The Supply of Skills in the Labor Force and Aggregate Output Volatility*

Steven Lugauer†

The cyclical volatility of U.S. gross domestic product suddenly declined during the early 1980's and remained low for over 20 years. I develop a labor search model with worker heterogeneity and match-specific costs to show how an increase in the supply of high-skill workers can contribute to a decrease in aggregate output volatility. In the model, firms react to changes in the distribution of skills by creating jobs designed specifically for high-skill workers. The new worker-firm matches are more profitable and less likely to break apart due to productivity shocks. Aggregate output volatility falls because the labor market stabilizes on the extensive margin. In a simple calibration exercise, the labor market based mechanism generates a substantial portion of the observed changes in output volatility.

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†University of Notre Dame; Department of Economics; 719 Flanner Hall; Notre Dame, IN 46556 USA website: http://www.nd.edu/~slugauer; phone: 1-574-631-1432; email: slugauer@nd.edu

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1 Introduction

The volatility of U.S. gross domestic product (GDP) sharply declined during the 1980's.[1] This 'Great Moderation' lasted at least 20 years (see Figure 1). A gradual increase in the supply of high-skill workers may have been a contributing cause. The number of college graduates, a proxy for the skill supply, has increased by an average of two percent per year for the past several decades (see Figure 2). I hypothesize that firms reacted to changes in the distribution of skills by creating new types of jobs and modifying their hiring strategies. As high-skill workers became plentiful, companies tailored jobs specifically to high-skill workers. These new positions generated more profits. The worker-firm decision to remain matched to one another reacted less to changes in productivity over the business cycle. Therefore, amplification of the shock along labor's extensive margin decreased, reducing aggregate output volatility.

[1] I use the term volatility to mean the magnitude of fluctuations at business cycle frequencies. For example, output volatility can be measured as the standard deviation of the deviations from trend output.

Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) were among the first published papers to document the sudden and prolonged drop in GDP volatility. Uncovering the cause of the moderation should shed light on whether the period of tranquility has now ended or if the current economic turbulence will be short in duration. Several explanations have been suggested; they can be categorized as changes in either policy, luck, or the structure of the economy (Stock and Watson 2002). See Davis and Kahn (2008) for why these explanations fail to be completely convincing. More recently, Jaimovich and Siu (2009) and Lugauer (2011) have argued that demographics should be added as a fourth potential explanation for the changes in business cycle fluctuations. What follows can be viewed as a theory of one way demographics affect the magnitude of the business cycle.

In the next section, I develop the intuition in a labor-search environment. The analysis begins with a static one-period model, originally introduced in Acemoglu (1999). In the model, firms select capital based on the skill distribution. When skills are scarce, firms choose a middling amount of capital and hire any worker. Firms do not target high-skill workers because they are difficult to find. Neither high- nor low-skill workers produce with the optimal amount of capital. Thus, matches tend to be close to a shutdown level of productivity, which leads to aggregate output volatility. When high-skill workers are abundant, firms create different jobs for workers of different types. Matches are less likely to break apart in response to productivity shocks because firm capacity and worker skill-level fit better together. Aggregate output volatility decreases when the supply of high-skill workers reaches a high enough threshold.

The demographic changes are taken as given. As in Shimer (2001), Lugauer (2010), and elsewhere I assume the demographic change reflects fertility, education investment, and related choices made (sometimes) decades earlier. In the long run, of course, skill acquisition and other demographic characteristics of the labor force react to higher returns to schooling. Endogenizing education decisions represents another potential mechanism connecting the supply of high-skill workers to aggregate output volatility, but I do not pursue such a channel here.[2]

My main contribution is demonstrating how aggregate output volatility reacts to exogenous changes in the supply of high-skill workers. In Section 3, I extend the basic set-up from Acemoglu (1999) to include match-specific costs and aggregate exogenous productivity shocks in a multi-period setting. Introducing heterogeneity into search models makes solutions notoriously difficult to compute. I follow Nagypál (2006) and compare steady state equilibria with different aggregate productivity levels as an approximation to the business cycle. The intuition and main findings remain the same as in the one-period model.

Throughout the paper, (aggregate and firm specific) productivity shocks enter the model exogenously. Following the real business cycle literature, the productivity innovations represent technological know-how or physical phenomena (like natural disasters), etc. I do not model output demand, so the shocks also could be interpreted as changes to demand. I keep the shock process constant even as firms open new types of jobs. In reality, however, the new jobs might use different types of capital, altering the nature of the productivity shocks. For

[2] See Cazzavillan and Olszewski (2011) for a related paper that does endogenize the supply of high-skill workers.

example, skill biased technological change might lead to a different relevant shock process, creating another channel by which the types of jobs affects output volatility. I do not pursue such a mechanism here, but, as with endogenizing the supply of high-skill workers, the model could be extended in this direction.[3]

Finally, in Section 4, I discuss the results from a simple calibration exercise, emphasizing the decrease in aggregate output volatility that occurs when the supply of high-skill workers is sufficiently high (but the productivity shock process remains the same). The calibration exercise indicates a quantitatively large effect relative to the observed data.

2 One-Period Model

The set-up builds on the search models of Mortensen and Pissarides (1994). Models of this type feature a labor market friction; it takes time for workers and firms to meet. Rogerson, Shimer, and Wright (2005) offers a review. The model is standard in most respects; however, I add worker heterogeneity in the manner of Acemoglu (1999). Even in the simple one-period case, the distribution of skills affects amplification of changes to productivity via the labor market. When the supply of high-skill workers becomes large, the economy switches to an equilibrium in which firms create jobs specifically for high-skill workers. The new jobs produce more profits and are therefore less likely to be destroyed by small declines in productivity; this finding is my key contribution. The initial approach closely follows

[3] The two channels could be linked. As the technology employed changes, potentially leading to a different shock process, workers education decisions might change in response. Technological progress also could affect which workers participate in the labor market. Coen-Pirani, Leon, and Lugauer (2010) provide evidence of this in relation to household appliances and female labor supply.

Acemoglu (1999). Then in Section 3, I extend the Acemoglu (1999) model to include match-specific costs and shocks to the aggregate production technology in a multi-period setting.

2.1 Model Environment

A unit mass of workers passively waits to be matched, one-to-one, with an equal number of vacant firms. A fraction ϕ of workers possess superior skills, and the rest are low-skill workers. I normalize the productivity of low-skill workers to $h = 1$, and high-skill workers have $h = \eta > 1$. Firms open jobs, meet workers, and then decide whether to hire a worker and produce. Vacant firms randomly match to a single worker, with no switching allowed. Workers receive share β of output.[4] The firm pays the production costs Ψk out of its share. The fees associated with k are the price for rental and operation of the capital; non-productive firms incur no cost.

Firms know ϕ and η; however, they select k prior to learning their match's labor productivity, h. The technology takes a Cobb-Douglas form. I denote the share of labor by α and normalize $\Psi = (1 - \beta)$. To reduce notational clutter, I suppress functional arguments throughout. Superscripts H and L indicate association with high- and low-skill workers, respectively. See Table 1 for a list of notation and the Appendix for all derivations. The expected value of an unmatched firm with capital k equals:

[4] The search literature frequently uses a 'Nash bargaining' wage rule (Rogerson, Shimer, and Wright 2005). Shimer (2005) attacks this rule for not delivering the wage rigidity necessary to generate the observed fluctuations in the vacancy-unemployment ratio. Other ways to set wages have been proposed. For example, Hall (2005) specifies a rule with more wage stickiness. Since neither wage negotiation nor the vacancy-unemployment ratio is a central concern here, I assume matched pairs split each period's output as in Acemoglu (1999).

$$V = (1-\beta)[\phi x^H(k^{1-\alpha}\eta^\alpha - k) + (1-\phi)x^L(k^{1-\alpha} - k)]. \quad (1)$$

The choice variables x^H and x^L stand for the agent's expected probability, once matched, of actually producing. Thus, a firm expects to produce with a high-skill worker with probability ϕx^H. Firms select k, x^H, and x^L to maximize equation (1). Firms must decide what type of job to create when posting a vacancy and prior to meeting a worker. This irreversible technology decision costs nothing. In a one-period model, workers have no outside option and accept any job. Figure 3 outlines the sequence of events.

2.2 Equilibria

As detailed in Acemoglu (1999), the optimal choice of capital depends on the distribution of skills as captured by ϕ and η. When ϕ and η are relatively low, firms create jobs suitable for either type of worker. If enough workers have sufficiently large productivity, then firms open jobs specifically for high-skill workers. Since workers passively accept any match, an equilibrium consists of firms maximizing their expected value (1). Two equilibrium types emerge. A "pooling" equilibrium prevails when ϕ and η have relatively small values. When ϕ and η are large, a "separating" equilibrium prevails, and firms target high-skill workers.[5]

[5] Acemoglu (1999) refers to one type of equilibrium as "separating" because firms select an amount of capital expecting to produce only when matched with a high-skill worker. Firms treat the two worker types in separate ways. In a "pooling" equilibrium, firms select a level of capital expecting to produce with either type of worker. In the multi-period model developed below, idiosyncratic shocks complicate matters somewhat. However, I continue to use the same labels as in Acemoglu (1999). Finally, the equilibria should not be confused with the pooling and separating concepts common to non-cooperative game theory.

The skill condition (2) dictates the prevailing equilibrium.

Skill Condition (Acemoglu 1999)

$$\eta > \left(\frac{1-\phi}{\phi^{\alpha}-\phi}\right)^{1/\alpha} = \Omega \tag{2}$$

When $\eta < \Omega$, the skill condition (2) fails, and if $\eta > \Omega$, then the skill condition (2) holds.

Letting $\Lambda = (1-\alpha)^{1/\alpha}$ and $\Pi = [\phi\eta^{\alpha} + 1 - \phi]^{1/\alpha}$, Proposition 1 describes the relationship shared by the skill condition (2), the prevailing equilibrium, and the choice of capital.

Proposition 1. *(Acemoglu 1999)*
If $\eta < \Omega$, then a Pooling Equilibrium prevails. Firms choose $k = k_P = \Lambda\Pi$ and $x^H = x^L = 1$.

If $\eta > \Omega$, then a Separating Equilibrium prevails. Firms choose $k = k_H = \Lambda\eta$, $x^H = 1$, and $x^L = 0$.

I take η as given and examine how the economy reacts to an exogenous increase in the supply of high-skill workers, ϕ. Firms select capacity $k = k_P = \Lambda\Pi$ or $k = k_H = \Lambda\eta$ depending on whether the skill condition (2) holds.[6] In a separating equilibrium, low-skill workers do not get hired. Both worker types find jobs in a pooling equilibrium.

[6] As mentioned in the Introduction, I assume that the type of capital and how capital interacts with worker skill or with the productivity shocks introduced in the next section does not change across job types.

2.3 Output and Labor's Extensive Margin

A firm with capital k matched to a worker with skill level h produces:

$$y = k^{1-\alpha} h^{\alpha}. \tag{3}$$

Firms decide whether to hire their match and produce (3) at cost Ψk, given h and k. A firm produces whenever revenues exceed costs. I refer to the hiring / production decision as labor's *extensive margin.* Decisions along the extensive margin are the critical mechanism amplifying shocks to firms' profits.

Figure 4 contains a stylized plot of profits against capacity for a firm with a high-skill worker. The optimal choice of capital is $k_H = \Lambda\eta$. Imagine an aggregate productivity shock shifting the entire profit curve up or down. If a firm selects the right amount of capital for its employee's skill type, then only a large negative shock can drop profits below the shutdown level. When profits are below the shutdown level, the match breaks apart. In a separating equilibrium, firms do pick the optimal capacity for a high-skill worker, $k = k_H = \Lambda\eta$, and profits equal $\frac{\alpha(1-\beta)}{1-\alpha}\Lambda\eta$. The shock would have to annihilate all this profit to disintegrate the match.[7] Only then would the shock generate movement along labor's extensive margin.

In a pooling equilibrium firms select $k = k_P = \Lambda\Pi$. This capacity choice is sub-optimal for

[7] See the Appendix for a derivation of a firm's profits in each equilibrium.

both low-skill workers and high-skill workers. When a firm has a sub-optimal (e.g. pooling) amount of capital for its employee's skill type, a relatively small change in productivity can drop profits to shutdown. For example, a match between a firm and a low-skill worker in a pooling equilibrium generates only $\frac{1-\beta}{1-\alpha}\Lambda\Pi\left(\Pi^{-\alpha}-1+\alpha\right)$ in profits. An aggregate shock impacts labor's extensive margin at less extreme values than in a separating equilibrium, so the pooling equilibrium generates more movement on labor's extensive margin. This finding encapsulates the main result of the paper. Labor's extensive margin connects the distribution of skills to aggregate output volatility. The next section extends the model to many periods and imbeds productivity shocks in order to quantify the difference in output volatility between the two equilibria.

3 Multi-Period Model

In a multi-period setting, the effect of an aggregate productivity shock depends on the distribution of skills in the labor force, and the mechanism works the same way as in the one-period model. When the model economy moves to a separating equilibrium, firms exploit the skill distribution by creating different jobs for workers of different skill types. Firms also modify their hiring strategies, and worker-firm pairs have better capacity-to-productivity matches. Only large shocks drop productivity below shutdown levels. The labor market gains stability along the extensive margin, reducing the amplification of aggregate productivity shocks. Thus, aggregate output has lower volatility in a separating equilibrium.

3.1 Model Environment and Steady State Equations

A unit mass of workers lives an infinite number of discrete periods. I define a period as the amount of time required to find a potential employer. Therefore, every unemployed worker meets a firm in every period, and all vacant firms meet an employee. As in the one-period model, firms choose a capacity k before matching. Firms consider a prospective match's lifetime value when deciding whether to hire a worker and produce. Workers also consider a match's expected lifetime value and do not necessarily accept every job. Workers have an outside option; they can wait for a better match. High-skill workers may have different job finding and unemployment rates than low-skill workers. The fraction of unemployed workers possessing superior skills is denoted by q; whereas, ϕ still denotes the fraction of high-skill workers in the entire population. Each firm knows q, ϕ, and η, the relative productivity of high-skill workers. If a pair does not mutually agree to produce, then the worker remains unemployed and the vacancy is destroyed. Agents discount future earnings at rate $(1 - \delta)$.

There exists a large number of inactive firms, but only measure one open lots for firms to operate. Inactive firms can pay c to post a vacancy on an open lot.[8] Posting a vacancy guarantees the firm meets a worker. The price c is determined in equilibrium, leaving firms indifferent between posting a vacancy and remaining inactive. The value of an inactive firm equals zero, and the value of a vacant firm equals c. In other respects, the matching process remains the same as in the one-period model. Firms entering the market create jobs and

[8] This payment can be considered a rental cost for one of the lots. Alternatively, the payment could be a function of a fixed cost and the probability of meeting a worker through a degenerate matching function, where the number of matches equals the number of unemployed workers. Either way, a free entry condition leaves firms indifferent between paying c and remaining inactive.

search for workers. Firms select k to maximize the expected value of an unmatched firm.

Firms pay all the costs. The period-by-period rental and operation payments, Ψk, depend on the firm's capacity. Initial set-up fees, $\Phi\epsilon$, are paid only once. The set-up costs could include match-specific training, human resources paperwork, moving fees, etc. Matched pairs draw this idiosyncratic shock from a uniform distribution on $[0, \tau]$, denoted by $F(\epsilon)$.

All agents face a common aggregate state, z. I interpret changes to the aggregate state as shocks to productivity. As mentioned in the Introduction, these 'shocks' also could be viewed as demand shocks. The nature of neither the aggregate nor idiosyncratic shocks vary (across equilibria or employee types). In particular, the firm cannot alter the shock processes through the choice of capital (or production technique).[9]

The timing within a period goes as follows. First, share σ of existing matches disintegrate for reasons exogenous to the model. Newly formed matches do not separate. Next, firms open vacancies and select a level of capital. Then, unemployed workers and vacant firms meet. Every unemployed worker meets a vacancy. Upon learning the properties of the match, agents decide whether to produce. The properties of the match include the worker's skill level h the firm's capacity k and the idiosyncratic match specific shock ϵ. If the pair does not produce, then the worker remains unemployed until the next period, and the vacancy ceases to exist. Finally, production (3) occurs. Agents split output; the worker receives share β.

[9] According to Comin and Philippon (2005), the variance of firm-specific output has increased over time. However, keeping the shock processes constant seems like a fair starting point for evaluating the model's main mechanisms. Future work could extend the model to include a deeper theory of how productivity or demand shocks might differentially affect firms employing high- or low-skill workers.

Following Nagypál (2006), and due to computational complexity, I only consider steady state equilibria.[10] The agents' value functions are defined prior to matching. The value of a vacancy with capacity k is:

$$V = qx^H\delta\int_0^{B_H}\left(J^H-\Phi\epsilon\right)dF\left(\epsilon\right)+\left(1-q\right)x^L\delta\int_0^{B_L}\left(J^L-\Phi\epsilon\right)dF\left(\epsilon\right).$$

Firms and workers mutually arrive at x^j, the probability a match with worker type $j\in\{H,L\}$ produces. Additionally, the firm must determine if the match will produce enough to justify paying the up-front fee, $\Phi\epsilon$. I normalize Φ to $\frac{1-\beta}{1-\delta(1-\sigma)}$. If a match produces, then the firm obtains the value of a matched firm, J^j. For example, an unmatched firm meets a low-skill worker with probability $(1-q)$. The pair agrees to produce with probability x^L, given $\epsilon<B_L$. Then, the firm gets J^L, the value of a matched firm. If the match-specific shock exceeds B_L, then the firm prefers to destroy the match. The terms B_H and B_L stand for the maximum idiosyncratic shocks with which a firm chooses to produce with high- and low-skill workers, respectively. Since the firm's outside option equals zero, a firm facing $\epsilon=B_j$ nets zero profits. The next equation encapsulates the value of a matched firm:

[10] In other words, I compare different steady state equilibria to assess the response of the model to aggregate shocks. Nagypál (2006) argues that in "the standard search model such a comparative static exercise invariably gives results that are very close to the dynamic response of the full stochastic model". See Shimer (2005) for an example.

$$J^j = (1-\beta)\, z \left(k^{1-\alpha} h_j^{\alpha} - k\right) + (1-\sigma)\, \delta J^j.$$

The value of a matched firm depends on its capacity, k, and the skill level of its worker, h_j. As before, $\Psi = (1-\beta)$. A match falls apart in any future period with probability σ. When a match breaks apart, the firm leaves the market, and the worker becomes unemployed.

Unemployed workers do not receive any payments. Although, including unemployment benefits would be straight forward. The next equation applies to unemployed workers:

$$U^j = \int_{\kappa} x^j \int_0^{B_j} dF(\epsilon)\, \delta W^j dG(k) + \left(1 - \int_{\kappa} x^j \int_0^{B_j} dF(\epsilon)\, dG(k)\right) \delta U^j.$$

Again, $j \in \{L, H\}$ represents a worker's skill level. An unemployed worker meets a firm with capacity k randomly drawn from the distribution $G(k)$ with support κ. The term $\int_0^{B_j} dF(\epsilon)$ represents the equilibrium probability of the firm actually hiring the worker and producing. Workers take the probability as given.

The following equation expresses the value of an employed worker producing with a firm of capacity k:

$$W^{j} = \beta z k^{1-\alpha} h^{j} + (1-\sigma)\,\delta W^{j} + \sigma \delta U^{j}.$$

As in the one-period model, the worker and the firm divide output with the worker obtaining share β. The firm pays the operating costs Ψk from its share. Each party must receive at least their outside option.

3.2 Pooling and Separating Equilibria

Again consider pooling and separating equilibrium.[11] The exogenous productivity shock z enters aggregate output through the production function and also via the employment level. The production function channel has the same effect across equilibria. The second channel operates through the labor market. Labor's extensive margin responds to changes in the aggregate state. The capital choice in a pooling equilibrium keeps profits closer to shutdown for both worker types. The quantitative analysis in Section 4 confirms that the extensive margin exhibits more volatility than in a pooling equilibrium.

In a pooling equilibrium, $x^{L} = x^{H} = 1$, and κ has only one element k_{P}. The percent of unemployed workers with high-skills q does not equal the population value ϕ because of idiosyncratic shocks. I derive the steady state value functions, the firm maximization problem, the employment level, and the supply of high-skill workers in the Appendix. The

[11] Mixed equilibria may exist for a given set of parameter values. I restrict attention to the pooling and separating cases studied in Acemoglu (1999).

optimal choice of capital must be found numerically. Aggregate output Y^P can be calculated easily given a solution for k_P:

$$Y^P = zk_P^{1-\alpha}\left(\phi\frac{k_P^{1-\alpha}\eta^\alpha - k_P}{k_P^{1-\alpha}\eta^\alpha - k_P + \frac{\sigma\tau}{z}}\eta^\alpha + (1-\phi)\frac{k_P^{1-\alpha} - k_P}{k_P^{1-\alpha} - k_P + \frac{\sigma\tau}{z}}\right). \quad (4)$$

In a separating equilibrium, share p of firms target high-skill workers and set $x^H = 1$ and $x^L = 0$. The remaining $(1-p)$ of firms face $x^H = 0$ and $x^L = 1$ and can only hire low-skill workers. Firms looking for high-skill workers select a high-capacity, and firms searching for low-skill workers pick a low level of capital. So κ has two elements, k_L and k_H. Again, I derive the steady state value functions, the firm maximization problem, the employment level, and the supply of high-skill workers in the Appendix.

The solution to the firms' problems can be found analytically. The choices are:

$$\begin{aligned} k_L &= \Lambda \\ k_H &= \Lambda\eta. \end{aligned}$$

For a separating equilibrium to exist, high-capacity firms should not be willing to hire low-skill workers even with the best possible idiosyncratic shock, $\epsilon = 0$. This technical condition implies $\eta > \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}}$. I assume $\eta > \left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}}$. The value of creating a low-capacity vacancy must be the same as the value of opening a high-capacity vacancy in equilibrium. In a steady state, the flows in and out of employment are equal. These two conditions pin down q, the percent of unemployed with high-skills, and p, the percent of vacant firms with high-capacities. The productivity shock z enters aggregate output (5) through the production function and through employment (see the Appendix for details):

$$\begin{aligned} Y^S &= z\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\{\frac{\phi\left(1-\sigma\right)}{\left(1-\sigma\right)+\frac{\tau\sigma(\phi\eta+1-\phi)}{\phi\eta^2\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}+(\phi\eta^2-1+\phi)\sigma\tau}}\eta \\ &\quad +\frac{\left(1-\phi\right)\left(1-\sigma\right)}{\left(1-\sigma\right)+\frac{\tau\sigma(\phi\eta+1-\phi)}{(1-\phi)\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}+(\phi\eta^2-1+\phi)\frac{\sigma\tau}{\eta}}}\}. \end{aligned} \tag{5}$$

In a pooling equilibrium, all firms choose capacity $k = k_P$. Firms agree to produce with any worker as long as the match-specific costs do not exceed the boundary B. Workers' outside options do not bind because $\kappa = \{k_P\}$. The multi-period version of the skill condition can be found numerically by setting $V^S = V^P$. When $V^S = V^L > V^P$, the economy is in a separating equilibrium. In a separating equilibrium, firms only produce with one of the two types of workers. A high-capacity firm will not hire a low-skill worker, and a high-skill worker would rather wait than produce with a low-capacity firm. Thus, $\kappa = \{k_L, k_H\}$.

4 Quantitative Results

This section reports the results from a simple quantitative exercise in order to assess how much of the drop in aggregate output volatility can be attributed to changes in the skill distribution.

4.1 Parameter Values and Main Results

There are only a few parameters to choose. Each period lasts one quarter. I set the exogenous separation rate σ equal to 0.1. This value generates the average job duration of about 2.5 years quoted in Shimer (2005). The supply of high-skill workers, ϕ, is set equal to the percentage of the labor force with more than a high-school education as reported in Acemoglu (2002).[12] The production function parameter α is set to 0.64 to match the long-run share of output going to labor (Kydland and Prescott 1982). Given α, the model implies that η must be 5 or higher for a separating equilibrium to exist (see Section 3). The share of output going to workers β and the discount rate δ only act to normalize the value of a matched firm; I set these parameters to 0.64 and 0.95, respectively. Table 2 lists the relevant parameter value choices. I discuss alternative parameter values below.

The separating case can be directly evaluated. To solve for the pooling equilibrium, I search over a coarse grid to find starting points. Then, I use a hill climber. Table 3 details firms' equilibrium capital choices.

[12] As already noted, I use the supply of college trained workers as a proxy for the supply of high-skill workers. Acemoglu (2002) lists this number at about 19.2% in 1980 and 24.0% in 1990.

The results from the multi-period model agree with the theory built up with the one-period model. In the pooling equilibrium firms optimally select a middling amount of capital, $k_P = 0.471$. This capital choice is sub-optimal for both worker types and generates relatively low profits. The value of a firm matched with either a high- or low-skill worker in a separating equilibrium exceeds the value of a firm matched with the same worker in a pooling equilibrium. In the separating case, workers produce with the optimal amount of capital for their skill type. High-skill workers produce with more capital, $k_H = 1.013$, while low-skill workers produce with less, $k_L = 0.203$. When the supply of high-skill workers gets large enough, firms have a profit incentive to design new types of jobs. Aggregate output volatility declines because matches are more stable on the extensive margin. The value, J, of being matched goes up for the firm. The model also features a change in the skill premium or wage inequality. This result follows directly from Acemoglu (1999). Wage inequality averages η^α in the pooling case and increases to η in a separating equilibrium.

Table 4 presents the model output and employment results with U.S. data in parentheses.[13] Aggregate output comes from equations (4) and (5). The comparison is over steady state equilibria as in Nagypál (2006) with the aggregate productivity variable z changing by 5 percent. When subjected to this 'shock', aggregate output changed by 6.9 percent less in the separating equilibrium than in the pooling equilibrium. The percent change in output can be interpreted as a measure of cyclical volatility. Thus, the change in equilibrium generates about 16 percent of the observed reduction in aggregate output volatility. This finding

[13] Table 4 reports the difference in output and employment across steady state equilibria, where z has been changed by 5%. The U.S. data from 1980 (for pooling) and 1990 (for separating) are given in parentheses. The U.S. data are the standard deviation of the logged, de-trended GDP and employment time series.

represents the main quantitative result.

4.2 Alternative Parameter Values

The relative productivity of high-skill workers could take on values above 5, but if η is too high, then the pooling equilibrium will fall apart (some firms will target high-skill workers, only). Increasing η moves the economy closer to the threshold (i.e. the skill condition) at which firms begin to treat workers separately because η increases the value of high-skill workers. Similarly, higher values of ϕ move the economy closer to a separating equilibrium because high-skill workers become easier to find.

Table 5 presents the results using alternate parameter choices. The parameters α and σ remain at their previous values, and ϕ still equals 0.24 in the separating case. Table 5 reports only the final results (i.e. the percent difference in 'volatility' between the two equilibria types in steady state). The difference between output volatility in the pooling case and separating case gets larger as the initial (pooling) value of ϕ gets smaller and as η grows larger. The benchmark results from Table 4 appear to be a lower bound. Across the different parameter value choices, the mechanism explains $16 - 28$ percent of the observed decline in aggregate output volatility.

4.3 Discussion

I have conjectured a link between the supply of high-skill workers and aggregate output volatility. The story goes as follows. The economy gained skilled workers throughout the

1970's. Most notably, the large, well-educated, baby-boom generation entered the workforce beginning around 1970. By the mid-1980's, firms reacted by altering their hiring strategies and by creating jobs tailored to workers of different skill types. The average worker became better suited to his or her job. The labor market's ability to amplify the aggregate shock declined, so GDP volatility fell. The drop corresponds to the switch from a pooling equilibrium to a separating equilibrium in the model economy.

In the model, the decline in output volatility occurs just as the economy moves from a pooling equilibrium to a separating equilibrium. The reason for the change in output volatility can be described as follows. In a pooling equilibrium, the proportion of high-skill workers, ϕ, is relatively small, and firms select $k = k_P$. Firms expect to produce with workers of either skill type. Small increases in ϕ or η lead to small changes in output. When ϕ exogenously increases enough to satisfy the skill condition, the economy moves into a separating equilibrium, and the composition of jobs changes. The equilibrium switch happens because firms respond to profit incentives created by the availability of high-skill workers. Firms open new high-capacity jobs and modify their hiring strategies. Labor and capital are better matched because firms select a level of capital suited to producing with only one type of worker. Workers in a separating equilibrium produce with the optimal amount of capital for their skill type, reducing the economy's responsiveness to productivity shocks along labor's extensive margin. Only large shocks disintegrate a match. Shocks have less impact on hiring and production decisions, decreasing aggregate output volatility, and the model economy generates the sudden and sustained business cycle moderation observed in the data.

The model has several other implications. If the model economy changes from a pooling to a separating equilibrium, then firms create different types of jobs regardless of business cycle fluctuations. The skill premium increases because low-skill workers produce with less capital than high-skill workers. In the main quantitative example, high-skill workers produce with $k_H = 1.013$, while low-skill workers produce with $k_L = 0.203$. Wage inequality among workers grew (Katz and Murphy 1992, Karoly 1992) over roughly the same time period as GDP volatility shrank, so it is tempting to imagine a connection between output volatility and income inequality. In the model, an exogenous progression in skills increases both macroeconomic stability and the skill premium.

The new composition of jobs and associated hiring strategies create the increase in the skill premium. Acemoglu (1999) lists several pieces of empirical evidence in this regard. The evidence includes measurable changes in recruitment practices, the capital-to-labor ratio, the distribution of jobs, the distribution of on the job training, and better employee-employer matching. The U.S. economy has also been moving away from manufacturing and towards service based industries such as information technologies. See Acemoglu (1999) for more details.

Closely related to the increased skill premium is the decrease in relative productivity between low- and high-skill workers. Again, the decrease occurs in the model because of the capital choice of firms. Low-skill workers produce with much less capital in the separating equilibrium. Cazzavillan and Olszewski (2011) offers evidence that the relative productivity of low-skill workers has decreased over the time. Cazzavillan and Olszewski (2011) view the change through the lens of skill biased technological change, but my model is also consistent (qualitatively) with the observed changes in relative productivity.

My model also predicts that the GDP volatility decrease will be accompanied by a decrease in employment volatility (see Table 4). As already noted, employment fluctuations have declined in the U.S. aggregate data; however, the drop in employment volatility has not been the same across skill groups. The decline has been greater for low-skill workers; see Castro and Coen-Pirani (2008). In my simulation exercise, employment volatility fell by 25 percent more for low-skill workers than for high-skill workers. In fact, when the model economy switches to a separating equilibrium, employment volatility among high-skill workers does not fall appreciably *relative to the observed decline in GDP volatility.* These results are not inconsistent with the observed changes in cyclical employment volatility by skill group reported in Castro and Coen-Pirani (2008).

Another key implication of the model is that workers are better matched to their jobs in the separating equilibrium, implying a drop in the overall job separation rate. I have calculated the separation rate (into unemployment) using Current Population Survey data. In 1982, about 2.5% of workers separated from their employer. By 1988, the separation rate had fallen to 1.6% and it remained low until the most recent recession. Fujita (2011) reports similar numbers and plots a time series of the separation rate. Although the drop in separations does happen as a trend break (as my model would suggest), the fall is quite rapid.

Finally, wages tend to be weakly pro-cyclical and unemployment moves counter-cyclically in the U.S. data. The model economy features both pro-cyclical wages and counter-cyclical unemployment. Wages equal a share of output, and output co-moves with the aggregate shock. Similarly, the employment rate moves in tandem with the aggregate shock because firms react to high realizations of z by becoming less selective employers.

5 Conclusion

This paper extends the heterogeneous agent labor search model developed in Acemoglu (1999) to a multi-period setting, building in match-specific and aggregate shocks. The model shows how a large increase in the supply of high-skill workers can cause a sudden decrease in output volatility. The supply of skills in the labor force has been dismissed as a cause of GDP volatility reduction because of an apparent timing problem. The stock of high-skill workers increased gradually, whereas GDP volatility experienced a dramatic break. However, a smooth increase in the proportion of high-skill workers causes an abrupt change in aggregate output volatility in the model economy developed in this paper.

In the model, firms react to an influx of skills by modifying both the composition of jobs and their hiring strategies. The labor market's responsiveness to the aggregate productivity shock declines when firms alter these extensive margin decisions. The economy moves to a separating equilibrium and enters a state of quiescence. The change corresponds to the sudden and sustained drop in U.S. GDP volatility, which occurred in the early 1980's. The results of a simple quantitative exercise indicate a large increase in the relative supply of high-skill workers can account for over 15 percent of the Great Moderation. The labor market based theory provides a single explanation for both the decrease in output volatility and the increase in wage inequality. Simulated data from a calibrated version of the model are consistent with the observed data along several other dimensions as well.

Throughout the paper I take the skill supply as given and examine the consequences for output volatility. This set-up keeps the model tractable, and taking demographics as given seems reasonable for short-run analysis. Also, there exists a growing literature studying how exogenous demographic changes affect the macro-economy, including Feyrer (2007), Lugauer and Redmond (2011), Curtis, Lugauer, and Mark (2011), and Jensen, Lugauer, and Sadler (2011) among others. Endogenizing human capital acquisition and other labor force demographic characteristics is a logical extension for this line of inquiry. I look forward to pursuing such an approach in future research.

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6 Appendix

The Appendix contains the algebraic derivations referenced throughout the paper.

6.1 Proposition 1 and the Skill Condition

Acemoglu (1999) contains a proof of Proposition 1. I replicate the proof using my notation for the sake of completeness, and I also derive the skill condition (2).

Workers accept all jobs because their outside option equals zero and wages are strictly positive. Thus, an equilibrium is a set, $\{k, x^H, x^L\}$, maximizing each firms expected value (1). Firms maximize (1) according to the first-order condition:

$$\frac{\partial}{\partial k}V(k, x^H, x^L) = (1-\beta)[\phi x^H((1-\alpha)k^{-\alpha}\eta^\alpha - 1) + (1-\phi)x^L((1-\alpha)k^{-\alpha} - 1)] = 0, \quad (6)$$

where x^H and x^L are considered fixed. Setting $x^H = x^L = 1$ and solving equation (6) for k_P gives:

$$\begin{aligned}
(1-\beta)[\phi((1-\alpha)k^{-\alpha}\eta^\alpha - 1) + (1-\phi)((1-\alpha)k^{-\alpha} - 1)] &= 0 \\
\phi((1-\alpha)k^{-\alpha}\eta^\alpha - 1) + (1-\phi)((1-\alpha)k^{-\alpha} - 1) &= 0 \\
\phi(1-\alpha)k^{-\alpha}\eta^\alpha - \phi + (1-\alpha)k^{-\alpha} - 1 - \phi(1-\alpha)k^{-\alpha} + \phi &= 0 \\
(1-\alpha)^{1/\alpha}[\phi\eta^\alpha - \phi + 1]^{1/\alpha} &= k \\
k_P &= \Lambda\Pi.
\end{aligned}$$

With $x^H = x^L = 1$ and $k = k_P$, the expected value of an unmatched firm is:

$$\begin{aligned}
V^P(k = \Lambda\Pi, x^H = 1, x^L = 1) &= (1-\beta)[\phi((\Lambda\Pi)^{1-\alpha}\eta^\alpha - \Lambda\Pi) + (1-\phi)((\Lambda\Pi)^{1-\alpha} - \Lambda\Pi)] \\
&= (1-\beta)[\phi(\Lambda\Pi)^{1-\alpha}\eta^\alpha + (\Lambda\Pi)^{1-\alpha} - \Lambda\Pi - \phi(\Lambda\Pi)^{1-\alpha}] \\
&= \Lambda\Pi(1-\beta)[\phi(\Lambda\Pi)^{-\alpha}\eta^\alpha + (\Lambda\Pi)^{-\alpha} - 1 - \phi(\Lambda\Pi)^{-\alpha}] \\
V^P &= \Lambda\Pi\alpha(1-\beta)/(1-\alpha).
\end{aligned}$$

Setting $x^H = 1$ and $x^L = 0$ and solving equation (6) for k_H gives:

$$\begin{aligned}
(1-\beta)[\phi((1-\alpha)k^{-\alpha}\eta^\alpha - 1)] &= 0 \\
(1-\alpha)k^{-\alpha}\eta^\alpha - 1 &= 0 \\
(1-\alpha)^{1/\alpha}\eta &= k \\
k_H &= \Lambda\eta.
\end{aligned}$$

With $x^H = 1$, $x^L = 0$, and $k = k_H$ the expected value of an unmatched firm equals:

$$\begin{aligned}
V^H(k = \Lambda\eta, x^H = 1, x^L = 0) &= (1-\beta)[\phi((\Lambda\eta)^{1-\alpha}\eta^\alpha - \Lambda\eta)] \\
&= \Lambda(1-\beta)\eta\phi[\Lambda^{-\alpha} - 1] \\
&= \Lambda(1-\beta)\eta\phi[1 - 1 + \alpha]/(1-\alpha) \\
V^H &= \Lambda\alpha(1-\beta)\eta\phi/(1-\alpha).
\end{aligned}$$

Note that $V(k_P, x^H < 1, x^L = 1) < V^P$ and $V(k_H, x^H < 1, x^L = 0) < V^H$.

Setting $V^P = V^H$ and solving for η gives the skill condition (2) :

$$\begin{aligned}
\Lambda\Pi\alpha(1-\beta)/(1-\alpha) &= \Lambda\alpha(1-\beta)\eta\phi/(1-\alpha) \\
\Pi &= \eta\phi \\
\left[\phi\eta^{\alpha}+1-\phi\right]^{1/\alpha} &= \eta\phi \\
1-\phi &= (\eta\phi)^{\alpha}-\phi\eta^{\alpha} \qquad (7) \\
\eta &= \left(\frac{1-\phi}{\phi^{\alpha}-\phi}\right)^{1/\alpha} \qquad (8) \\
\eta &= \Omega.
\end{aligned}$$

When the skill condition (2) does not hold (i.e. $\eta < d$), then $V(k_P, x^H \leq 1, x^L < 1) < V^P$; also, when the skill condition (2) holds (i.e. $\eta > d$), then $V(k_H, x^H \leq 1, x^L < 1) < V^H$. Thus, either the pooling equilibrium is the unique equilibrium or the separating equilibrium is the unique equilibrium in the one-period model.

6.2 Firm Profits in the One-Period Model

Firm profits in a one-period model can be calculated by subbing in the firm's choice of capital. Consider first a pooling equilibrium:

$$\begin{aligned}
\text{Profit} &= (1-\beta)\left(k_P^{1-\alpha}h^{\alpha} - k_P\right) \\
\text{note:} &\quad k_p = \Lambda\Pi, \quad \Lambda = (1-\alpha)^{\frac{1}{\alpha}}, \quad \Pi = (\phi\eta^{\alpha} + 1 - \phi)^{\frac{1}{\alpha}} \\
\text{Profit} &= (1-\beta)\left((\Lambda\Pi)^{1-\alpha} h^{\alpha} - \Lambda\Pi\right) \\
&= (1-\beta)\Lambda\Pi\left((\Lambda\Pi)^{-\alpha} h^{\alpha} - 1\right) \\
&= \frac{1-\beta}{1-\alpha}\Lambda\Pi\left(\Pi^{-\alpha}h^{\alpha} - 1 + \alpha\right) \\
\text{Profit} &= \frac{1-\beta}{1-\alpha}\Lambda\left(\phi\eta^{\alpha} + 1 - \phi\right)^{\frac{1}{\alpha}}\left(\frac{h^{\alpha}}{\phi\eta^{\alpha} + 1 - \phi} - 1 + \alpha\right).
\end{aligned}$$

Similarly, in a separating equilibrium profits are:

$$\begin{aligned}
\text{Profit} &= (1-\beta)\, z\left(k_H^{1-\alpha}\eta^{\alpha} - k_H\right) \\
\text{note:} &\quad k_H = \Lambda\eta, \quad \Lambda = (1-\alpha)^{\frac{1}{\alpha}} \\
\text{Profit} &= (1-\beta)\, z\left((\Lambda\eta)^{1-\alpha}\eta^{\alpha} - \Lambda\eta\right) \\
&= (1-\beta)\, z\Lambda\eta\left((\Lambda\eta)^{-\alpha}\eta^{\alpha} - 1\right) \\
\text{Profit} &= \frac{\alpha(1-\beta)}{1-\alpha}\Lambda\eta.
\end{aligned}$$

The claim in the main body of the paper is that minimum profits in a separating equilibrium are larger than in a pooling equilibrium. This fact can be shown analytically:

$$\begin{aligned}
\frac{\alpha(1-\beta)}{1-\alpha}\Lambda\eta &> \frac{1-\beta}{1-\alpha}\Lambda\left(\phi\eta^{\alpha}+1-\phi\right)^{\frac{1}{\alpha}}\left(\frac{1}{\phi\eta^{\alpha}+1-\phi}-1+\alpha\right) \\
\alpha\eta &> \left(\phi\eta^{\alpha}+1-\phi\right)^{\frac{1}{\alpha}}\left(\frac{1}{\phi\eta^{\alpha}+1-\phi}-1+\alpha\right) \\
\text{note} &: 1<\left(\phi\eta^{\alpha}+1-\phi\right)^{\frac{1}{\alpha}}<\eta.
\end{aligned}$$

The result follows immediately.

The result can also be seen by using the parameter values from the first numerical example in Section 5:

$$\begin{aligned}
\alpha\eta &> \left(\phi\eta^{\alpha}+1-\phi\right)^{\frac{1}{\alpha}}\left(\frac{1}{\phi\eta^{\alpha}+1-\phi}-1+\alpha\right) \\
(.64)\,5 &> \left((.192)\,5^{.64}+1-.192\right)^{\frac{1}{.64}}\left(\frac{1}{(.192)\,5^{.64}+1-.192}-1+.64\right) \\
3.20 &> .61\,.
\end{aligned}$$

So, in the simple one-period model, a separating equilibrium requires a shock of about five times the magnitude to generate movement on the extensive margin.

6.3 Solution to the Pooling Equilibrium

The firm's choice of capital in a pooling equilibrium can only be found numerically in the multi-period model. In this section, I derive the equations used to find the numerical solution.

The following system of equations (9) defines the economy when in a steady state pooling equilibrium:

$$\begin{aligned}
V^P &= q\delta \int_0^{B_H^P} \left(J^H - \Phi\epsilon\right) dF(\epsilon) + (1-q)\,\delta \int_0^{B_L^P} \left(J^L - \Phi\epsilon\right) dF(\epsilon) \qquad (9) \\
J^H &= (1-\beta)\,z\left(k_P^{1-\alpha}\eta^\alpha - k_P\right) + (1-\sigma)\,\delta J^H \\
U^H &= \delta \int_0^{B_H^P} dF(\epsilon)\, W^H + \left(1 - \int_0^{B_H^P} dF(\epsilon)\right) \delta U^H \\
W^H &= \beta z k_P^{1-\alpha}\eta^\alpha + (1-\sigma)\,\delta W^H + \sigma\delta U^H \\
J^L &= (1-\beta)\,z\left(k_P^{1-\alpha} - k_P\right) + (1-\sigma)\,\delta J^L \\
U^L &= \delta \int_0^{B_L^P} dF(\epsilon)\, W^L + \left(1 - \int_0^{B_L^P} dF(\epsilon)\right) \delta U^L \\
W^L &= \beta z k_P^{1-\alpha} + (1-\sigma)\,\delta W^L + \sigma\delta U^L.
\end{aligned}$$

In equilibrium, each firm must be choosing the optimal amount of capital given the steady state equations (9). This level of capital can be found by letting $k_P = k$ and substituting:

$$\begin{aligned}
J^H &= \frac{(1-\beta)\,z\left(k^{1-\alpha}\eta^\alpha - k\right)}{(1-(1-\sigma)\,\delta)} \\
J^L &= \frac{(1-\beta)\,z\left(k^{1-\alpha} - k\right)}{(1-(1-\sigma)\,\delta)}
\end{aligned}$$

into V^P and integrating. The idiosyncratic shock is uniformly distributed between zero and τ. Thus:

$$\begin{aligned}
V^P &= q\int_0^{B_H^P} \delta\left(J^H - \frac{(1-\beta)\,\epsilon}{(1-(1-\sigma)\,\delta)}\right) dF(\epsilon) + (1-q)\int_0^{B_L^P} \delta\left(J^L - \frac{(1-\beta)\,\epsilon}{(1-(1-\sigma)\,\delta)}\right) dF(\epsilon) \\
V^P &= \frac{(1-\beta)\,\delta}{\tau\,(1-(1-\sigma)\,\delta)}\left(q\left(B_H^P z\left(k^{1-\alpha}\eta^{\alpha}-k\right) - \frac{1}{2}\left(B_H^P\right)^2\right)\right) \\
&\quad + (1-q)\left(B_L^P z\left(k^{1-\alpha}-k\right) - \frac{1}{2}\left(B_L^P\right)^2\right).
\end{aligned}$$

In a pooling equilibrium, each firm chooses an optimal amount of capital given the above equations (9). So, k_P is the solution to:

$$\begin{aligned}
\max_{k_P}\left\{V^P\right\} &= \max_{k_P}\{\frac{(1-\beta)\,\delta}{\tau\,(1-(1-\sigma)\,\delta)}\{q\left(B_H^P z\left(k^{1-\alpha}\eta^{\alpha}-k\right) - \frac{1}{2}\left(B_H^P\right)^2\right) \\
&\quad + (1-q)\left(B_L^P z\left(k^{1-\alpha}-k\right) - \frac{1}{2}\left(B_L^P\right)^2\right)\}\}.
\end{aligned} \tag{10}$$

The first-order condition of equation (10) captures the optimal level of capital, k_P. The first-order condition is:

$$0 = qB_H^P\left((1-\alpha)\,k_P^{-\alpha}\eta^{\alpha}-1\right) + (1-q)\,B_L^P\left((1-\alpha)\,k_P^{-\alpha}-1\right). \tag{11}$$

Not every match produces. A firm hires its match and produces for idiosyncratic shocks, ϵ, where $J^j - \Phi\epsilon$ is greater than zero (the outside option). In other words, a firm only hires a worker and produces if the idiosyncratic shock is low enough.

The threshold values, B, are given by:

$$
\begin{aligned}
J^j - \Phi B_H^P &= 0 \\
\frac{(1-\beta)\, z\left(k^{1-\alpha}\eta^{\alpha} - k\right)}{(1-(1-\sigma)\,\delta)} - \Phi B_H^P &= 0 \\
z\left(k^{1-\alpha}\eta^{\alpha} - k\right) - B_H^P &= 0
\end{aligned}
$$

$$
B_H^P = z\left(k^{1-\alpha}\eta^{\alpha} - k\right).
$$

Similarly:

$$
B_L^P = z\left(k^{1-\alpha} - k\right). \tag{12}
$$

The flows of workers in and out of employment in the steady state pin down the employment levels and the value of q, the percent of unemployed with high-skills. Let e^j and u^j denote the number (not percent) of employed and unemployed, respectively.

By definition:

$$
1 - \phi = u^L + e^L
$$

$$
\phi = u^H + e^H
$$

$$
q = \frac{u^H}{u^H + u^L}.
$$

The flow equations in steady state are:

$$\begin{aligned} e^H &= e^H + u^H \int_0^{B_H^P} dF(\epsilon) - \sigma e^H \\ e^L &= e^L + u^L \int_0^{B_L^P} dF(\epsilon) - \sigma e^L. \end{aligned}$$

Thus:

$$\begin{aligned} e^H &= \phi \frac{z\left(k^{1-\alpha}\eta^\alpha - k\right)}{z\left(k^{1-\alpha}\eta^\alpha - k\right) + \sigma\tau} \\ e^L &= (1-\phi)\frac{z\left(k^{1-\alpha} - k\right)}{z\left(k^{1-\alpha} - k\right) + \sigma\tau}, \end{aligned}$$

and:

$$q = \frac{1}{1 + \frac{(1-\phi)\left(z\left(k^{1-\alpha}\eta^\alpha - k\right) + \sigma\tau\right)}{\phi\left(z\left(k^{1-\alpha} - k\right) + \sigma\tau\right)}}.$$

For a pooling equilibrium in a steady state:

$$\begin{aligned} e^H &= \phi \frac{z\left(k_P^{1-\alpha}\eta^\alpha - k_P\right)}{z\left(k_P^{1-\alpha}\eta^\alpha - k_P\right) + \sigma\tau} \\ e^L &= (1-\phi)\frac{z\left(k_P^{1-\alpha} - k_P\right)}{z\left(k_P^{1-\alpha} - k_P\right) + \sigma\tau}, \end{aligned}$$

and:

$$q = \frac{1}{1 + \frac{(1-\phi)\left(z\left(k_P^{1-\alpha}\eta^\alpha - k_P\right) + \sigma\tau\right)}{\phi\left(z\left(k_P^{1-\alpha} - k_P\right) + \sigma\tau\right)}}. \tag{13}$$

Equations (11), (12), and (13) can be combined to find a numerical solution to the model economy in a pooling equilibrium and to calculate aggregate output (4) for a given set of parameter values.

6.4 Solution to the Separating Equilibrium

When in a separating equilibrium the model can be solved analytically. The steady state is characterized by the following equations (14):

$$
\begin{aligned}
V_H^S &= q\delta \int_0^{B_H^S} \left(J^H - \Phi\epsilon\right) dF(\epsilon) \qquad (14)\\
J^H &= (1-\beta) z \left(k_H^{1-\alpha}\eta^\alpha - k_H\right) + (1-\sigma)\delta J^H \\
U^H &= p\delta \int_0^{B_H^S} dF(\epsilon)\, W^H + \left(1 - p\int_0^{B_H^S} dF(\epsilon)\right)\delta U^H \\
W^H &= \beta z k_H^{1-\alpha}\eta^\alpha + (1-\sigma)\delta W^H + \sigma\delta U^H \\
V_L^S &= (1-q)\,\delta \int_0^{B_L^S} \left(J^L - \Phi\epsilon\right) dF(\epsilon) \\
J^L &= (1-\beta) z \left(k_L^{1-\alpha} - k_L\right) + (1-\sigma)\delta J^L \\
U^L &= (1-p)\,\delta \int_0^{B_L^S} dF(\epsilon)\, W^L + \left(1 - (1-p)\int_0^{B_L^S} dF(\epsilon)\right)\delta U^L \\
W^L &= \beta z k_L^{1-\alpha} + (1-\sigma)\delta W^L + \sigma\delta U^L.
\end{aligned}
$$

The value function for a vacant high-capacity firm can be rewritten by letting $k_H = k$ and $\Phi = \frac{(1-\beta)}{(1-(1-\sigma)\delta)}$:

$$
V_H^S = q \int_0^{B_H^S} \delta\left(J^H - \frac{(1-\beta)}{(1-(1-\sigma)\delta)}\epsilon\right) dF(\epsilon)
$$

$$
J^H = \frac{(1-\beta)\, z\left(k^{1-\alpha}\eta^\alpha - k\right)}{(1-(1-\sigma)\delta)}.
$$

Subbing in and evaluating the integral gives the following:

$$
\begin{aligned}
V_H^S &= q\delta \int_0^{B_H^S} \left(\frac{(1-\beta)\, z\left(k^{1-\alpha}\eta^\alpha - k\right)}{(1-(1-\sigma)\,\delta)} - \frac{(1-\beta)}{(1-(1-\sigma)\,\delta)}\epsilon \right) dF(\epsilon) \\
V_H^S &= \frac{q(1-\beta)\,\delta}{\tau(1-(1-\sigma)\,\delta)} \left(B_H^S z\left(k^{1-\alpha}\eta^\alpha - k\right) - \frac{1}{2}\left(B_H^S\right)^2 \right).
\end{aligned}
$$

So, k_H is the solution to:

$$
\max_{k_H}\left\{V_H^S\right\} = \max_{k_H}\left\{ \frac{q(1-\beta)\,\delta}{\tau(1-(1-\sigma)\,\delta)} \left(B_H^S z\left(k^{1-\alpha}\eta^\alpha - k\right) - \frac{1}{2}\left(B_H^S\right)^2 \right) \right\}.
$$

Not every match produces. A firm hires its match when the idiosyncratic shock ϵ is such that $J^j - \Phi\epsilon$ is greater than zero (the outside option). In other words, a firm only hires a worker and produces if the idiosyncratic shock is low enough. The threshold values, B, are given by:

$$
\begin{aligned}
0 &= zk^{1-\alpha}\eta^\alpha - zk - B_H^S \\
B_H^S &= z\left(k^{1-\alpha}\eta^\alpha - k\right) \\
B_H^S &= \alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}\eta
\end{aligned}
$$

$$
B_L^S = \alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}.
$$

Then, from the first-order condition:

$$1 = (1-\alpha)\,k^{-\alpha}\eta^{\alpha}$$

$$\begin{aligned} k_H^S &= (1-\alpha)^{\frac{1}{\alpha}}\,\eta \\ k_H &= \Lambda\eta. \end{aligned}$$

Similarly, let $k_L = k$ and $\Phi = \frac{(1-\beta)}{(1-(1-\sigma)\delta)}$. Then, from the steady-state equations (14) :

$$V_L^S = (1-q)\int_0^{B_L^S} \delta\left(J^L - \frac{(1-\beta)}{(1-(1-\sigma)\delta)}\epsilon\right) dF(\epsilon)$$

$$J^L = \frac{(1-\beta)}{(1-(1-\sigma)\delta)} z\left(k^{1-\alpha} - k\right).$$

Combining and evaluating the integral gives the following:

$$V_L^S = \frac{(1-q)(1-\beta)\delta}{(1-(1-\sigma)\delta)}\int_0^{B_L^S}\left(z\left(k^{1-\alpha}-k\right) - \epsilon\right) dF(\epsilon)$$

$$V_L^S = \frac{(1-q)(1-\beta)\delta}{(1-(1-\sigma)\delta)\tau}\left(B_L^S z\left(k^{1-\alpha}-k\right) - \frac{1}{2}\left(B_L^S\right)^2\right),$$

and k_L solves:

$$\max_{k_L}\left\{V_L^S\right\} = \max_{k_L}\left\{\frac{q(1-\beta)\delta}{\tau(1-(1-\sigma)\delta)}\left(B_L^S z\left(k^{1-\alpha}-k\right) - \frac{1}{2}\left(B_L^S\right)^2\right)\right\}.$$

The first-order condition implies:

$$0 = (1 - \alpha)\, k^{-\alpha} - 1$$

$$\begin{aligned} k_L^S &= (1-\alpha)^{\frac{1}{\alpha}} \\ k_L &= \Lambda. \end{aligned}$$

Also, a technical condition for a separating equilibrium is high-capacity firms must not be willing to hire low-skill workers even with the best possible idiosyncratic shock, $\epsilon = 0$ (the firm's outside option). This implies the following:

$$(1-\beta)\, z \left(k_S^{1-\alpha} - k_S\right) < 0$$

$$\text{note:} \;\; k_S = (1-\alpha)^{\frac{1}{\alpha}}\, \eta.$$

So, it must be that:

$$\begin{aligned} (1-\alpha)^{\frac{1-\alpha}{\alpha}}\, \eta^{1-\alpha} - (1-\alpha)^{\frac{1}{\alpha}}\, \eta &< 0 \\ 1 - (1-\alpha)\, \eta^{\alpha} &< 0 \end{aligned}$$

$$\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\alpha}} < \eta.$$

The value of creating a low-capacity vacancy must be the same as the value of opening a high-capacity vacancy in equilibrium. In a steady state, the flows in and out of employment are equal. These two conditions pin down q, the percent of unemployed with high-skills, and p, the percent of vacant firms with high-capacities as follows.

Note:

$$V_L^S = \frac{(1-q)(1-\beta)\delta}{(1-(1-\sigma)\delta)\tau}\frac{1}{2}\left(\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right)^2$$

$$V_H^S = \frac{q(1-\beta)\delta}{\tau(1-(1-\sigma)\delta)}\frac{1}{2}\left(\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right)^2\eta^2.$$

Setting $V_L^S = V_H^S$ requires:

$$\frac{(1-q)(1-\beta)\delta}{(1-(1-\sigma)\delta)\tau}\frac{1}{2}\left(\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right)^2 = \frac{q(1-\beta)\delta}{\tau(1-(1-\sigma)\delta)}\frac{1}{2}\left(\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right)^2\eta^2$$

$$q = \frac{1}{\eta^2+1}.$$

The flow equations in steady state can be used to calculate the percent p of vacant firms with a high-capacity level of capital, k_H^S.

By definition:

$$1-\phi = u^L + e^L$$

$$\phi = u^H + e^H$$

$$q = \frac{u^H}{u^H+u^L}.$$

The steady state flow equations are:

$$
\begin{aligned}
e^H &= e^H + u^H p \int_0^{B_H^S} dF(\epsilon) - \sigma e^H \\
e^L &= e^L + u^L (1-p) \int_0^{B_L^S} dF(\epsilon) - \sigma e^L.
\end{aligned}
$$

Thus:

$$
\begin{aligned}
u^H &= \frac{\sigma\phi}{(1-\sigma)\, p \frac{B_H^S}{\tau} + \sigma} \\
u^L &= \frac{\sigma(1-\phi)}{(1-\sigma)(1-p)\frac{B_L^S}{\tau} + \sigma} \\
e^H &= \frac{\phi(1-\sigma)}{(1-\sigma) + \frac{\tau\sigma}{pB_H^S}} \\
e^L &= \frac{(1-\phi)(1-\sigma)}{(1-\sigma) + \frac{\tau\sigma}{(1-p)B_L^S}},
\end{aligned}
$$

and:

$$
q = \frac{\frac{\sigma\phi}{p\frac{B_H^S}{\tau}+\sigma}}{\frac{\sigma\phi}{p\frac{B_H^S}{\tau}+\sigma} + \frac{\sigma(1-\phi)}{(1-p)\frac{B_L^S}{\tau}+\sigma}}.
$$

Subbing in p and solving:

$$\frac{1}{\eta^2+1} = \frac{1}{1+\frac{(1-\phi)pB_H^S+(1-\phi)\sigma\tau}{\phi(1-p)B_L^S+\phi\sigma\tau}}$$

$$(1-\phi)\,pB_H^S+(1-\phi)\,\sigma\tau = \eta^2\phi\,(1-p)\,B_L^S+\eta^2\phi\sigma\tau$$

$$p=\frac{\phi\eta+\frac{\left(\phi\eta^2-1+\phi\right)\sigma\tau}{\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}\eta}}{(\phi\eta+1-\phi)}.$$

which gives:

$$e^H = \frac{\phi\,(1-\sigma)}{(1-\sigma)+\frac{\tau\sigma(\phi\eta+1-\phi)}{\phi\eta^2\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}+(\phi\eta^2-1+\phi)\sigma\tau}}$$

$$e^L = \frac{(1-\phi)\,(1-\sigma)}{(1-\sigma)+\frac{\tau\sigma(\phi\eta+1-\phi)}{(1-\phi)\alpha z(1-\alpha)^{\frac{1-\alpha}{\alpha}}+(\phi\eta^2-1+\phi)\frac{\sigma\tau}{\eta}}}.$$

Finally, the above equations can be combined into $Y^S=e^Hzk_H^{1-\alpha}\eta^\alpha+e^Lzk_L^{1-\alpha}$ to form an expression for aggregate output (5).

7 Tables and Figures

Table 1: *Notation*

Symbol	Description	Symbol	Description
B	Boundary for producing	α	Production function parameter
c	Cost to meet worker	β	Worker's share of output
F	Idiosyncratic shock distribution	δ	Time discounting parameter
G	Firm capacity distribution	ϵ	Match-specific idiosyncratic shock
h	Worker skill level	η	High-skill productivity
H	High-skill worker	κ	Support of capacity distribution
J	Value of active firm	σ	Exogenous separation rate
k	Capital or firm capacity	τ	Maximum idiosyncratic cost
L	Low-skill worker	ϕ	% High-skill in population
P	Pooling equilibrium	Φ	$\frac{1-\beta}{1-\delta(1-\sigma)}$, set-up price
p	% vacant firms w/ high-capacity	Λ	$(1-\alpha)^{\frac{1}{\alpha}}$
q	% of unemployed w/ high-skill	Π	$(\phi\eta^{\alpha}+1-\phi)^{\frac{1}{\alpha}}$
S	Separating equilibrium	Ψ	$1-\beta$, rental price
U	Value of unemployed worker	Ω	$\left[(1-\phi)\left(\phi^{\alpha}-\phi\right)^{-1}\right]^{\frac{1}{\alpha}}$
V	Value of vacant firm		
W	Value of employed worker		
x	Probability of producing		
Y	Aggregate output		
z	Aggregate state		

Table 2: *Parameter Values*

Symbol	Description	1980	1990
α	Production function parameter	.64	.64
η	High-skill productivity	5	5
σ	Exogenous separation rate	.1	.1
ϕ	% High-skill in population	19.2	24.0

Table 2 lists the parameter values used in the benchmark analysis. Only steady states of the model economy are considered. The 1980 column represents the economy in a pooling equilibrium, and the 1990 column captures the separating case.

Table 3: *Solutions for Pooling and Separating Equilibria*

Symbol	Description	Pooling	Separating
k^P	Capital - employ all	0.471	-
k^H	Capital - employ high-skill	-	1.013
k^L	Capital - employ low-skill	-	0.203
J^L	Value of matched firm	0.292	0.360
J^H	Value of matched firm	1.665	1.801
	Skill Premium	2.8	5.0

Table 3 lists the firms' capital choices and associated valuations in pooling and separating equilibria. See Table 2 for the parameter values used to obtain these results.

Table 4: *Results - Pooling and Separating Equilibrium*

	Pooling	Separating	Decline in Volatility
Change in Output	6.23%	5.80%	6.90%
(U.S. Data)	(2.65%)	(1.51%)	(43.02%)
Change in Employment	6.33%	4.40%	30.49%
(U.S. Data)	(1.36%)	(0.76%)	(44.12%)

Table 4 reports the percent decline in aggregate output and total employment after reducing the aggregate productivity variable z by 5% for both the pooling and separating equilibrium. The U.S. data row (in parentheses) provides the standard deviation of the deviations from trend over the relevant time period. The last column lists the percent decline in aggregate cyclical volatility that occurs after moving from the pooling equilibrium to the separating case. See the text for more details.

Table 5: *Results using Alternate Parameter Values*

η	ϕ (pooling)	Percent decline in Output Volatility
5.0	0.192	6.90%
5.5	0.192	7.06%
6.0	0.192	7.78%
5.0	0.150	9.38%
5.0	0.100	11.04%
6.0	0.100	11.92%

Table 5 presents the results for alternative parameter value choices. See the main text and Table 4 for more on how these results were calculated.

Figure 1: *Real GDP Growth*

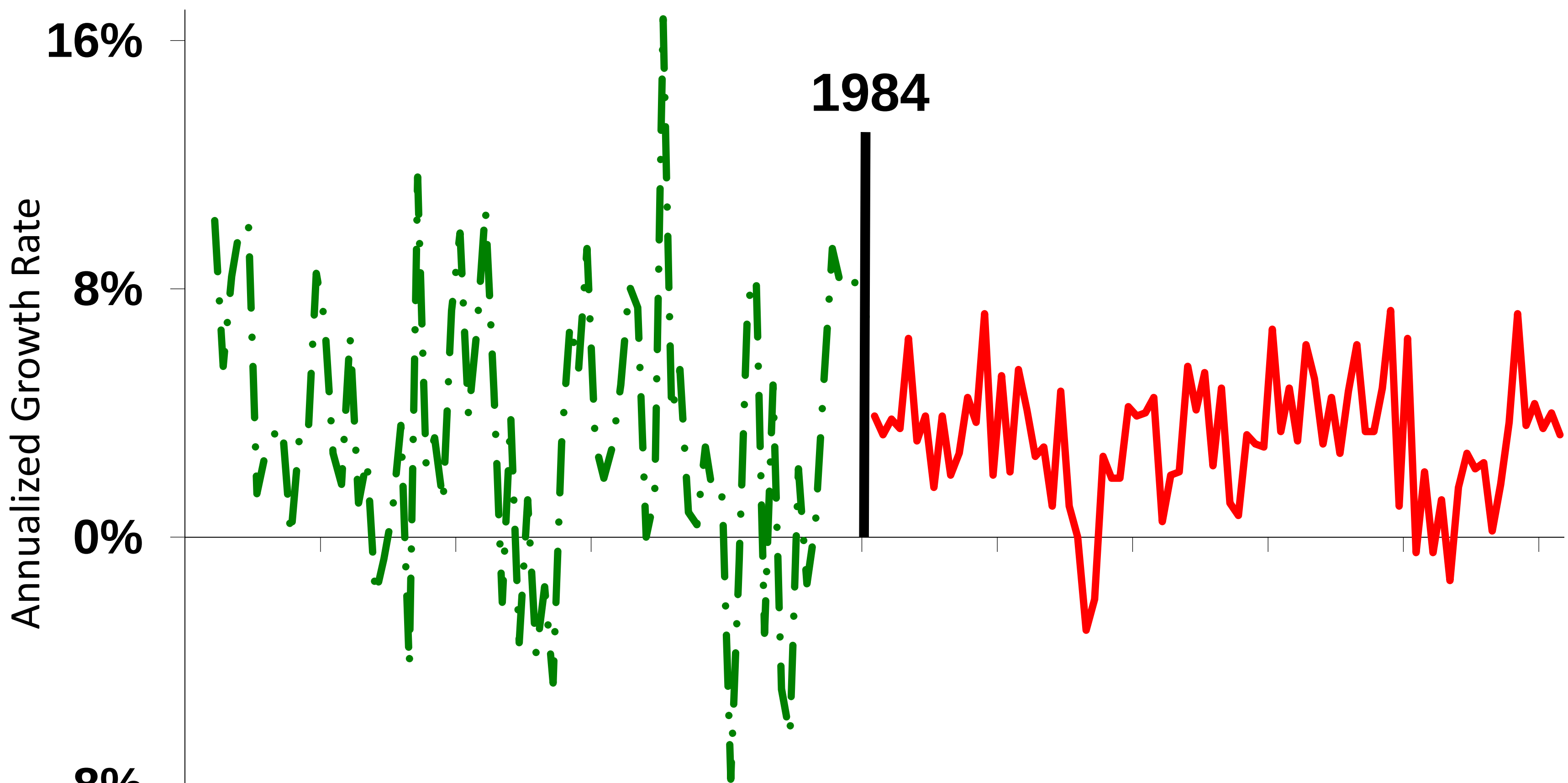

Figure 1 was created using data from the BEA.

Figure 2: *Relative Supply of College Graduates*

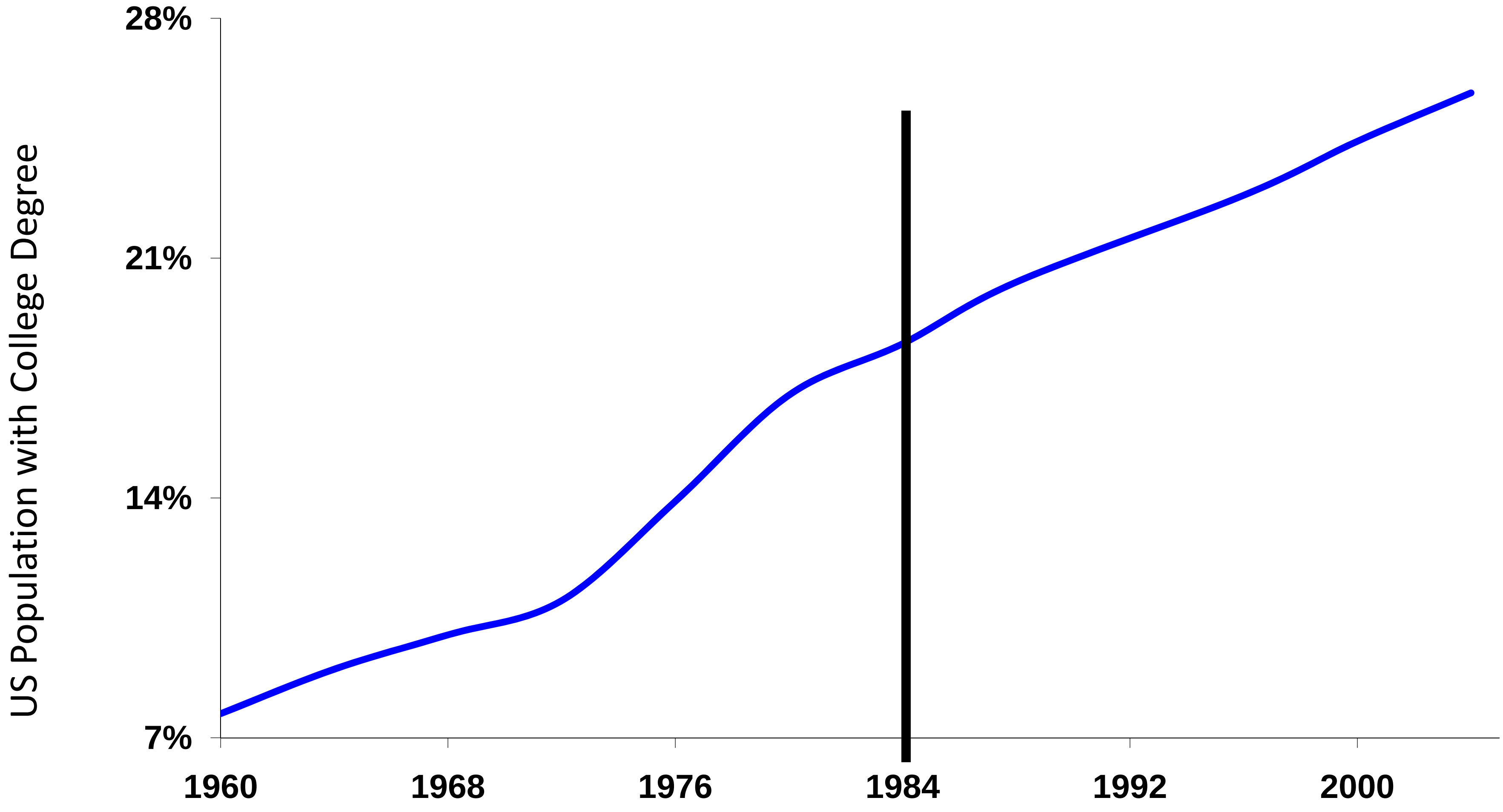

Figure 2 was created from CPS and BLS data.

Figure 3: *Firm's Sequence of Events*

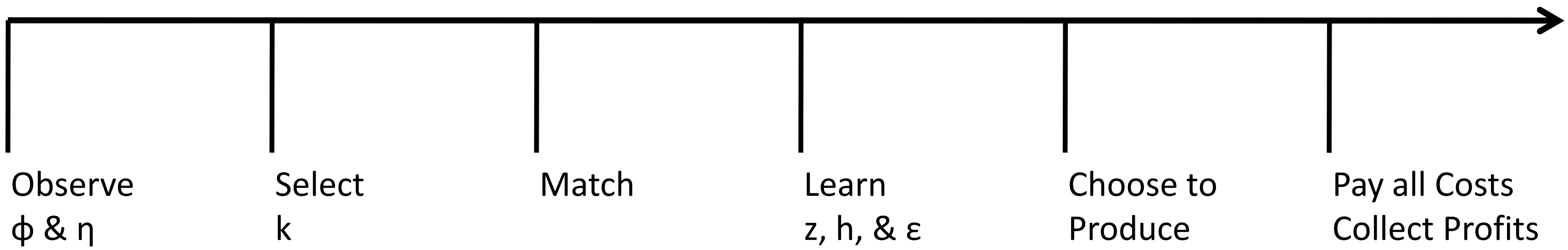

Figure 3 details the sequence of events within a period of the model economy.

Figure 4: *Firm Profits*

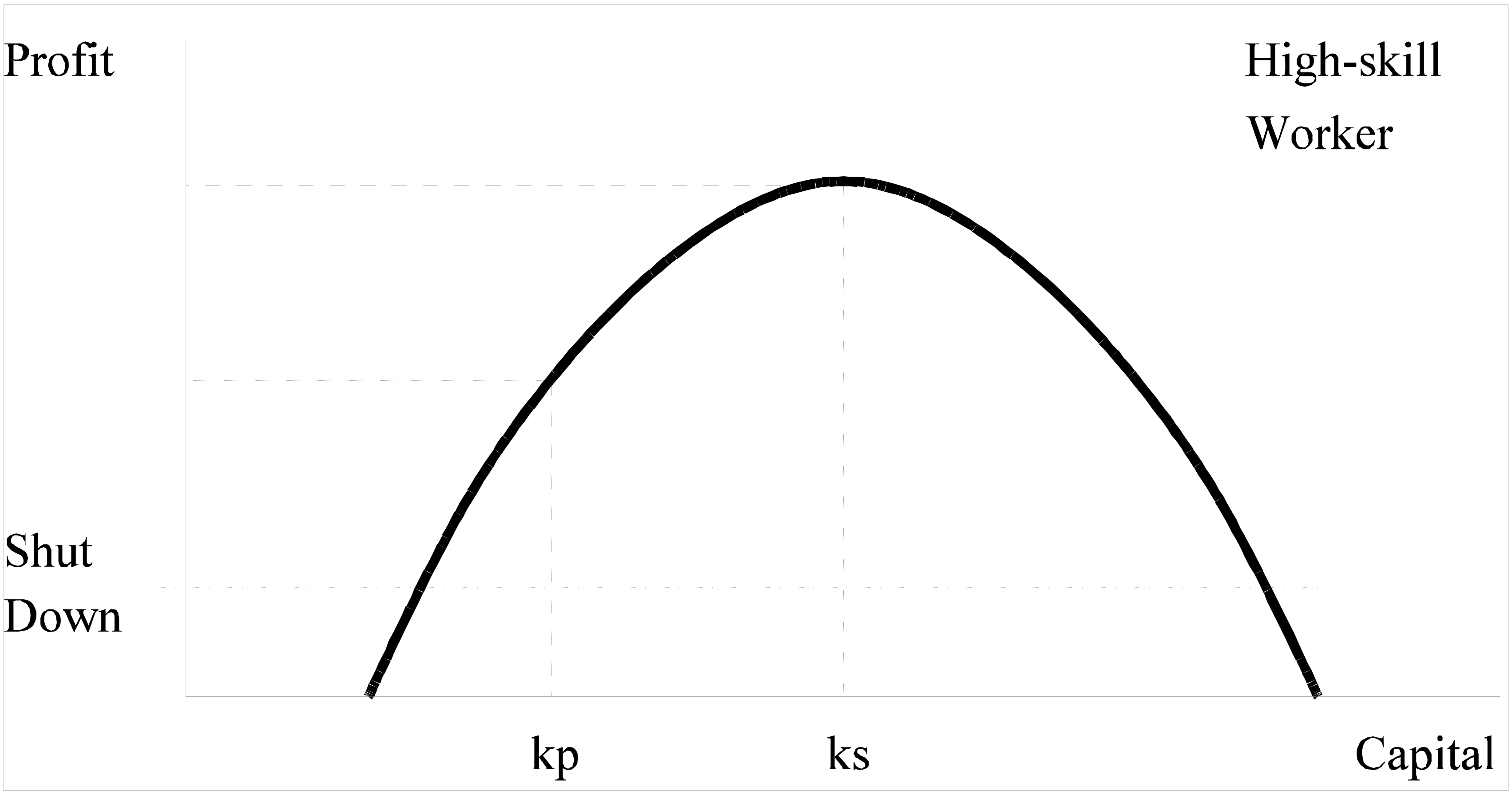

Figure 4 depicts the potential profits for a firm with different choices of capital when matched with a high-skill worker.