to continue using strategies that are successful. Thus if the task of '16 - 12' is placed later in the interview, a child who has used counting-by-ones successfully, to solve an earlier subtractive task might use counting-by-ones out of habit almost, rather than a more advanced strategy, to solve '16 - 12'.

**Missing Subtrahend**

A purpose of the missing subtrahend tasks is to elicit the strategy of 'counting-down-to'. The task involves displaying and screening a collection and while the child is looking away to reduce the collection to a certain amount. The child has to calculate by how many the set was reduced, that is, the missing subtrahend. This is a challenging task for most young children so an introductory item is provided, followed by two entry tasks. If the child gets both correct a more advanced item is available.

- **Missing subtrahend**
  - **Introductory task**: 5 to 3 \[5 - \_ = 3\]
  - **Entry tasks**: 10 to 6 \[10 - \_ = 6\] 12 to 9 \[12 - \_ = 9\]
  - **More advanced task**: 15 to 11 \[15 - \_ = 11\]

**Removed Items**

The purpose of the removed items tasks is to elicit the strategy of 'counting-down-from'. As with all of the assessment tasks it is important for the interviewer to maintain an open frame of mind about the types of strategies the children use to solve these subtractive tasks. The task is simple to administer because the assessor reduces the first screened collection by a stated amount and asks the child how many will remain under the screen. The removed quantity is briefly displayed to the child and then screened. As above there is an introductory task followed by entry and advanced tasks.

- **Removed items**
  - **Introductory task**: 3 - 1
  - **Entry tasks**: 6 - 2 \[6 - 2 = 4\] 9 - 4 \[9 - 4 = 5\] 15 - 3 \[15 - 3 = 12\]
  - **More advanced task**: 27 - 4

Earlier, when we presented partially screened, additive tasks, 8(b) we indicated that if a child responded correctly to all three items then the assessor could present the subtractive tasks but commencing with removed items, 9(c). It is anticipated that such children would have difficulty with the subtractive tasks beyond removed items.

In the next chapter we explain and exemplify the Stages of Early Arithmetical Learning.

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**The Stages of Early Arithmetical Learning (SEAL)**

**Summary**

The Stages of Early Arithmetical Learning are defined and illustrated by profiles of typical children. These are explained further by the presentation of scenarios taken from assessment and teaching sessions. Additional guidance is provided to analyze what might be termed 'difficult cases'.

**INTRODUCTION**

In the previous chapter we showed how to administer the Assessment Interview 1.1. We presented information and models for the analysis and allocation of levels for the Number Words and Numerals Strand of the LFIN and explained how to administer the additive and subtractive tasks. In this chapter we focus on the analysis of the child's performance on these tasks by explaining and exemplifying the five Stages of Early Arithmetical Learning (SEAL). Table 4.1 show the significant task groups in Assessment Schedule 1.1 for a given Stage of Early Arithmetical Learning. Assessment 1.1 will place a child in one of six Stages. More detailed information on whether a child is Stage 5 can be gleaned by the application of Assessment Interview 1.2, Task Group 3: Non-Count-by-Ones Strategies which is explained in Chapter 6.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Significant tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Emergent Counting</td>
<td>8: Additive tasks</td>
</tr>
<tr>
<td>1: Conceptual Counting</td>
<td>8: Additive tasks</td>
</tr>
<tr>
<td>2: Figurative Counting</td>
<td>8: Additive tasks</td>
</tr>
<tr>
<td>3: Initial Number Sequence</td>
<td>8, 9: Additive and Subtractive tasks</td>
</tr>
<tr>
<td>4: Intermediate Number Sequence</td>
<td>8, 9: Additive and Subtractive tasks</td>
</tr>
<tr>
<td>5: Fast Number Sequence</td>
<td>8, 9, Additive and Subtractive tasks and Task Group 3, Non-Count-by-Ones Strategies, Assessment Interview 1.2</td>
</tr>
</tbody>
</table>

Progression across the stages involves the child using counting in increasingly sophisticated ways to solve additive and subtractive tasks. For each of the stages, we present two examples of children's solution attempts. Each example consists of an objective description of the child's solution of one or more problems, and a discussion of the strategies used by the child in terms of SEAL.

The model of the Stages of Early Arithmetical Learning was introduced in Chapter 2. The Stage model constitutes the primary and most important aspect of the Learning Framework in Number, and provides a model of increasingly advanced numerical strategies used by children in
situations that involve counting, adding or subtracting. In this chapter the origin and background of SEAL is explained. This presentation includes an introduction to the particular sense in which the term 'counting' is used in SEAL, and explanations of the terms 'stage' and 'level.' Together with a discussion of the means by which children's numerical strategies may be observed. This is followed by a detailed description of each of the stages. Included in the description of each stage are examples of children's problem-solving activity relevant to the stage. These examples take the form of video excerpts, that is, objective descriptions of the child's problem-solving activity from real classroom situations. Each video excerpt is followed by a discussion of the problem-solving activity portrayed in the excerpt.

Overview of SEAL

The model of the Stages of Early Arithmetical Learning is based on a coherent body of research into young children's number learning undertaken by Steffe (Steffe, 1992a; Steffe and Cobb, 1988; Steffe et al., 1983) and related research by Wright (1989; 1991a). This research involved longitudinal studies in which children were taught several times per week during teaching cycles of up to 20 weeks duration, in their first and second years of school. An important focus of this research was the ways in which children's numerical strategies arose during inquiry-based teaching, and how these strategies developed and changed over the course of one or two years of school.

Teachers who have learned about and applied SEAL have found that it provides crucial directionality to their teaching. By applying SEAL in the observation and assessment of children, teachers have a sound knowledge of the child's current level in terms of the development of early arithmetical strategies. This is very important when teachers are being required to have enhanced assessment skills in deciding standards of performance. Also, SEAL provides a clear framework for determining teaching activities that are optimal in the sense of leading to advancement. By this we mean advancements in which children recognize their numerical thinking and construct novel strategies that, in a mathematical sense, are more sophisticated than their previous strategies. For a full discussion of how SEAL relates to teaching see Wright et al. (2002).

COUNTING

Steffe et al. (1983) make a crucial distinction between counting and the mere utterance of number words in sequence. Counting in the terms of Steffe et al., arises in problem-solving contexts, for example, additive or subtractive situations. In these situations it is assumed that the child has a goal, for example, figuring out how many counters are in two screened collections or how many counters remain under the screen when some have been removed. In the terms of Steffe et al. (1983) counting involves the coordination of each uttered number word with the conceptual production of 'unit items'. The nature of this unit item changes according to the child's stage in SEAL. Steffe's notion of 'unit item' is particularly useful for observing and understanding the counting behavior of children who are in Stage 2. This is discussed later in this chapter.

STAGES AND LEVELS

In the work of Steffe et al. (1983) the term 'stage' is used in a formal and theoretical sense to encompass four characteristics:

1. A characteristic remains constant for a period of time.
2. The stages form an invariant sequence.

3. Each stage builds on and incorporates the previous stage.
4. Each new stage involves a significant conceptual reorganization.

A level is regarded as a point in time rather than a period of time. The child has attained a level when he or she satisfies specific performance criteria for that level. This distinction between stage and level is explained in detail by von Glasersfeld and Kelley (1982). In the Learning Framework in Number, the term 'level' is used in the models which focus on the development of number word sequences, numeral identification and knowledge of tens and ones, and the term 'stage' is used in the SEAL model, which is the focus of this chapter.

OBSERVING CHILDREN'S STRATEGIES

Fundamental to the research on which SEAL is based is the notion that, in order to understand children's mathematical learning, it is necessary to closely observe their behavior in problem-solving situations. In studying children's early numerical learning in particular, this observation focuses on children's verbal and non-verbal responses in situations in which they solve problems involving counting, addition or subtraction. These problems typically involve presenting children with a range of tasks involving counters that may be displayed or screened. These types of situations, that is, involving what adults might regard as trivial number problems, have been found to be extremely useful for the purpose of studying children's early numerical learning.

EXPLAINING THE STAGES

In the remainder of this chapter, each of the Stages of Early Arithmetical Learning is explained. First we present the definition of the Stage and then a profile of a typical child at each stage and how this relates the models in Part A and Part B of the LEFlN. We use the profile of a typical child for convenience here to exemplify the stage. Of course, knowing that a child is at a given stage in terms of SEAL does not necessarily determine the child's level of the other models of early number knowledge. Nevertheless, it is feasible to assume levels on the other models that frequently appear for a child at a given stage. The numerical profile, derived from the models, is further fleshed out by describing the behaviors of a typical child at each stage and how their performance can be linked to the assessment schedules. Two illustrations of child problem-solving behavior, each in the form of an objective description of an excerpt from a videotape of an assessment or teaching session, are presented. Each excerpt is followed by a discussion. In writing the discussion the author takes account of the videotape excerpt per se, rather than the description. Ten of the excerpts are based on Mathematics Recovery assessment sessions and two on MR teaching sessions. For purposes of clarification, this is stated at the beginning of the excerpt in the case of the two teaching sessions.

STAGE 0: EMERGENT COUNTING

Stage 0: Emergent Counting. The child cannot count visible items. The child either does not know the number words or cannot coordinate the number words with items.

The work of Steffe et al. (1983) and Steffe and Cobb (1988) resulted in a five-stage model of the development of early arithmetical strategies, that is, strategies for counting, adding and subtract-
The first of these stages is given the label of 'perceptual'. In research projects conducted by Wright (1991b; 1994) prior to MR, it became apparent that among children in the first two years of school, there are a significant number who are not able to count a visible collection, for example a collection of 12, 15 or 18 counters. In MR the label 'emergent' was adopted for such children. The label 'Stage 0' is used to indicate children at this emergent level.

**The Typical Emergent Child**

In Table 4.2 we set out the stage and indicative levels for the emergent child on the models pertaining to FNWSSs, BNWSSs, numerical identification and tens and ones.

<table>
<thead>
<tr>
<th>Model</th>
<th>Stage/level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage of Early Arithmetical Learning (SEAL)</td>
<td>0</td>
</tr>
<tr>
<td>Level of Forward Number Word Sequences (FNWSS)</td>
<td>1</td>
</tr>
<tr>
<td>Level of Backward Number Word Sequences (BNWSS)</td>
<td>0</td>
</tr>
<tr>
<td>Level of Numerical Identification</td>
<td>0</td>
</tr>
<tr>
<td>Level of Tens and Ones Knowledge</td>
<td>0</td>
</tr>
</tbody>
</table>

**Stage of Early Arithmetical Learning**

The child at the Emergent Stage (Stage 0) is not able to count a collection of counters, for example 13 or 18 counters. The child might not know the forward number word sequence from one to ten or beyond ten. Alternatively the child might know the forward number words but not be able to correctly coordinate a number word with each counter. This might result in omitting one or more counters, or perhaps pointing to one or more counters twice during counting. More frequently the child simply lacks the ability to coordinate each spoken number word with each item to be counted. In many cases the child's pointing actions seem to upset their production of the number words in turn. The child might be able to count the counters in smaller collections, for example, collections up to 10. Some emergent children seemed to interpret questions such as: 'How many counters are there?' literally as an instruction to say the number word sequence from one. It is as if the literal meaning for the child is to say the words 'one, two, three, and so on', while pointing at the collection of items in question. This might involve pointing generally in the direction of the collection, rather than pointing at each item in turn.

**FNWSSs, BNWSSs and Numerical Identification**

The emergent child might well be able to say the forward number word sequence from one to ten, but it is probably they will not be able to say, immediately, the number word after a given number word in the range one to ten. Also, they probably will not be able to use a dropping back strategy, to say the number word after a given number word, for each of the numbers in the range one to ten. The emergent child typically has difficulty in saying backward number word sequences. For example, the child might not be able to say the words from ten to one, and might even have difficulty in saying the words from three to one. As well, the child typically cannot say the number word before a given number word, even permitting use of the dropping back strategy. The emergent child can typically name some, but not all, of the numerals in the range one to ten. They might be able to recognize most of the numerals in the range 1 to 5, but might be unable to name numerals beyond five, or might confuse numerals such as 6 and 8 or 6 and 9.

**Spatial Patterns and Finger Patterns**

The emergent child might be able to recognize some but not all of the regular spatial patterns (for example, the domino patterns) in the range 2 to 6. In many cases they will try to count the dots in a spatial pattern rather than immediately assign a number name to the pattern. The emergent child might be able to make finger patterns corresponding to the numbers from one to five. This typically will involve looking at the fingers and raising them slowly and sequentially, for example, the child might slowly raise three fingers sequentially, in response to a request to show three fingers. The emergent child might be able to copy or count temporal sequences of two or three but not larger (for example, on hearing a sequence of three sounds the child could tap a similar sequence).

Two illustrations of the problem-solving behavior of a child at Stage 0 now follow, each in the form of an objective description of an excerpt from a videotape of an assessment or teaching session. Each excerpt is followed by a discussion.

**Video Excerpt 0.1: Heather and Larry**

H: (Places out five red counters and screens them. Then places out four blue counters and screens them.) Five red counters and four blue counters. Can you tell me how many I have altogether this time?

L: (Immediately.) Six.

H: Are you thinking? Five and four more. (Points to the screens in turn.)

L: (Thinks for a few seconds, looking up.) Umm.

H: How are you going to work out how many that makes?

L: (Sequentially raises the five fingers of his left hand. Then looks at his right hand while raising one finger.) How many is that?

H: Five on this one and four on this one. (Points to the screens in turn as before.)

L: (Sequentially raises four fingers on his right hand and then looks at his left hand.) One, two, three, four (wagging a finger in coordination with each count. Now looks at his right hand.) Five. (Pauses.) Five, six, seven, eight! (Again wagging fingers in turn.)

H: (Removes both screens.) Have a look.

L: (Looks at the collection of five counters.) One, two, three, four. (In co-ordination with four points over the collection of five counters. Now looks at the collection of four counters. Points at one counter.) Five. (Pauses.) Five, six, seven, eight! (In coordination with four points over the collection of four counters.)

H: (Places out 13 counters.) Can you tell me how many counters there are there altogether?

L: One, two, three . . . twelve. (Moves each of the 13 counters in turn but makes an error in coordinating his counts with his points.)

**Discussion of Video Excerpt 0.1**

Significant in the above excerpt is that, on three occasions, Larry does not correctly count visible items. When the screened collections of five and four are unscreened Larry counts 'eight' rather than 'nine'. When counting the collection of 13 counters, Larry counts 'twelve'. Additionally, Larry counts eight when counting his raised fingers, that is five on one hand and four on the other. On each of the three occasions Larry is unable to properly coordinate his counts with the perceptual items, that is, counters. Counting behavior of this kind is illustrative of the child at the Emergent Stage, Stage 0.
Video Excerpt 0.2: Terry and Rita

T: What's three plus two? (Points to three unscreened counters and then to two unscreened counters.) How many would that be?  
R: (Looks at Terry.) Equals.  
T: What does it equal?  
R: (Immediately) Equals nine.  
T: Nine do you think? (Points to both collections.) Do you think there are nine counters there?  
R: Uh, yeah!  
T: I'm going to give you a group of counters Rita. (Places out 13 counters.) Can you tell me how many counters there are?  
R: One, two, three ... eleven. (Coordinates a number word with each count but omits to point at two of the counters.)

Discussion of Video Excerpt 0.2

In the above protocol Rita is presented with the task of 3 counters plus 2 counters with both collections unscreened. It is interesting to observe that Rita does not use a counting strategy to figure out how many counters altogether. She answers 'equals nine' but does not count. She does not spontaneously count the counters in this particular situation, that is, with two separate collections – even though they are very small collections – 3 and 2. She does not seem to have this strategy available, that is, she does not seem able to conceive of the counters as forming or being reformed into one collection, for the purposes of establishing how many altogether. At this point Terry places out 13 counters and asks Rita to tell her how many counters there are. Rita counts from 'one' but does not correctly coordinate the number words with the counters. For this reason Rita is classified as Stage O: Emergent Counting.  

Children who are classified at Stage O are those who are not able to count perceptual items, that is, they cannot coordinate number words with items when they are present. When attempting to count 13 counters Larry points 13 times while saying the words from one to twelve. Larry's error is one of not coordinating his number words with his points. By way of contrast Rita correctly coordinates her number words with her points but omits two of the counters. A child may also be classified as Stage O if they cannot say the forward number word sequence, for example, the child cannot count 13 counters because they do not know the number word sequence from one to thirteen. Rita's difficulty on the first task, that is, not having a strategy to establish the numerosity of two collections, for example, not counting the collections, has been observed among children classified as being at Stage O (and also some children at Stage 1). Their lack of counting to solve these tasks is possibly attributable to an inability to conceptualize that two collections can alternatively be regarded as one collection.

STAGE 1: PERCEPTUAL COUNTING

Stage 1: Perceptual Counting. Can count perceived items but not those in screened (that is concealed) collections. This may involve seeing, hearing or feeling items.

Children at Stage 1: Perceptual Counting are able to solve additive tasks involving material (for example, counters) which is perceptually available (for example, visible), that is, tasks involving one collection or two collections of counters that are displayed rather than screened (Steffe et al., 1988, pp. 22-3). Children at Stage 1 are unable to solve additive or subtractive tasks involving screened items. Children at Stage 1 are said to count 'perceptual unit items.'

Table 4.3 sets out the stage and indicative levels for the perceptual child on the models pertaining to FNWSs, BNWSs, numeral identification and tens and ones.

<table>
<thead>
<tr>
<th>Model</th>
<th>Stage/level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage of Early Arithmetical Learning (SEAL)</td>
<td>1</td>
</tr>
<tr>
<td>Level of Forward Number Word Sequences (FNWSs)</td>
<td>3</td>
</tr>
<tr>
<td>Level of Backward Number Word Sequences (BNWSs)</td>
<td>1</td>
</tr>
<tr>
<td>Level of Numeral Identification</td>
<td>1</td>
</tr>
<tr>
<td>Level of Tens and Ones Knowledge</td>
<td>0</td>
</tr>
</tbody>
</table>

The Typical Perceptual Child

The child at the Perceptual Stage is able to count a collection of counters, for example, 13 or 18 counters, but is not able to solve additive tasks involving two collections in cases where one or both of the collections are screened. Some children at this stage seem to have difficulty establishing the numerosity of two unscreened collections taken together. In the case of a task such as establishing the numerosity of a single collection, for example, a collection of 18 counters, the child can correctly count the counters by ones from one to 18. In the case of two separate collections, for example a collection of 9 red counters and a collection of 6 blue counters, some perceptual children do not seem to alternatively regard the two collections as one for the purposes of establishing the overall numerosity. This difficulty has been observed to arise even in cases involving very small numbers of counters (for example, five and two). Thus, when asked how many counters in all the child answers 'five' or 'two', but does not seem to realize that they can make one count from one to count the counters in both collections. As well, in the situation just described (that is, a collection of 5 counters and a collection of 2 counters — labeled verbally with the number words 'five' and 'two' respectively), some children seem to regard the number 7, and 5 and 2, as constituting a 2-digit number, that is 'fifteen.' The latter behavior can result from premature teaching of place value ideas.

FNWSs and BNWSs and Numeral Identification

The child at the Perceptual Stage typically has good facility with FNWS in the range 1 to 10. As well, the child might be able to say the FNWS into the twenties, sometimes stopping at 29 or might say the number word sequence beyond 30. Nevertheless the child might well have difficulty with tasks involving saying immediately the number word after a number beyond 10 (for example, say the number word after 13). Thus the child might use a dropping back strategy or not be able to solve the task at all. In the case of BNWS the child typically will be able to say the number words from 10 to 1, but might have difficulty with tasks involving saying immediately the number word before a number in the range one to ten. Again, the child might not be able to solve the task at all, or might use a dropping back strategy.

Children at this stage typically can recognize and identify numerals in the range 1 to 10, but might not be able to do so with numerals in the teens. They might incorrectly identify '12' as 'twenty' or 'twenty-one,' or a numeral in the range '13' to '19' as the corresponding decade number (for exam-
Spatial Patterns and Finger Patterns

Children at this stage typically can subitize in the range one to four, that is, they can correctly ascribe number to spatial patterns, particularly regular patterns in the range one to four, and typically do not count from one when doing so. Some children at this stage have facile finger patterns for numbers in the range 1 to 5, and might use their finger patterns to solve additive tasks in cases where both numbers are in the range 1 to 5. This frequently occurring strategy involves making a finger pattern on one hand to correspond with the first addend and similarly making a finger pattern on the other hand for the second addend. The child then counts their raised fingers from one to obtain the answer. Establishing the finger patterns might involve sequentially raising fingers in coordination with counting to the number of the corresponding addend. Children with more facile finger patterns will raise fingers sequentially to correspond to an addend, and do not count from one when doing so. At the same time, these children typically will not have facile finger patterns for the numbers in the range 6 to 10. Thus if asked to show 7 on their fingers they will raise seven fingers sequentially while counting from one to seven.

In Video Excerpt 1.1, Amy solves tasks in which the counters are displayed but is generally unsuccessful on tasks in which counters are screened.

**Video Excerpt 1.1: Mary and Amy**

M: (Places out five blue counters and screens them.) Five counters there. (Places out two red counters leaving them unscreened.) And two counters there. How many altogether?
A: (Quickly.) Five. Eight.
M: (Removes the screen covering the five blue counters.) Want to check?
A: (Looks steadily at the desk on which counters are placed for five seconds and then looks up.) One.
M: (Waves hand over the counters.) How many altogether?
A: (Immediately.) Three.
M: Would you like to count them up?
A: (Points at one blue counter.) One. (Pauses and then moves the collection of two red counters adjacent to the collection of five blue counters.) One, two . . . seven. (Counts quickly while pointing to each counter in turn.)
M: This time, there are four counters there. (Places out four red counters and screens them.) And four counters there. (Places out a second collection of five red counters and leaves them unscreened.) How many altogether?
A: (Looks at the unscreened collection for five seconds.) One.
M: (Places her hand on the screen.) How many are under here?
A: (Immediately.) Four.
M: (Waves her hand over the unscreened collection.) How many here?
A: (Immediately.) Four.
M: (Waves her hand over both collections.) How many altogether?
A: (Looks at the desk for three seconds.) Eight.

Discussion of Video Excerpt 1.1

In the first task Amy answers quickly 'five' and then 'eight'. She does not appear to have a strategy involving counting. When Mary removes the screen, Amy correctly counts the seven counters. In the second task Amy looks at the unscreened counters for five seconds and then answers 'one'. Mary re-poses the task and Amy on this occasion answers 'eight' but no counting strategy is apparent. It seems that Amy recalls that 4 + 4 makes 8 and hence is able to answer correctly on this occasion. On the third task Amy again does not have a counting strategy and her answers seem to be no more than guesses. When the screen is removed Amy seems to be immediately aware of what she is expected to do. She pushes the two collections together and correctly counts the 12 counters. Amy’s act of pushing all the counters together seems to be of cognitive significance for her. It seems that, from her point of view, it now makes sense to count a collection whereas, when the counters were in two collections, regarding all of the counters as a collection, that is, including the screened and unscreened counters, did not make sense. In summary, Amy does not have a strategy for solving additive tasks involving two collections, one of which is screened. This is characteristic of children classified as Stage 1: Perceptual Counting.

**Video Excerpt 1.2: Heather and William**

H: (Places out three red counters and screens them.) Three red counters, William. I'm going to put them under there. (Places out two blue counters and screens them.) Two blue counters and I'm going to put them under there. How many counters is that altogether?
W: (Immediately.) Four.
H: Would you like to look and check?
W: (Rises screen and points to three counters in turn.) One, two, three. (Rises the other screen and points to two counters in turn.) Four, five!
H: Were you right?
W: (Shakes his head indicating that he was not correct.)
H: We'll try another one. I'll make it five red counters this time. (Places out two more red counters and screens them.) And four blue counters. (Places out two more blue counters and screens them.) How many does that make altogether?
W: (Immediately.) Eleven.
H: Eleven. How did you work that out?
Stage 2: FIGURATIVE COUNTING

Stage 2: Figurative Counting. Can count the items in a screened collection but counting typically includes what adults might regard as redundant activity. For example, when presented with two screened collections, tell how many in each collection, and asked how many counters in all, the child will count from ‘one’ instead of counting on.

Table 4.4 sets out the stage and indicative levels for the perceptual child on the models pertaining to FNWSs, BNWSs, numerical identity and tens and ones.

<table>
<thead>
<tr>
<th>Model</th>
<th>Stage/level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage of Early Arithmetical Learning (SEAL)</td>
<td>2</td>
</tr>
<tr>
<td>Level of Forward Number Word Sequences (FNWSs)</td>
<td>4</td>
</tr>
<tr>
<td>Level of Backward Number Word Sequences (BNWSs)</td>
<td>3</td>
</tr>
<tr>
<td>Level of Numerical Identification</td>
<td>2</td>
</tr>
<tr>
<td>Level of Tens and Ones Knowledge</td>
<td>0</td>
</tr>
</tbody>
</table>

The Typical Figurative Child

The child at the Figurative Stage can solve additive tasks involving two screened collections, and when doing so, counts from one. Thus on an additive task such as a collection of 5 red counters and a collection of 4 yellow counters, both of which are screened, the child might count from one to five to count the counters in the first collection, and then continue counting from six to nine to count the counters in the second collection. From a cognitive perspective it seems necessary for the child to count from one in order to give meaning to six, that is, the number of counters in the first collection. Also, the child typically cannot make sense of subtractive tasks such as Missing Addend, Missing Subtrahend or Comparison tasks.

FNWSs, BNWSs and Numerical Identification

Children at this stage typically are facile with FNWSs in the range 1 to 30 but might not know the FNWS to 100. They might not be able to say the number word after some of the numbers beyond 30, for example, 49 or 80. Typically these children are facile with BNWSs to 10, but might have difficulty in producing the BNWSs from 23 to 16 and from 15 to 10. In similar vein they might have difficulty saying the word before number words such as 12, 15, 20, 21 and 30. They might use a dropping back strategy to work out the number word before a given number word. Some children at this stage say an incorrect decade number when saying BNWSs, for example, ‘52, 51, 49, 49’, ‘48’. Children at this stage typically can recognize and identify one-digit numerals and numerals in the teens, although the numeral ‘12’ might be incorrectly identified as ‘twenty’ or ‘twenty-one’. Also common at this stage is the error of ‘digit reversal’ when identifying two-digit numbers. Thus, for example, ‘27’ is named as ‘seventy-two’.

Spatial Patterns and Finger Patterns

Children at this stage typically will have facile finger patterns for numbers in the range 1 to 5 and some children will also have finger patterns for numbers in the range 6 to 10. Children may also have sound knowledge relating to spatial patterns (for example, pair-wise patterns for the numbers from 1 to 10) and are able to subitize random arrays of up to 5 items. They can regard a pattern for 6 for example, alternatively as 3 twos or 2 threes, and a pattern for 8, as 2 fours or 4 twos.

Figural, Motor and Verbal Counting

Children at Stage 2 have been observed to use one or more of three types of counting, that is, figural, motor or verbal. Alternatively, one could say the child counts figural, motor or verbal unit items (as discussed earlier in this chapter). Typically these types of counting arise when the child is counting the second, screened collection. In the case of figural unit items, visualized items (for example, counters) are regarded as being significant for the child when counting. Thus, in solving the above task, the child may be observed to be apparently visualizing a collection of four counters. In the case of motor unit items, movements (for example, sequentially raising fingers) are regarded as significant because the child utters the corresponding number word after making each movement. Finally, in the case of verbal unit items, the uttered number words are regarded as significant. For example, in solving the above task the child will use double counting. Thus after counting from one to eight, the child may say ‘one more is nine’, and so on, and thereby keep track of the four counters in the second screened collection.

The label of ‘figurative’ is applied collectively to figural, motor and verbal counting because each involves counting based on re-presentation (like a mental replay) of sensory-motor material, rather than direct perception of sensory-motor material which occurs at Stage 1. Figural is
regarded as the least advanced of these and verbal is regarded as the most advanced, and advancement from figural to motor and motor to verbal involve progressively less dependence on re-presented sensory material. Thus the child whose most advanced strategy involves counting figural unit items is referred to as 'early Stage 2' and the child whose most advanced strategy involves counting verbal unit items is referred to as 'late Stage 2'.

Video Excerpt 2.1: Jane and Shirley

The following video excerpt is taken from an MR teaching session.

J: (briefly displays and then screens three counters. Then screens two counters.) And two here. How many have I got altogether? Three and two?
S: (looks at the screen covering three counters and makes three points in coordination with counting.) One, two, three (turns to the other screen and make two points in coordination with counting), four, five.
J: (removes both screens.) Show me.
S: (Points at each counter in turn.) One, two, three - four, five.
J: Right. What about if I put five here (screens five counters) and three here? (screens three counters.)
S: (Looks at the screen covering three counters and makes three points in coordination with counting.) One, two, three (looks at the other screen and again coordinates a point with each number word), four, five, six, seven (pauses and looks up), eight!
J: (removes both screens.) Show me.
S: (Points at each counter in turn.) One, two, three - four, five, six, seven, eight!
J: What about if I move ten here (screens ten counters) and three here (screens three counters).
S: (Looks at the screen covering ten counters and makes three points in coordination with counting.) One, two, three (looks at the other screen and again coordinates a point with each number word), four, five, six, seven, eight, nine (pauses and looks up) - ten - eleven, twelve.
J: Have another think. (Places her hand on the screen covering ten counters.) Start with this side. How many here?
S: Ten.
J: Right. Start with this side.
S: (looking up) One, two, three, four (in coordination with four points), five, six (looks at the screen and appears to feel the counters).
J: (Interrupting.) No, don't feel them. How many are here?
S: (Places left hand on the screen covering ten counters.) Ten there -.
J: Alright and three there.
S: (Places right hand on other screen.) Three there. (Looks at the screen covering ten counters and coordinates a point with each number word.) One, two, three, four, five, six (looks at the second screen) seven -.
J: (Interrupting) No, ten here (points to the screen).
S: Ten. (Places a hand on each screen.
J: Start at ten and count these ones on (points to each screen in turn).
S: (Looks at the screen covering ten counters and then touches the screen four times in coordination with counting.) One, two, three, four.
J: (Interrupting) No, don't touch them. We know how many are there. How many are there?
S: (Removes the screen.) You count them.
J: (Rescreens the counters.) Now do we have to count them again? How many are there?
S: Ten.
J: Now let's just say ten.
S: (Points to the screen covering three counters.)
J: (Points to the screen covering ten counters.) Ten - (Points to the screen covering three counters.)
S: Three.
J: (Points to the screen covering ten counters.) Ten - (Points to the screen covering ten counters) Ten -.
S: (Looks at Jane and then looks at the screen covering ten counters and makes ten points in coordination with counting.) One, two, ten (looks at the second screen and continues to make points in coordination with counting), eleven, twelve, thirteen!

Discussion of Video Excerpt 2.1

In the first task (3 and 2) Shirley counts aloud from one and keeps track of the second collection (that is, two counters). In the second task (5 and 3) Shirley counts from one, first counting the collection of three counters and then the collection of five. Thus her solution involves keeping track of five counts. Because Shirley solves tasks involving screened collections and, in doing so, counts from one she is classified as Stage 2: Figurative Counting. Shirley's strategy seems to involve visualizing, that is, figural counting or counting figural unit items. She seems to visualize the second collection to keep track of the items to be counted. Although her strategy involves movements, that is, pointing at the counters, her visualizing seems to be the most significant aspect of her counting. In the last task (10 and 3) we see that Shirley seems necessarily to have to count from 'one' and doing so provides meaning to 'ten' for Shirley. This is typical of children at Stage 2. As in the earlier tasks, Shirley's strategy seems to involve visualizing the second collection when continuing after counting 'ten'.

Video Excerpt 2.2: Molly and Tom

M: (Places out five red counters and screens them and then places out four blue counters and screens them.) Now if I've got five counters there Tom and I cover them up and four counters here and I cover them up, how many counters would I have altogether there?
T: (Smiles and shifts in his seat.) These are the hard ones. (Thinks for three seconds.) Umm.
M: (Nods and points at each collection in turn.) Five and four?
T: (Looks momentarily at the screens and then looks up to his right. Then counts subvocally for seven seconds.) Nine!
M: Why do you think it's nine?
T: Because I counted through them.
M: Did you? How did you count? Could you just tell me?
T: I went one, two, three, four, five (points at the screen covering five counters and then looks at the second screen), and then one, two, three, four. (Pauses.) So it's seven. I can tell that's seven -
M: (Interrupts Tom's thinking and removes the screens.) Umm.
T: So I went - (counts subvocally from one to nine in coordination with moving each counter in turn). Yes!
**Stage 3: Initial Number Sequence**

Stage 3: Initial Number Sequence. Child uses counting-on rather than counting from 'one', to solve addition or missing addend tasks (for example, $6 + 2 = 9$). The child may use a counting-down-from strategy to solve removed items tasks (for example, $17 - 3 = 16$, $15, 14 - 17 = 14$ as $15, 14 - 17 = 3$).

Children at Stage 3 use counting-on to solve additive and missing addend tasks and may also use counting-down-from to solve removed items tasks. When solving an additive task such as 8 and 4, for example, presented as two screened collections, the child seems to be aware that, when counting-on, the number word 'eight' signifies the act of having counted the first collection from 'one' to 'eight'.

Table 4.5 sets out the stage and indicative levels for the counting-on child on the models pertaining to FNWSs, BNWSs, numeral identification and tens and ones.

<table>
<thead>
<tr>
<th>Table 4.5 Stage and levels of a typical initial number sequence child</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>Stage of Early Arithmetical Learning (SEAL)</td>
</tr>
<tr>
<td>Level of Forward Number Word Sequences (FNWSs)</td>
</tr>
<tr>
<td>Level of Backward Number Word Sequences (BNWSs)</td>
</tr>
<tr>
<td>Level of Numeral Identification</td>
</tr>
<tr>
<td>Level of Tens and Ones Knowledge</td>
</tr>
<tr>
<td>Level of Early Multiplication and Division Knowledge</td>
</tr>
</tbody>
</table>

**The Typical Initial Number Sequence Child**

The child at the Initial Number Sequence Stage (Stage 3) has developed one or more of the advanced counting-by-ones strategies of counting-up-from, counting-to, and counting-down-from. Thus children at these stages will solve additive tasks by counting-up-from, and typically counts-up-from the first mentioned or larger addend. On additive tasks involving two covered collections children are able to keep track of six or more counts, when counting the second collection. In many cases children are able to use these counting-on strategies in the range 1 to 100. Eighty-seven and five for example, is solved by counting-on five from 87. The child knows to stop at ninety-two because they realize they have made five counts. Thus prior to commencing the count from 87, the child anticipates that they can keep track of the number of counts. This anticipation and capacity to keep track of the number of counts is the hallmark of this stage.

**Counting-up-to and Counting-down-from**

The counting-up-from strategy described in the previous paragraph is also referred to simply as counting-on. Counting-on also includes counting-up-to which typically arises when solving tasks involving a small unknown addend, for example, $7 + x = 10$, presented using counters of two colours and referred to as missing addend tasks. Some children who count-on to solve additive tasks might not have developed the counting-up-to strategy. They might be unfamiliar with missing addend tasks, and it is common for children to misinterpret a missing addend task as an additive task (for example, in the task just described they might attempt to start from seven and count-on ten). In the case of most children though, counting-up-to emerges at the same time as, or soon after, counting-up-from. At around the same period children learn to use counting-down-from to solve subtractive tasks with small known differences (referred to as removed items tasks). Thus on a task such as removing three counters from a collection of 15 (with the counters concealed), the child will count down three from 15. As before, the child anticipates that they can keep track of the number of counts, and thus stops after three counts.

**FNWSs, BNWSs and Numerical Identification**

It is typical for the child at this stage to be facile with FNWSs to 100 and beyond. In the case of BNWSs children might be facile to beyond 30 but not necessarily to 100. A reasonably common mistake in the case of saying BNWSs is the following: when saying the words backward from 53 for example, the child says '53, 52, 51, 40, 49, 48...'. This kind of mistake occurs at any decade number and a likely explanation is that the child counts backwards by ten from the decade number in order to determine the next decade (for example, counts '50, 40' to figure out the decade number before the fifties). Children at this stage typically can recognize and identify most or all of the numerals in the range 1 to 100 and beyond although some digit reversal errors (for example, '27' is named as 'seventy-two') may persist. Children might also be able to name some 3-digit numerals. The 3-digit numerals with a zero in the ones or tens (for example, 620, 407) are less likely to be correctly identified.

**Tens and Ones**

The child at this stage typically has made little progress in developing knowledge of tens and ones. They are likely to be able to count forwards and backwards by tens on the decade (10, 20,
Early Multiplication and Division

The child at this stage might also have developed initial multiplication and division knowledge relating to equal groups, equal shares and arrays, for example, combining equal groups and finding the total, sharing equally and finding the number in one share, making an array or determining how many dots in an array.

The assessment of knowledge and strategies used in the solving of tasks in tens and ones is discussed in detail in Chapter 6 and Early and Advanced Multiplication and Division in Chapter 8.

Video Excerpt 3.1: Julie and Tania

J: I've got five under there (points to a screen), and four under there (points to a second screen).
T: (Counts subvocally in coordination with pointing in turn at four fingers on her left hand.) Nine.
J: How did you work that out?
T: Counted on my fingers.
J: What numbers did you count?
T: (Points to her one finger.) Six - (pauses), five, six, seven, eight, nine. (Coordinates the last four number words with pointing to each finger in turn.)
J: I've got nine under there and six under there. How many altogether?
T: Nine, ten, eleven, twelve, thirteen, fourteen, fifteen (points in turn at the five fingers on her left hand and one finger on her right hand).

Discussion of Video Excerpt 3.1

Tania's strategy for solving these tasks involves counting-on and using her fingers to keep track of the number of her counts, that is, when counting the second collection. In the first task, for example, she does not raise her four fingers prior to commencing to count, that is, it is not necessary for her to raise four fingers in advance in order to keep track of counting-on. Rather, her strategy involves recognizing when she has raised four fingers to keep track of the second collection. In the second task, in similar vein, Tania knows when she has raised six fingers to keep track. Her counting-on strategy includes a quite facile (that is, skillful) use of finger patterns.

Video Excerpt 3.2: Jane and Shirley

The following video excerpt is taken from an MR teaching session.
J: (Points to a screen and then covers four counters with a second screen.) Twelve, and I've got four more under here?
S: (Places her hand on the first screen and looks ahead.) Twelve. (Moves her hand to the second screen. After three seconds during which time she moves her fingers over the second screen.) Sixteen!

STAGE 4: INTERMEDIATE NUMBER SEQUENCE

Stage 4: Intermediate Number Sequence. The child counts-down-to to solve missing subtrahend tasks (for example, 17 - 14 as 16, 15, 14 - answer 3). The child can choose the more efficient of count-down-from and count-down-to strategies.

The child at Stage 4 has developed counting-down-to as well as the other advanced counting-by-ones strategies. Counting-down-to is not easy to detect and it is useful to discuss this strategy in more detail. The tasks involve small, unknown subtrahends, for example remove some counters from a collection of eleven to leave eight (11 - 1 = 8), known as missing subtrahend tasks. Tasks of this kind can evoke the counting-down-to strategy. Thus the child counts back from 11 until they reach 8, and keeps track of the number of counters after saying eleven, that is, three counts. In the terms of the Learning Framework in Number counting-down-to is regarded as more
This task involves facile use of 'finger patterns.' Thus her solution in the first task does not provide an indication of whether she might use advanced counting strategies in some tasks.

The second task involves 16 - 12 and is presented with plastic numerals. It seems reasonable to suggest that Kelley cannot use finger patterns on this task because she cannot readily make a pattern to signify 16. In other words, 16 is beyond the finger range in terms of signifying numbers in the same way as done in the task of 10 - 7. Kelley seems to spend around 20 seconds focusing on trying to solve the problem after which she seems to suddenly become aware of a strategy she can use. Kelley uses a counting-down-to strategy to solve the task. She begins at 16 and then keeps track of the number of counts until she gets to 12 and stops, and she uses her fingers on her right hand to keep track. A plausible explanation is that, prior to beginning to count down, Kelley was aware of the 12 as a part of the 16. She was aware of 12 as a whole, or as a unit within the larger unit of 16. It is this kind of thinking that is the hallmark of Stage 4. She is able to conceptualize the smaller unit within the larger unit prior to commencing her count. It is also interesting to note that she raises her thumb and fingers and then deftly switches the pattern by lowering her thumb and raising her little finger. She then recognizes immediately her finger pattern for 4. She does not have to count from 'one' to 'four.' In summary, Kelley uses counting-down-to and in doing so uses her fingers to keep track of four counts. For this reason Kelley is classified as Stage 4: Intermediate Number Sequence.

The Typical Initial Number Sequence Child

The typical behaviors are similar to those for the Stage 3 child described above.

**Video Excerpt 4.1: Libby and Kelley**

**T:** (Places eight counters under a screen.) I'm taking some away. (Points at the screen and removes two counters without displaying them.) I've got six left. I'm taking some away. (Points at the screen and removes three counters without displaying them.) How many did I take away?

**S:** (Immediatly.) Two.

**T:** How did you do that so quickly?

**S:** (Shirking her shoulders.) I don't know.

**T:** Twelve, okay! There's twelve there this time. (Places twelve counters under a screen.) I am going to take some away. (Points at the screen and removes three counters without displaying them.) And there are nine left. I'm taking some away. (Points at the screen and removes three counters without displaying them.) I have five left.

**S:** (Sequentially raises three fingers.) Three.

**T:** Okay, now what number were you counting to yourself to get those three?

**S:** I was saying twelve, and I took away three, and I counted to nine, so I went, twelve, eleven, ten (in coordination with raising three fingers), and that was nine (moves her fourth finger). I put that down and then I worked it out.

**Discussion of Video Excerpt 4.1**

In the first task, 10 - 7, presented with plastic numerals, Kelley uses a strategy that we might suppose is convenient for her. She simultaneously raises 10 fingers and then looks carefully at her fingers and after a little while lowers 2 fingers on her left hand and then the 5 fingers on her right hand. A plausible explanation is that, at this point she has established a finger pattern that stands for 7 for her; that is, for the 7 being taken away. She immediately knows that now she has 3 fingers remaining, that is, she recognizes a finger pattern for three and it is not necessary for her to count from one. In summary, Kelley knows the patterns for 10 and 3, and she builds a pattern for 7 by lowering two fingers on one hand and then five on the other. Kelley's strategy on
presumption, and Sarah's strategy unfolds. This is another excellent example of a count-down-on strategy that is characteristic of Stage 4.

STAGE 5: FACILE NUMBER SEQUENCE

Stage 5: Facile Number Sequence. The child uses a range of what are referred to as non-count-by-ones strategies. These strategies involve procedures other than counting by ones but may also involve some counting by ones. This is in additive and subtractive situations the child uses strategies such as compensation, using a known result, adding to ten, commutativity, subtractions as the inverse of addition and awareness of the 'een' in a teen number.

This section provides an overview of a typical child at the Facile Stage. That is at Stage 5 of the Stages of Early Arithmetical Learning. Table 4.7 sets out the stage and indicative levels for the counting on child on the models pertaining to FNWSs, BNWSs, numeral identification, tens and ones, and multiplication and division. The overview discusses the six aspects of early number knowledge listed in Table 4.7 and grouping by fives and tens.

Table 4.7 Stage and levels of a typical facile child

<table>
<thead>
<tr>
<th>Model</th>
<th>Stage/level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage of Early Arithmetical Learning (SEAL)</td>
<td>5</td>
</tr>
<tr>
<td>Level of Forward Number Word Sequences (FNWSs)</td>
<td>5</td>
</tr>
<tr>
<td>Level of Backward Number Word Sequences (BNWSs)</td>
<td>5</td>
</tr>
<tr>
<td>Level of Numeral Identification</td>
<td>4</td>
</tr>
<tr>
<td>Level of Tens and Ones Knowledge</td>
<td>2</td>
</tr>
<tr>
<td>Level of Early Multiplication and Division Knowledge</td>
<td>3</td>
</tr>
</tbody>
</table>

The Typical Facile Child

The child at the Facile Stage typically has developed strong facility in a range of aspects of the learning framework in Number. Facile arithmetical strategies might include knowing a range of doubles and using doubles to work out other facts, for example, using 5 + 5 to work out 4 + 5. These strategies can also incorporate knowledge of the numbers in the teens in terms of a ten and ones, for example, the child uses knowledge of 3 + 3 to work out 13 + 3. Understanding of the inverse relationship between addition and subtraction is also common at this stage. Thus the child might use knowledge of 9 + 3 to work out 12 - 3, or knowledge of 10 + 5 to work out 15 - 11.

Grouping by Fives and Tens

The child at this stage might use grouping by fives and tens when adding or subtracting in the range 1 to 20 (for example, 4 + 3 is found by partitioning 3 into 1 and 2, and then adding 2 to 5, and 13 - 8 is found by partitioning 8 into 3 and 5, and then subtracting 5 from 10). As well,

they might solve without counting by ones, additions such as 40 + 8 and subtractions such as 28 - x = 30, 87 - x = 80, and 30 - 2. Finally, the child might use the strategy of adding to ten, for addition (for example, 18 + 6 is found by finding the complement of 8 in 10, that is, 2, then partitioning 8 into 2 and 4, and then adding 4 to 20, and subtracting to 10 for subtraction (for example, 42 - 6 is found by partitioning 6 into 2 and 4, and then subtracting 4 from 40).

FNWSs, BNWSs and Numeral Identification

The child at this stage typically is facile with FNWSs and BNWSs in the range 1 to 100 and beyond, and can count forwards and backwards by 2s, 10s, 5s, 3s, and 4s. Also, the child can recognize and identify numerals in the range 1 to 100. In many cases the child can also recognize and identify many three-digit numerals as well, although numerals with the digit '0' in the right-hand or middle place (for example, 602, 803) are likely to be more difficult.

Tens and Ones

The child at this stage is likely to be able to increment and decrement in the range 1 to 100, by tens on and off the decade (for example, 40 and 10, 70 less 10, 32 and 10, 88 less 10), and by tens and ones on and off the decade (for example, 32 and 2 tens and 3 ones, 87 less 2 tens less 2 ones).

Early Multiplication and Division

The child at this stage is likely to be able to use skip counting and repeated addition or subtraction to solve multiplicative and divisional tasks involving equal groups and screened arrays. Each of these tasks are, determining the number of items altogether in equal groups or an array and, given the number of items altogether, determining the number of groups, the number in each group or the number in each row of an array.

Video Excerpt 5.1: Robin and Allan

R: (Places 12 counters under a screen.) There were twelve counters and I took away some (removes and screens three counters), and I have nine left. How many did I take away?
A: (Looks at Robin.) Three.
R: How did you work that out?
A: Nine plus three is twelve, and then you take away, it's nine.
R: (Places 15 counters under a screen and then removes and screens four counters.) This time I had fifteen counters. I took some away and I have eleven left. Fifteen, took some away and I've got eleven left.
A: (Looks ahead for nine seconds.) Four!
R: Tell me how you did that one.
A: Umm, you had fifteen and took some away to make eleven because you just take away four, 'cause if you took away five it would be ten and you just plus it on.
R: (Places counters under a screen.) What if I start off with fifteen under there (removes, displays and then screens three counters), take three out, how many are left under there?
A: Twelve.
R: Hmm, how did you do that one?
A: Because five take away three is two, and then you just, cause there was fifteen so you just take away three and it's twelve.
Discussion of Video Excerpt 5.1

Allan shows a good number of non-count-by-ones strategies on these tasks. In the case of the first task – 12 take away some, now I have 9, how many did I take away? – it becomes apparent in Allan’s explanation that he worked this Missing Subtrahend task out by thinking of the addition 9 + 3 = 12. This is a very advanced strategy for a 6-year-old and is an excellent example of a non-count-by-ones strategy. Strategies such as this are commonly referred to as thinking strategies for the basic facts, that is, using addition to work out subtraction, or exploiting knowledge that subtraction and addition are inverse operations. Allan’s solution of the second task involves knowledge of the tens and ones structure of the number 15. Allan realizes that 15 is 10 and 5. Again this is very advanced for a 6-year-old and not typical of children lower than Stage 1. Allan also uses what is commonly referred to as a compensation strategy – if 5 are taken from 15, the answer is 10, thus, if the answer is 11, only 4 are taken away. These strategies are very advanced and are referred to as non-count-by-ones strategies.

In the final task in this excerpt, 15 take away 3, Allan in explanation indicates that he has worked the task out by thinking of 5 take away 3. As with the above strategies, this is an example of a non-count-by-ones strategy. He seems to be aware of a correspondence of the relationship between 5 and 15, on the one hand, with the relationship between 2 and 12, on the other hand, that is, in both cases the second number is 10 more than the first. Given the problem of the names of these numbers, for example ‘twelve’, which does not give any clue of the relationship between 2 and 12, it seems plausible to suggest that Allan is thinking in terms of written symbols perhaps even written symbols arranged in columns. This corresponds with how we might expect an advanced child, or a child beyond Year 1, to work such a subtraction. Whatever the precise nature of Allan’s thinking, it is quite advanced. In Allan’s solutions we see strategies well beyond counting-by-ones. His strategies, considered across all of these tasks, are characteristic of Stage 5: Facile Number Sequence, where non-count-by-ones strategies are used spontaneously.

Video Excerpt 5.2: Kathryn and Loretta

K: If we have five under there (places five counters under a screen) and four under there (places four counters under a second screen), how many would we have?
L: (Immediately.) Five and four more is nine.
K: And how do you know that?
L: Well, five and three more is eight and so one more is nine.

Discussion of Video Excerpt 5.2

Loretta, in explanation, reveals a strategy where 5 plus 4 is worked out from 5 plus 3. This kind of strategy, that is, using a known fact to work out an unknown and making an appropriate adjustment – one more in this case – is well known and is referred to as a ‘thinking strategy’ for the basic facts. Because Loretta uses a strategy that has features other than counting-by-ones she is classified as Stage 5: Facile Number Sequence.

CONCLUSION

The model of the Stages of Early Arithmetical Learning presented in this chapter provides a means of understanding the progression of children’s early number learning from perceptual counting strategies in which children are reliant on seeing materials to the point where children have a relatively sophisticated knowledge of addition and subtraction, in the range 1 to 100 and beyond. Children at the Perceptual Stage (Stage 1) can solve problems involving visible items, whereas children at the Figurative Stage (Stage 2) can solve problems involving hidden items but count from one when doing so. Children at the stage of the Initial Number Sequence (Stage 3) use counting-on to solve additive and/or missing addend tasks, and may use counting-down-from to solve removed items tasks, while children at the stage of the Intermediate Number Sequence (Stage 4) use counting-down-to to solve missing subtrahend tasks. Finally, children at the stage of Facile Number Sequence (Stage 5) use a range of strategies other than counting-by-ones to solve additive and subtractive tasks.

Two examples of children’s attainment were presented to further exemplify the stages. We are aware, however, that more experience and practice may be necessary before one is confident to recognize the stages. Therefore, we now provide two support mechanisms. First, we conclude this chapter by presenting a discussion of the common difficulties which trainee assessors face when trying to allocate a stage. Secondly, in Chapter 5 we present twelve scenarios, two for each of the Stages 0–5, but not in any defined order. We challenge you to identify the stages. The answer for each scenario is given at the end of the chapter with an explanation why each child is placed at a particular stage.

Determining the Child’s Stage: Judging on the Basis of the Most Advanced Strategy

When determining the child’s stage it is important to remember that the child is judged on the basis of the most advanced strategy they use. In all the additive and subtractive tasks, the interviewer should attempt to elicit the most advanced strategies. In the discussions in the following paragraphs the term ‘at least’ is often used as a qualifier, for example, ‘the child who correctly counts these [visible] collections is judged to be at Stage 1 at least’ and ‘the child who counts-down-to to solve missing subtrahend tasks is judged to be at Stage 4 at least’. A child might count-down-to to solve missing subtrahend tasks and use Stage 5 strategies on other tasks. In this case the child is judged to be Stage 5. In similar vein, the child who counts on from one to solve additive tasks, counts-on to solve missing addend and counts-down-from to solve removed items is judged to be at Stage 3 (assuming they do not use Stage 4 or Stage 5 strategies).

Why 5, at least advanced strategies exist but they are not necessarily used?

Stage 0 or Stage 1?

The need to distinguish between Stage 0 and Stage 1 is likely to arise in cases where the interviewer has presented the two items in Assessment Interview 1.1. Task Group 8(d), perceptual counting, that involve counting unscreened collections of 13 and 18 counters. The child who is unable to count the collections because they omit some of the counters, or do not correctly coordinate the number words with the counters, or because they apparently do not know the FNWS, is judged to be at Stage 0. The child who correctly counts these collections is judged to be at Stage 1 at least. In most cases, because the interviewer decided to present these tasks on the basis of their performance on the Entry Tasks, the child is no more advanced than Stage 1.
Stage 1 or Stage 2?

The need to distinguish between Stage 1 and Stage 2 arises when the child uses particular finger patterns to solve the additive tasks of 3 and 2 (that is, the introductory example) and 5 and 2 and unsuccessfully attempts to use finger patterns to solve tasks such as 9 and 6, 8 and 5, 9 and 4, 1.1.8(a) and 8(c). Matthew in Chapter 5 exemplifies this case. His raised fingers constitute perceptual replacements for the screened counters and he establishes his finger patterns prior to commencing to count both collections. In this case the child is classified at Stage 1 only. Distinguishing between Stage 1 and Stage 2 also arises in cases where the child solves tasks involving one screened collection 1.1.8(a) and one unscreened collection 1.1.8(b). If the child counts the unscreened collection first, and then keeps track while continuing in order to count the screened collection, the child is judged to be at Stage 2, with the proviso that the child’s use of this strategy is not limited to the first, partially screened, additive task of 5 and 2. Thus it is necessary that the child use this strategy on one or both of the tasks of 7 and 3 and 9 and 4. In the case where the child counts the screened collection first and then continues in order to count the unscreened collection, the child is judged to be at Stage 1. In this case the means by which the child has counted the screened collection does not constitute the notion of counting.

Stage 2 or Stage 3?

Distinguishing between Stage 2 and Stage 3 is usually unproblematic. In the case where the child counts on to solve additive tasks but does not seem to understand the missing addend task 1.1.8(f), the child is judged to be at Stage 3. Thus, it is sufficient to use counting on to solve additive tasks only. In many cases, children who do not seem to understand missing addend tasks will count down from the removed items tasks 1.1.9(c) as well as count on for additive tasks.

Stage 3 or Stage 4?

The child who counts down to solve missing subtrahend tasks 1.1.9(b) is judged to be at Stage 4 at least. Counting down to solve the rewritten task of 16 - 12 (Task Group 9) is also indicative of Stage 4. Attempting to solve missing subtrahend tasks or the rewritten task of 16 - 12 by counting down from is not indicative of Stage 4, for example, the child attempts to count down 12 counts from 16. Children may count up to solve missing subtrahend tasks although this is not a common occurrence. One might say these children interpret a missing subtrahend task as they might interpret a missing addend task. In such cases the child is judged to be at least Stage 4. Their ability to conceptualize the task in this way is regarded as indicative of Stage 4 rather than Stage 3.

We recognize that additional exemplification and practice may be necessary in order to become skilled in analysis, and this is provided next in Chapters 5 and 6.