Counting and Composition

1. CI and The Counting Objection

Most of us¹ think that ordinary objects are composed of parts—you are composed of body parts, which are in turn composed of molecular parts, etc. But imagine that we are to create the universe from scratch, given only the parts, first. We get our molecules or atoms or quarks or whatever you think are the smallest bits that compose ordinary objects. Once we’ve got these bit parts, do we need something in addition to get the bigger objects these parts compose? Are wholes something over and above their parts? For those who think not, you might be tempted by the claim that composition is identity.

Composition is Identity (CI): Any composite object O is (collectively) identical to its parts \(O_1, O_2, O_3, \ldots O_n\). ²

If composition is identity, then we have a ready explanation for how it is that whenever we have some parts, we get the whole for free. It’s the same reason why whenever we have Hesperus, we get Phosphorus for free: the relation between the object(s) in question is identity, so of course when you have ‘one’, you have the ‘other’.

There are many who object to composition is identity (CI). Notably, Peter van Inwagen has argued that an examination of how we count and quantify over objects in our ontology shows that CI must be false. Borrowing an example from Lewis (who borrows it from Baxter), van Inwagen

---


² I have in mind here the strongest form of Composition is Identity (CI): viz., where the predicate ‘are’ used to indicate the composition relation is literally another form of the ‘is’ of identity. See Lewis (1991) for an example of a weaker form of CI. Baxter (1988a), (1988b), (1999) also endorses a CI view, though it differs from the one I endorse here. Given the hybrid identity predicate I will introduce below, CI is: any composite object, \(O\), is (hybrid) identical to the objects \(O_1, O_2, O_3, \ldots, O_n\) that are its parts; \(O \equiv_{h} O_1, O_2, \ldots, O_n\).
puts an objection against CI as follows: imagine that there is one big parcel of land, divided neatly into six smaller-parcel parts. Van Inwagen argues,

Suppose that we have a batch of sentences containing quantifiers, and that we want to determine their truth values: \( \exists x \exists y \exists z (y \text{ is a part of } x \& z \text{ is a part of } x \& y \text{ is not the same size as } z) \); that sort of thing. How many items are in our domain of quantification? Seven, right? That is, there are seven objects, and not six objects or one object, that are possible values of our variables, and which we must take account of when we are determining the truth value of our sentences.

Given how we usually quantify over objects in the world—i.e., with a singular existential quantifier—there will be no way to quantify over wholes without adding to the number of things in our ontology. And if we are adding to the things in our ontology when we accept any whole, then wholes are not ontologically innocent. Let us call this the **Counting Objection**.

Suppose you are someone who believes in unrestricted mereology. You think that for any \( x \) whatsoever, those \( x \) compose one and only one fusion. We could imagine van Inwagen presenting the Counting Objection against your view as follows. “You grant that there are two quarters in your pocket. You also grant that there is a mereological sum of the two quarters. So let’s take a count of all of the things that there are in your pocket. We existentially quantify over all of the things, together with the relevant non-identity claims. We will get a statement as follows (where ‘\( P_x \)’ is read as ‘\( x \) is in your pocket’):

\[
(1) \quad \exists x \exists y \exists z (P_x \& P_y \& P_z \& x \neq y \& x \neq z \& y \neq z)
\]

Since each of the individual two quarters is non-identical to the mereological sum of the quarters, then there are the two things (the quarters) plus the one thing (the mereological sum of the quarters)

---

3 Van Inwagen assumes that the smaller parcels are simples, and ignores (for brevity’s sake) many of the overlapping parts. I will do likewise.

and, hence, there are three things in your pocket, not two. This shows that composition is not ontologically innocent. If it were, then we wouldn't get more entities when we count the whole as distinct from the parts. If the whole just is the parts, then our counts should bottom out at the level of parts. So a commitment to wholes is an additional commitment to parts, in a very literal sense of the word additional: we can see that it is one more item in our domain whenever we try to take a count of all of the things that there are! Thus, composition is not ontologically innocent, and hence, CI is false.”

Is this right? Can it be this easy? Can we really yield such weighty metaphysical conclusions—i.e., that composition is not identity—just by using some simple tools in our logic book? I doubt it. In the next section I explain why.

2. Logic Book Counting and Plural Counting

Let us consider what a proponent of CI (and unrestricted mereology) would say in response to van Inwagen’s Counting Objection. A CI theorist maintains that the two quarters just are the mereological sum, and that the mereological sum just is the two quarters. But how could she possibly

---

5 Actually, there are plenty more than three things in your pocket: each quarter has a right half and a left half, each of those halves has halves, and so on. Let us ignore this complication for now and imagine that each of the quarters is mereological simple—i.e., they have no parts.

6 In conversation and correspondence, van Inwagen has explained that he did not intend to be arguing for a weighty metaphysical conclusion (for he did not intend to be making a metaphysical point at all). Rather he intended that “the conclusion of [his 1994] paper…was a thesis about words.” In particular, he took the conclusion of what I am calling the Counting Objection to show that CI sentences such as the parts are (collectively) identical to the whole “mean nothing at all…they are…not even false.” If so, then one can take my points here as an effort to show how CI identity statements are coherent and do mean something. In answering what I took to be a metaphysical point about the consequences of CI and counting, I can do double duty: I can (i) answer the metaphysical point and in so doing (ii) further flesh out the meaning and coherence of CI as a philosophical position. Even if van Inwagen did not intend the Counting Objection to yield a metaphysical conclusion, such an argument is available and is initially challenging, and so it is incumbent on the CI theorist to answer the objection, no matter who endorses it (or doesn’t). Finally, van Inwagen has suggested in conversation that he intended some of his arguments in van Inwagen (1994) to be a version of a Leibniz’s Law objection against CI. I deal with these sorts of objections elsewhere [reference omitted for blind review], so I will not address them here.
maintain this in light of the above sort of reasoning? Must a CI theorist deny the truth of statements like (1)? Is a CI theorist maintaining her view at the cost of giving up well-entrenched rules of logic?

Not at all. For she could grant that statements such as (1) are true, yet insist that it does not follow from this that there are three things in someone's pocket. This is because, she argues, the truth of (1) is independent from whether we think that (1) is always an appropriate representation of “there are three things in my pocket.” (1) is true. It is true that there is one quarter, and another distinct from the first, and that there is a mereological sum of these quarters that is not identical to the first quarter, nor is it identical to the second quarter. But, she insists, it is also true that the one mereological sum is identical to the two quarters; the mereological sum is identical to both of the quarters taken together, which is perfectly compatible with the sum being distinct from either quarter individually.

The problem, our CI theorist will insist, is that the identity predicate of first order logic, which we used in (1), does not allow us express a statement such as “one mereological sum is identical to two quarters.” This is because the only terms allowed to flank the first-order logic identity predicate are singular ones. By the first-order grammatical rules alone, then, one is prohibited from accurately representing the claim one thing is identical to many; one doesn’t even have a way of referring to many objects at once in classical first-order logic, so we certainly couldn’t say of many things that they are identical to one.

But there is an easy fix. Let us introduce a way of creating plural terms out of singular ones such that we can refer to many objects at once. Let us use ‘,’ as a way of concatenating singular terms, where, for example ‘x,y’ means something like “x and y, taken together.” Then we could have a sentence such as (2):

7 Such plural terms are irreducibly plural, and are not translatable into singular terms or any first-order sentence. For more on plural logic and plural languages, see Boolos (1984), McKay (2006), et. al.
Importantly, (2) is not equivalent to (3), which is a statement expressible in first-order logic:

(2) \( \exists x \exists y \exists z \left( z = x, y \right) \)

(3) \( \exists x \exists y \exists z \left( z = x \land z = y \right) \)

(3) claims that there is something, \( z \), that is identical to \( x \), and identical to \( y \); it says that something, \( z \), is identical to \( x \) and \( y \) taken individually. (2), in contrast, claims that something, \( z \), is identical to \( x \) and \( y \) taken together. Also, (2) is not equivalent to (4), which is also something that a singular logic can say

(4) \( \exists x \exists y \exists z \left( S z \land x M z \land y M z \land x \neq y \land x \neq z \land y \neq z \right) \land \forall x \forall y \forall z \forall w \left( \left( S z \land x M z \land y M z \land x \neq y \land w M z \right) \rightarrow \left( w = x \lor w = y \right) \right) \)

(4) says that there is something, \( z \), that is a set that has only \( x \) and \( y \) as members. But this means that there is one thing—a set, \( z \)—which is identical to the mereological sum. But the relation between a whole and the set of some parts is not a many-one relation, but a one-one relation. (2) explicitly posits a many-one relation, not one-one, and so (4) is not equivalent to sentence (2).

Granted, (2) is ill-formed in first-order logic. But this is exactly how a CI theorist might wish to represent “there is one thing (the mereological sum) which is identical to two things (the quarters).” Moreover, a CI theorist will want to have some way to talk about objects plurally, not just singularly as first-order logic does.

However, we can introduce terminology that will capture the metaphysical facts that the CI theorists wishes to express. One way to do this is by introducing a singular/plural hybrid two-place identity predicate, ‘\( =_h \)’, that takes either plurals or singulars as argument places—i.e., \( \alpha =_h \beta \), where \( \alpha \) and \( \beta \) can be either plural or singular terms. We can also allow the concatenation of singular

---

8 It is one thing to think that CI is false, it is quite another to it is incoherent. If CI were utter nonsense, we wouldn’t be able to coherently deny the view. So we can at least try to understand what CI is, and attempt to represent her position semi-formally. (But see van Inwagen (1994), and see above fn. 6).
terms—e.g., ‘x,y,z,’ etc.—into plural terms, with the use of commas, as we did in (2), and as demonstrated on the right side of the hybrid identity symbol in (2h):

\[(2h) \; \exists x \exists y \exists z (z =_h x, y)\]

The adoption of the hybrid identity predicate, =_h, will not force us to abandon the singular identity predicate used in traditional first-order logic, or in sentences such as (1). For singular identity statements are just a special case of hybrid identity statements. We can incorporate singular identity as follows:

\[(i) \; \alpha = \beta \equivdf \alpha =_h \beta, \text{ where } \alpha \text{ and } \beta \text{ are singular terms}\]

I intend for hybrid identity to be the classical identity relation.\(^\text{10}\) We can re-interpret (1) in terms of the plural identity predicate, =_h, to yield (1h):

\[(1h) \; \exists x \exists y \exists z (P_x \& P_y \& P_z \& x \neq_h y \& x \neq_h z \& y \neq_h z)\]

We can further provide an acceptable and well-formed interpretation of (2), as shown above in (2h). Thus, we can now describe the CI theorist as one who accepts the following sort of sentence, (5):

\[(5) \; \exists x \exists y \exists z (P_x \& P_y \& P_z \& \& x \neq_h y \& x \neq_h z \& y \neq_h z \& \& z =_h x, y)\]

(5) is simply (1h) and (2h) combined (where the amendment to (1h) in (5) is in bold typeface). Since the singular identity relation is special case of the hybrid identity relation, we can think of (5) as involving the singular non-identity statements of first-order logic together with the hybrid identity statement that is endorsed by a CI theorist. Because statements such as (5) allow and include plural

\[^9\text{[Acknowledgment omitted for blind review].}\]

\[^{10}\text{With only one exception: hybrid identity is transitive, reflexive, symmetric, it obeys Leibniz’s Law, etc.; the exception is that the hybrid identity relation allows us to claim that many things can be identical to a singular thing, where letting many things be identical to one is not itself be a violation of Leibniz’s Law (which I defend elsewhere).}\]
subject terms such as ‘x,y’, let us call this \textbf{Plural Counting}. Contrast this with the first-order, singularly quantified count statements such as (1), which I will call \textbf{Logic Book Counting}.

We now have a way of expressing what the CI theorist believes is going on with the various things in your pocket. But how does this address the original question: how many things are in your pocket? In the case of Logic Book Counting, we had an easy inference from statement (1) to a statement such as “there are three things in my pocket” because we simply took (1) to be the correct representation of the sentence “there are three things in your pocket.” Yet the CI theorist grants the truth of (1)’s equivalent—i.e., (1h)—but denies that this always correctly expresses a count statement. This is because she believes there is more to the story. According to CI, one of the items quantified over in (1h) is identical to some of the others, and this singular/plural identity statement cannot be ignored if we want to keep our counts accurate. Thus, we get a statement such as (5).

Yet suppose we grant all of this to the CI theorist. How, exactly, if we utilize sentences such as (5), is Plural Counting supposed to yield a \textit{count}?

\section{Relative Counting and Plural Counting}

We will be better able to answer this question if we examine one more kind of counting: \textbf{Relative Counting}. Relative Counting claims that we cannot determine how many things there are until we have been given a sortal or concept or kind under which to count by. This view of counting is suggested by Frege in \textit{The Foundations of Arithmetic} where he claims:

\begin{quote}
The Illiad, for example, can be thought of as one poem, or as twenty-four Books, or as some large Number of verses; and a pile of cards can be thought of as one pack or as fifty-two cards (§22). \textit{One} pair of boots can be thought of as \textit{two} boots (§25).
\end{quote}

In §46, Frege continues,

\begin{quote}
…it will help to consider number in the context of a judgment that brings out its ordinary use. If, in looking at the same external phenomenon, I can say with equal truth ‘This is a copse’ and ‘These are five trees’, or ‘Here are four companies’ and ‘Here are 500 men’, then what changes here is neither the individual nor the whole, the aggregate, but rather my
\end{quote}
terminology. But that is only the sign of the replacement of one concept by another. This suggests... that a statement of number contains an assertion about a concept.

The suggestion here is that we can think of thing(s) in various different ways—e.g., as cards, decks, complete sets of suits, etc.—and depending on these various ways of thinking about thing(s), we can yield different numbers or counts in answer to the question *how many?* We can talk about how many $F$s or $G$s are there, where $F$ and $G$ stand in for specific sortals, concepts, or kinds. But one can only take a count *relative* to these sortals, concepts, or kinds, but never a count *tout court*. Let us call this

*Relative Counting.*

As concerns the number of things in your pocket, the relative counter maintains that a non-relativized question such as “how many things are in your pocket?” is an ill-formed question. The only legitimate counting questions are ones that provide us with a sortal or concept or kind to count by such as “How many *quarters* are in your pocket?” or “Or how many *coins* are in your pocket?” etc. That we sometimes *do* give answers to unqualified *how many?* questions can be explained, perhaps, by the fact that the sortals we are interested in are often implicit or pragmatically understood. But a bit of reflection reveals that we seem to always have some sortal or concept or kind in mind when we answer a seemingly unrelativized counting question. Thus, Relative Counting is appealing because, on reflection, that’s how it seems we *do* in fact count.

This does not mean that there is not an answer to the question *how many things are there?,* and it doesn’t mean that the answer is somehow indeterminate. But it does mean that the answer won’t be a single numerical value. There will always be a maximum to the number of things there are—

---

11 I am leaving the exact details of Relative Counting intentionally vague, since I can imagine many variations on the Fregean theme suggested above. All that matters for my purposes, however, is that a theory of counting qualifies as Relative Counting if it claims (i) that there cannot be a unique numerical answer (e.g., ‘52’) to the question *how many things are there?,* and (ii) that there can only be a unique numerical answer to questions that include a legitimate sortal, concept, or kind term (e.g., ‘how many *cards* are there?’).

12 Except in the sad, lonely world that contains just one mereological simple.
the number of simples, say—and there will always be a minimum—one mereological sum, say. And then there will also be all of the identity statements that hold between everything that is between the upper and lower bounds. We may have in front of us 1 deck of cards, which is identical to 4 sets of suits, which is identical to 52 cards. And let us imagine for now that that’s all there is. If I ask how many things there are in front of us?, then the answer will be something like: there are 52 cards, and 1 deck, and 4 sets of suits, and the 52 cards is identical to the 1 deck, which is identical to the 4 sets of suits.” So there is an answer to the question how many?, it’s just that the answer is slightly more complicated than we may have first suspected. And this is what seems right about relative counting.13

But what goes for cards and decks and sets of suits, goes for simples and mereological sums as well. Imagine a world with 2 simples. Imagine that the two simples are identical to one mereological sum. Never mind for now whether ‘simple’ and ‘sum’ qualify as sortals or not. We should be able to count up how many things there are, even if the story is complicated, and we think that the two simples are identical to the mereological sum, for example. Analogous to the card/deck case, we should be able to say something like ‘there are two simples and one sum, and the two simples are identical to the one sum’ in answer to a question such as how many things are in this world?

True, there may not be a single numerical value; we can’t say ‘one’ or ‘two’ or ‘three’ and be right. But that’s because the metaphysical facts are more intricate than we may have first supposed. Even so, there is a determinate answer, albeit a slightly complicated one.

This is exactly what I think Plural Counting can capture. But let us return to the question above: how does a Plural Counter, if she is utilizing a sentence such as (5), take a count?

(5) \( \exists x \exists y \exists z (Px \& Py \& Pz \& x \neq h y \& x \neq h z \& y \neq h z \& z = h x y) \)

13 Despite its initial intuitiveness, Relative Counting has some substantial worries. Unfortunately, I do not have the space here to address them. But it doesn’t matter. The main point I wish to make in this paper is that CI can avoid the Counting Objection. She could do this by either adopting Relative Counting or Plural Counting. I will detail Plural Counting in response to van Inwagen’s counting objection, and the reader can surmise for himself how a Relative Counter could make an analogous move.
I suggest she borrow a bit from each of Logic Book Counting and Relative Counting. From Logic Book Counting, she will take the ability to singularly existentially quantify over some objects, together with the identity and non-identity claims about those objects. Only instead of using Logic Book Counting to range over objects in our usual domain—the universe—I suggest she use it to range over the distinct variables in her singular/plural hybrid identity statements, which correspond directly to objects in our ontology. The distinguishing (hybrid) identity claim that falls out of (5) is: $\chi =_h x \land y$. We can take such a hybrid identity statement and count—i.e., Logic Book Count—all of the variables on either side of the identity predicate. Imagine that all of the variables on the left-hand side of the symbol ‘$=_h$’ are one domain, and that the variables on the right-hand side of the hybrid identity symbol are another domain. Then let us Logic Book Count all of the variables on first one side, and then the other, using ‘$V_L$’ and ‘$V_R$’ for “is a left-hand variable” and “is a right-hand variable” respectively:

**Left-hand-side Domain:** $\exists x \, (V_L x \land \forall x \forall y(V_L x \land V_L y \rightarrow x = y))$

**Right-hand-side Domain:** $\exists x \exists y \, (V_R x \land V_R y \land x \neq y) \land \forall x \forall y \forall \chi (V_R x \land V_R y \land V_R \chi \rightarrow (\chi = y) \lor (\chi = x))$

In the first case we get a count of one, and on the other we get a count of two. (It is important to note that, in this particular example, we never get a count of three.)

We still have yet to show how a count of variables could yield a count simpliciter of object in our domain. For this, I suggest the Plural Counter borrow a technique used by the Relative Counter: the Plural Counter should borrow the intuitive procedure of allowing more complicated answers to questions such as *how many?*

So, for example, we might take a statement such as (5), logic book count all of the variables on either side of any of the identity statements that fall out of (5), and produce a count such as: “there is one thing and two things, and the one thing is identical to the two things.” The Plural
Counter grants (in this case) that there is at least one thing, and also that there is at most two things. But she also endorses an identity claim that cannot be ignored in our count. Thus, similar to the relative counter, she will deny that there is a flat-out, singular numerical value. Rather, she will claim that there is one of something, and two of some other things, but also that the one thing is identical to the two things. Thus, her answer to how many? in this case will reflect this, and will be something like: there is one thing and two things, and the one thing is identical to the two things.14

Let’s return to van Inwagen’s Counting Objection. The Plural Counter does not count by singular existential statements and singular identity and non-identity claims, so she will reject van Inwagen’s suggestion that this is the correct way to count. Rather, she counts by plural terms and variables, uses a hybrid identity predicate, ‘=h’, and Logic Book counts at the level of variables. So in the land parcel example, the Plural Counter would existentially quantify over all six parcels of land, and the mereological sum of the six parcels to yield (6):

\[
\exists x \exists y \exists z \exists w \exists v \exists u \exists t (x \neq_h y \land x \neq_h z \land x \neq_h w \land x \neq_h v \land x \neq_h t \land y \neq_h z \land y \neq_h w \land y \neq_h v \land y \neq_h t)
\]

(6) expresses exactly what the CI theorist thinks is going on with the parcels of land: there are six smaller parcels, \(x, y, z, w, v, u\), that make up one larger parcel, \(t\), and the six are identical to the one, which is expressed by the identity claim ‘\(t =_h x, y, z, w, v, u\)’ In order to get a count, the Plural Counter takes as her domain the right hand side of this identity claim, and then the left hand side of this identity claim, and then logic book counts all of the variables in these domains, separately:

**Left-hand-side Domain:** \(\exists x \ (V_1 x \land \forall x \forall y (V_1 x \land V_1 y \rightarrow x =_h y))\)

14 Because the answer includes the hybrid identity claims that the Plural Counter accepts, we eliminate confused cases of double counting whereby someone might think there is one and two and three things, and then would add all of these things up, yielding a total of six things. This would be just as illegitimate as thinking that sitting in front of us is one deck of cards and fifty-two cards and four complete sets of suits, yielding a total of fifty-seven things in front of us, etc.
Right-hand-side Domain: \( \exists x \exists y \exists z \exists w \exists v \exists u (V_k x \& V_k y \& V_k z \& V_k w \& V_k u \& x \neq y \& x \neq h z \& x \neq h w \& x \neq h v \& x \neq h u \& y \neq z \& y \neq w \& y \neq v \& y \neq u \& z \neq w \& z \neq v \& z \neq u \& w \neq v \& w \neq u \& v \neq u) \)
\[ \forall x \forall y \forall z \forall w \forall v \forall u \forall s (V_k x \& V_k y \& V_k z \& V_k w \& V_k v \& V_k u \& V_k s \rightarrow (s = h y) \vee (s = h x) \vee (s = h z) \vee (s = h w) \vee (s = h v) \vee (s = h u)) \]

In the first case we get a count of one, and in the second, six. So in answer to the question how many parcels of land are there?, the Plural Counter insists: “there is one parcel of land, and six others, and the one is identical to the six.” Thus, we never get a count of seven, as van Inwagen claims. And so the CI theorist can avoid the Counting Objection by adopting Plural Counting.

There may be other reasons to reject CI, and other arguments that succeed in showing CI is false. But I hope I have convinced you that the Counting Objection is not among them. Moreover, I hope I have shown how, armed with a plural logic, a CI theorist can coherently state her view.

References


