEASURES OF AVERAGE STAND DENSITY

Measures of stand density can conveniently be grouped as measures of average stand density or as point density measures, depending on whether they express...
average overall crowding or the competitive stress affecting a particular tree (Spurr, 1962). Some of the measures can be directly calculated from data collected in the stand of interest, while others require additional reference to previously determined relationships. In this section, we consider some commonly used measures of average stand density. Methods for expressing point density are considered in Section 3.3.

The degree of crowding in a stand is determined by the number of trees present, their respective sizes, and their spatial distribution. Distribution is not considered explicitly in existing measures of average stand density. For timber management purposes in general, and for the prediction of stand growth and yield in particular, a measure of stand density should be easily and objectively measurable, biologically meaningful, and highly correlated with both stand growth and yield. Some commonly used measures of average stand density are discussed in the remainder of this section.

### 3.2.1 Number of Trees per Acre

In homogeneous all-aged stands and in unthinned even-aged stands of given age and site quality, the average number of trees per acre is a useful measure of stand density. Number of trees per acre is also meaningful when used for thinned stands if the thinning history is known in detail. Many plantation yield studies have used number of trees per acre as a measure of stand density in the development of yield equations and tables (Dull et al., 1979).

### 3.2.2 Basal Area per Acre

The cumulative cross-sectional area, per acre, of tree stems at 4.5 feet above ground level (basal area) is a widely used measure of stand density. Basal area per acre is easily and objectively measured and its value depends on both the number of trees and their respective sizes. It is often used as a relative measure of stocking by expressing the basal area per acre in a given stand either as a percentage of some appropriately chosen norm or as a percentage of average basal area for comparable stands.

As with number of trees per acre, the utility of basal area per acre as an indicator of crowding is limited when the prior stand history is unknown. However, when used in the proper context, such as for a given age and site in unthinned even-aged stands or plantations, or in all-aged stands with a reasonably stable age distribution, basal area has proved useful in yield estimation. Since basal area per acre and the number of trees per acre specify average tree size, the use of both will often give improved yield estimates in comparison to those obtained with the use of only one of these measures.

The basal area $B_i$ in square feet for a tree of breast height diameter $D_i$ inches is

$$B_i = \pi D_i^2/36 = 0.005454 D_i^2$$

(3.1)

and the total basal area $B$ for a sample of $n$ trees is

$$B = \sum_{i=1}^{n} B_i = 0.005454 \sum_{i=1}^{n} D_i^2$$

(3.2)

with a mean per-tree basal area of

$$\bar{B} = \frac{1}{n} \sum_{i=1}^{n} B_i = 0.005454 \frac{\sum_{i=1}^{n} D_i^2}{n}$$

(3.3)

The quadratic mean breast height diameter $\bar{D}$ is defined as

$$\bar{D} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} D_i^2}$$

(3.4)

so that

$$\bar{B} = 0.005454 \bar{D}^2$$
and
\[ B = 0.005454x \overline{D}_3^2 \]  

(3.5)

In other words, \( \overline{D}_3 \) is the diameter of the tree of mean basal area. It should also be noted that
\[ \sum_{i=1}^{n} (D_i - \overline{D})^2 = \sum_{i=1}^{n} D_i^2 - \left( \sum_{i=1}^{n} D_i \right)^2 / n \]

and
\[ \frac{1}{n} \sum_{i=1}^{n} (D_i - \overline{D})^2 = \frac{1}{n} \sum_{i=1}^{n} D_i^2 - \left( \sum_{i=1}^{n} D_i / n \right)^2 \]

so that
\[ \frac{1}{n} \sum_{i=1}^{n} D_i^2 = \frac{1}{n} \sum_{i=1}^{n} (D_i - \overline{D})^2 + \left( \sum_{i=1}^{n} D_i / n \right)^2 \]

or
\[ \overline{D}_n = \sqrt{\left( \frac{n-1}{n} \right)s^2 + \overline{D}^2} \]  

(3.6)

where \( s^2 \) is the sample variance of the \( D_i \) values.

### 3.2.3 Stand Density Index

Stand density index (SDI) is a measure of average stand density that can only be obtained with reference to a predetermined limiting relationship between the number of trees per acre and the average tree size. Frothingham (1914) used this relationship in the development of a white pine yield table and Reineke (1933) later used the same relationship as the basis for a stand density index.

In fully stocked even-aged stands the limiting relationship between the number of trees per acre \( N \) and the quadratic mean dbh \( \overline{D}_n \) often appears linear in logarithmic coordinates, as shown in Figure 3.6. For any given \( \overline{D}_n \), there exists a limit to the expected number of trees per acre in even-aged stands regardless of the age or site quality. Stands at the limit are assumed to experience the same

**Figure 3.6** The relationship between number of trees per acre and quadratic mean dbh in fully stocked even-aged stands.

degree of crowding. Reineke observed this relationship in even-aged stands of several species, both conifers and hardwoods. For the cases considered, the slope of the limiting line was approximately -1.6.

The limiting relationship is of the form
\[ N = \alpha \overline{D}_n^\beta \]  

(3.7)

where \( \alpha \) and \( \beta \) are the parameters that define the relationship. For stands in this limiting condition the common degree of crowding is expressed numerically as the expected number of trees per acre when \( \overline{D}_n \) is 10 inches, so that
\[ SDI = N(10) \overline{D}_n^\beta \]  

(3.8)

where \( SDI \) is the stand density index. For any stand of known \( N \) and \( \overline{D}_n \), Reineke defines the stand density index as
\[ SDI = N(10) \overline{D}_n^\beta \]  

(3.9)

which implies that all stands with the same proportion of the limiting number of trees per acre have the same stand density index regardless of average stand
GROWTH AND YIELD PREDICTION

For stands of given $D_x$, the stand density index increases proportionally with the number of trees per acre and, therefore, also with the basal area per acre.

The standard procedure for estimating the parameters $\alpha$ and $\beta$ requires appropriate sample data from fully stocked even-aged stands. Linear least squares estimates of the parameters are obtained from observations of the number of trees per acre and the quadratic mean $D_b$ for each stand using a logarithmic transformation of equation (3.7).

$$\log(N) = \log(\alpha) + \beta \log(D_x)$$  \hspace{1cm} (3.10)

For example, suppose

$$\hat{\alpha} = 10.119$$
$$\hat{\beta} = -1.5$$

so that the predicted number of trees per acre is given by

$$N = 10.119D_x^{-1.5}$$

and the full stocking stand density index value is

$$SDI = 10.119(10)^{-1.5} = 320$$

For a given stand with $N = 600$ and $D_x = 5.0$ inches,

$$SDI = 600(10)^{-1.5} = 212$$

The full stocking (limiting) number of trees per acre when $D_x = 5.0$ inches is

$$N = 10.119(5.0)^{-1.5} = 905.07$$

while the number of trees per acre in a full stocking (limiting) stand with $D_x = 15.0$ inches would be

$$N = 10.119(15.0)^{-1.5} = 174.18$$

GROWING STOCK AND STAND DENSITY

Thus, a stand with $D_x = 15.0$ inches would have the same stand density index as another stand with $D_x = 5.0$ inches and $N = 600$ trees per acre if the number of trees per acre is

$$N = (600/905.07) \times 174.18 = 115.35$$

This is easily verified by computing

$$SDI = 115.35(10/15.0)^{-1.5} = 212$$

The limiting relationship between number of trees per acre and quadratic mean $D_b$ in fully stocked stands as expressed in equation (3.7) assumes that the relative rate of mortality in such stands and the relative rate of increase in quadratic mean $D_b$ have the relationship

$$\frac{dN}{dt} = \beta \left( \frac{dD_b}{dt} \right)$$  \hspace{1cm} (3.11)

Before a stand reaches this limiting condition, mortality will be less than in stands of comparable quadratic mean $D_b$ at the limit, and the constant proportionality between the rates will not yet apply. A typical example of a stand approaching full stocking over time is shown in Figure 3.7. Parker (1976) used data from remeasured Monterey pine plots to explore the change in stand density index values over time and reported a new procedure for estimating the slope of the limiting relationship. An approach to stand density control of Douglas-fir plantations by Drew and Flewelling (1979) is based on stand density index as the measure of stand density. In an analysis of the utility of various stand density measures for the prediction of cubic volume growth in natural even-aged loblolly pine stands, Nelson and Border (1963) found no advantage for stand density index over the simpler basal area per acre when both were used in combination with information on age and site index.

3.2.4 Tree-Area Ratio

Clutter and Schumacher (1940) proposed a measure of stand density based on the assumption that the land area $A$ occupied by any given tree in a stand can be represented by the equation

$$A = \beta_1 + \beta_2D + \beta_3D^2$$  \hspace{1cm} (3.12)
where $D$ is tree diameter (dbh). The total area occupied by $n$ trees on one acre of land is then

$$ \sum_{i=1}^{n} A_i = \beta_0 n + \beta_1 \sum_{i=1}^{n} D_i + \beta_2 \sum_{i=1}^{n} D_i^2 \quad (3.13) $$

where the summation is made over the $n$ trees growing on the acre. Suppose $N$ one-acre sample plots were selected from a population of interest, and on each plot the values $n_i, \sum_{j} D_{ij}$ and $\sum_{j} D_{ij}^2$ ($i = 1, 2, \ldots, N$) were obtained, where $D_{ij}$ is the dbh of the $j$th tree on the $i$th sample plot and $n_i$ is the number of trees on sample plot $i$. Estimates of the parameters $\beta_0, \beta_1$, and $\beta_2$ are then obtained by minimizing

$$ \sum_{i=1}^{N} \left( 1 - \beta_0 n_i - \beta_1 \sum_{j} D_{ij} - \beta_2 \sum_{j} D_{ij}^2 \right)^2 \quad (3.14) $$

These least squares estimates can then be used to evaluate equation (3.13) for any given stand with known $n_i, \sum_{j} D_{ij}$, and $\sum_{j} D_{ij}^2$. The resultant tree-area ratio

$$( \sum_{i=1}^{N} A_i )$$

is a measure of stand density relative to the average relationship in

The estimated average area occupied by a tree of diameter $D = 10$ inches, for example, is calculated from equation (3.12) as

$$ A = 0.000200 + 0.000100(10) + 0.0000250(10)^2 $$

$$ = 0.0057 \text{ acres.} $$

For a stand with an average of 300 trees per acre whose diameters sum to 2400 and whose squared diameters sum to 19,200, the tree-area ratio (TAR) is calculated as

$$ TAR = \frac{0.000200(300) + 0.000100(2400) + 0.0000250(19,200)}{300} $$

$$ = 0.78 $$

### 3.2.5 Crown Competition Factor

Krajicek et al. (1961) proposed a measure of stand density that is appropriate for both even-aged and all-aged stands. It is based on the horizontal projection of the crown area of trees of given diameter relative to the maximum crown area for open-grown trees of the same diameter and is another measure of stand density based on a predetermined relationship. The relationship between crown width (CW) and diameter (D) for open-grown trees is assumed to be of the form

$$ CW = \alpha + \beta D \quad (3.15) $$

GROWING STOCK AND STAND DENSITY
If \( CW \) is in feet, the crown area \( CA \) in square feet is given by

\[
CA = \frac{(\pi/4)(CW)^2}{3.14}
\]

or

\[
CA = \frac{(\pi/4)(\alpha + \beta D)^2}{3.14}
\] (3.16)

The maximum crown area \((MCA)\), that is, the crown area for an open-grown tree of diameter \(D\), expressed as a percentage of an acre, is given by

\[
MCA = \frac{100(\pi/4)(\alpha + \beta D)^2}{43,560}
\]

which is equivalent to

\[
MCA = 0.001803(\alpha + \beta D)^2
\] (3.17)

The parameters \(\alpha\) and \(\beta\) are estimated from measurements of crown width and diameter on open-grown trees. An \(MCA\) value is then calculated for each tree in a stand, and the sum of all these values, on an acre basis, is the crown competition factor \((CCF)\). The \(CCF\) has been used as a measure of stand density in growth and yield estimation (Stapp, 1973).

As an example of how to calculate the crown competition factor, suppose a sample of open-grown trees of a given species was measured for average crown width \(CW\) in feet and diameter \(D\) in inches. From these observations, the parameter estimates obtained by least squares procedures were

\[\hat{\alpha} = 5.0\]
\[\hat{\beta} = 2.0\]

so that

\[CW = 5.0 + 2.0(D)\]

Maximum crown area \((MCA)\) is then given by

\[MCA = 0.001803(5.0 + 2.0D)^2\]
\[= 0.007212D^2 + 0.03606D + 0.04507\]

Computation of the \(CCF\) for an example stand is shown in Table 3.1.

<table>
<thead>
<tr>
<th>(D) (inches)</th>
<th>Number of Trees/ Acre</th>
<th>(MCA) (%)</th>
<th>(MCA \times) Trees/Acre (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>0.3047</td>
<td>3.047</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>0.4057</td>
<td>12.171</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>0.5211</td>
<td>26.655</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
<td>0.6599</td>
<td>39.054</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>0.7951</td>
<td>39.755</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>0.9538</td>
<td>28.614</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1.1269</td>
<td>11.269</td>
</tr>
</tbody>
</table>

\[CCF = \frac{150}{1000}\]%

### 3.2.6 Spacing Index or Relative Spacing

A density measure based on the average distance between trees and the average height of the dominant canopy was first proposed for plantations by Hart (1928) and was later referred to as a spacing index (Beekhuis, 1954; Hummel, 1954) and then as relative spacing (Beekhuis, 1966). Apparently, this statistic evolved from the observation that stands judged to have desirable stocking shared similar ratios of average distance between trees to average dominant height. Some reflection on this observation led to the suggested use of the ratio as a measure of stand density. Spacing index or relative spacing is defined as

\[
RS = \frac{\text{Average distance between trees}}{\text{Average height of dominant canopy}}
\]

which, if square spacing is assumed, can be written as

\[
RS = \frac{\sqrt{3.14\times N}}{H}
\] (3.18)

where

\[N = \text{number of trees per acre} \]
\[H = \text{average dominant height}\]

Some have argued that triangular spacing is a more realistic assumption for calculating the average distance between trees, but since any triangular-spacing
RS value is a constant proportion of the corresponding square-spacing RS value, the distinction is irrelevant as far as a density measure is concerned.

An interesting feature of relative spacing is illustrated in Figure 3.8, which shows the development through time of relative spacing values in a close-spacing and a wide-spacing stand. Regardless of site quality and initial age, all stands of a given species seem to asymptotically approach a common minimum relative spacing value as they grow older. For any given average dominant height growth curve, this lower limit establishes a maximum number of trees per acre to be expected at any given age. It should also be noted that a combination of the height growth curve and the RS time trend line implicitly defines a survival curve (Parker, 1978). Relative spacing has been used to control density in intensively managed plantations. For example, thinning regimes have been designed to attain prespecified values of relative spacing at each thinning, and thinning intervals have been determined to coincide with specified increments in dominant height. Yield tables have been developed in which average dominant height replaces both age and site quality in yield prediction (Beekhuis, 1966).

As an example of the application of the relative spacing index, consider an even-aged stand that has an average dominant height of 44 feet and 300 trees per acre. With square spacing, the relative spacing value is

\[ RS = \sqrt{\frac{43,560N}{44}} \]

\[ = 0.25 \]

As an illustration of the use of relative spacing in developing thinning specifications, consider a thinning schedule that involves two thinnings. Both are to leave relative spacing values of 0.275 in the residual stand. The first thinning will be imposed when average dominant heights are 40 feet, and the second thinning will be made when average dominant height reaches 60 feet. The numbers of trees per acre to be left after thinning are:

1. When average dominant height is 40 feet,
   \[ N = 43,560(40 \times 0.275)^2 = 360 \]

2. When average dominant height is 60 feet,
   \[ N = 43,560(60 \times 0.275)^2 = 160 \]

3.3 MEASURES OF POINT DENSITY

The data-handling capacity and speed of modern computers have made possible the development of complex stand growth and yield simulation models. Most detailed are the so-called single-tree, distance-dependent tree growth simulators which take into account the actual spatial distribution of trees as well as their numbers and sizes. Stem maps, in the form of location coordinates, are maintained during growth simulation, and individual trees are characterized in terms of several measures of size and form, which are then projected over time.

Point density measures are attempts to refine average stand density measures so that the varying degrees of crowding experienced by individual trees in a stand can be incorporated into the procedure of projecting individual tree growth. These measures can also be used in the prediction of mortality probabilities (Keister and Tidwell, 1975), and possibly for the introduction of natural regeneration into the stand during simulation (Spurr, 1962). Measures of point density for single-tree growth simulators are based on the dimensions of the subject tree and on the dimensions of, and distances to, competitors. The utility of a measure is judged in terms of its correlation with observed tree growth and its computational simplicity (Daniels, 1976).

Lemmon and Schumacher (1962) proposed a measure that used Bitterlich's angle-count method to provide an estimate of basal area per acre in the vicinity of the subject tree. An obvious shortcoming of this approach is its inability to
take into account the relative sizes of the subject and competing trees and their spatial distribution. Spurr (1962) suggested a method of estimating point density that essentially estimates the subtended angles for a fixed number of surrounding trees. The measure is expressed in basal area per acre and is obtained by associating with the tree that subtends the largest angle. A basal area per acre estimate is obtained from the distance to the tree and the angle it subtends. A second basal area per acre estimate is obtained by accumulating similar basal area per acre estimates using the two trees with largest subtended angles. The process continues using 3, 4, . . . , and finally n trees. The point estimate of density is then computed as the mean of the n basal area per acre estimates.

Most of the recent studies of intertree competition and tree growth have used a different approach that involves the concept of a competition influence zone around each tree and assumes the area, in horizontal projection, over which a tree competes for all site resources can be represented by a circle whose radius is a function of tree size. The competitive stress experienced by a given tree is then assumed to be a function of the extent to which its competition circle overlaps those of neighboring trees (Staal, 1951). Different functions of tree size for determining the radius of the competition circle and different procedures for approximating or calculating overlap areas and weighting these by relative sizes of competitors have led to a rapid proliferation of competition indexes (Newirth, 1966; Opie, 1968; Gerrard, 1969; Bell, 1971; Arney, 1973; Ek and Monsrud, 1974). For example, Gerrard (1969) suggested that competition index be defined as

\[
CI_i = \frac{1}{A_i} \sum_{j=1}^{n} a_j
\]  

(3.19)

Figure 3.9 An illustration of the competition zone overlap used in the definition of point density measures.
GROWTH AND YIELD PREDICTION

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GROWTH AND YIELD PREDICTION


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