Estimated Proven Reserves in the Devonian Ohio/New Albany/Chattanooga Black Shale of Kentucky

Compiled by: Brandon C. Nuttall, Kentucky Geological Survey  
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Method: Volumetric  

Proven Reserves:  
9.81 trillion cubic feet (using industry standard 300 MMcf per well)  
12.07 ± 1 trillion cubic feet at the 95 percent level of confidence (average 350.7 MMcf per well)  

Discussion of method

Initial assumptions  
1. For the purpose of defining "proven" reserves, a 1-minute Carter coordinate section will be included in the analysis if there is at least one successful Devonian shale completion within the section.  
2. Proven reserves for the Illinois Basin will be proportionally small enough that the analysis can be combined with the Appalachian Basin data without significant overestimation.  
3. The dimensions of each 1-minute Carter coordinate section are roughly 6,060 feet by 4850 feet for a total of 674.7 acres. If the spacing between gas wells is 1000 feet, there is ideally a maximum of 29.4 drillable locations per 1-minute section. Assuming there are always some locations in a section that are undrillable for any number of reasons, 20 drillable locations per 1-minute section will be assumed.  
4. For a 1-minute Carter coordinate section, the total proven gas-in-place can be estimated to be equal to the product of the reserves per well and the number of wells in the section where the number of wells includes both existing wells and an estimate of the number of future successful wells.  
5. Parameters to be estimated are log normally distributed. That is, most of the values are relatively small, but a few large ones will occur. This is typical of petroleum related data.
**Discussion**

A Delphi method based on modeling the triangular distributions of parameters for the standard volumetric equation was used to investigate the recoverable gas-in-place per acre-foot by well. The standard equation is:

\[
GIP = 43560 \phi (1 - S_w) \left[ \frac{520}{460 + T} \right] \cdot \frac{P}{14.73} \cdot \frac{1}{0.89} \cdot RF \quad \text{(Eq. 1)}
\]

where

- \( \phi \) = porosity (decimal fraction)
- \( S_w \) = water saturation (decimal fraction)
- \( T \) = reservoir temperature (°F)
- \( P \) = reservoir pressure (psia)
- \( RF \) = recovery factor (decimal fraction)

note

- 0.89 is a "typical" gas compressibility factor

In this application of the Delphi method, the literature and the Kentucky Geological Survey database were consulted to provide the range of values for the parameters in the equation. Based on information on the Devonian shale play in the Atlas of Major Appalachian Gas Plays, parameters for the triangular distributions for each of the parameters are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Most likely value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>0.015</td>
<td>0.025</td>
<td>0.11</td>
</tr>
<tr>
<td>Water saturation(^1)</td>
<td>0</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>Temperature</td>
<td>60</td>
<td>75</td>
<td>112</td>
</tr>
<tr>
<td>Pressure</td>
<td>100</td>
<td>300</td>
<td>1000</td>
</tr>
<tr>
<td>Recovery Factor</td>
<td>0.4</td>
<td>0.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Thickness(^2)</td>
<td>1</td>
<td>400</td>
<td>1100</td>
</tr>
</tbody>
</table>

\(^1\) In Kentucky, water is seldom encountered in the Devonian shale. In fact, two fields had \( S_w \) data and one of them was 10%.

\(^2\) Used for calculating the reserves per well (see discussion).

The distribution for each parameter was simulated by generating 1,000 observations using random numbers as input to the equations describing the triangular distribution. Each of these sets of data were described by compiling frequency statistics and visually comparing a graph of the frequency classes to standard normal and log normal distributions.

**Thickness of the completion interval**

Thickness data proved to be a difficult distribution to simulate. Raw completion thickness data for were extracted from the main KGS well record data base (well_pay_information table). Zero and negative thickness values (6) were discarded; they were assumed to be a
typographic error in the data. The resulting data set contained 9,512 thickness data points. A frequency analysis of these data revealed the completion dataset to be bimodal and the log transform of the data to be skewed to the right. Figure 1 shows the log transform of the thickness data distribution ("Freq") compared to the log normal distribution ("NormFreq") and the triangular distribution ("TDFreq") used in the simulation.

This bimodal distribution is best understood in terms of the historic change of completion practices in the shale. Until the mid-1970's, it was standard practice to stimulate shale wells by using explosive fracturing in the open hole over the entire thickness of shale. After the introduction of hydraulic fracturing, it was realized that the maximum results could be obtained with the least expense by targeting particular organic-rich zones within the shale sequence, particularly the Cleveland and Lower Huron Members of the shale. Thus, there are many large completion intervals from the explosive fracturing days and many small completion intervals characteristic of more modern completions.

The shape of the triangular distribution was manipulated to best visually match the observed frequency distribution. The maximum value parameter for the triangular distribution was set to 1,100 (greater than or equal to 98 percent of the observed values). The most likely value was set to 400, slightly smaller than the observed median value of 430. Figure 2 shows histograms of the original and simulated frequency distributions of the thickness data.

![Log Transforms of the Thickness Data Distribution](image)

Figure 1.
Figure 2.

Calculating gas-in-place (per acre-foot)
A series of 1,000 gas-in-place per acre-foot values were calculated by substituting values for each of the parameters from the triangular distributions into the standard equation. The most frequently occurring value (the mode) of this distribution was examined for use in reserves calculations. Note that in the workbook, anytime the value of any cell was changed, the workbook was recalculated. With each workbook recalculation, new random numbers were generated and the statistics of each parameter changed slightly. Figure 3 shows the distribution of the gas-in-place estimates. Recoverable gas-in-place per well was calculated assuming a spacing of 22.95 acres per well (1,000 feet between wells), the mode of the gas-inplace distribution, and the 50\textsuperscript{th}-percentile value of the thickness distribution as a constant according to the formula.

\[ \text{Reserves} = \text{GIP} \times 22.95 \times \text{Thickness} \]  
(Eq. 2)

Multiple re-calculations of the spreadsheet revealed this reserves figure to vary approximately over the range 290 to 350 million cubic feet. This range is sufficiently representative of the rule of thumb value of 300 million cubic feet to confirm its adoption as a conservative value.
To calculate a statistical mean and confidence interval, the simulated thickness values were used to generate a distribution of gas-in-place per well values, again using equation 2. The log transform of this distribution is normally distributed enabling the application of standard statistics. Figure 4 shows the distribution of the (transformed) gas-in-place estimates per well. The average of the log normal distribution is $2.54 \pm 0.0241$ at the 95 percent level of confidence. These values correspond to an average of 350.7 MMcf and a range of from 331.8 and 370.8 MMcf gas-in-place per well.

Figure 3.
A database of well completions by play was compiled. The database included reported initial open flow or initial potential for each interval calculated to barrels of oil equivalents (boeq). All locations in the Devonian black shale play were extracted from this data set and imported into ArcView. The number of these existing completions in each 1-minute Carter coordinate section was counted. Only those sections with at least one successful completion were considered for inclusion in "proved" reserves. Multiple completions within the same well were ignored for the purpose of determining how many of the available, drillable locations within the section were, in fact, drilled. Each completion was assumed to occupy one drillable location.

A drilling success rate was determined for each 1-minute Carter coordinate section. The success rate was established as the number of successful completions divided by the total number of Devonian shale or deeper penetrations. When multiple completions are considered, however, the number of completions may exceed the total number of wells. In this instance, the success rate was assumed to be 100 percent.
The number of new locations available for future drilling was calculated:

\[ Future = (20 - \text{completions}) \times \text{success rate} \]

A negative result was interpreted to indicate the particular 1-minute Carter coordinate section was "drilled-up," with no future locations possible. Thus, the total wells in a 1-minute section is:

\[ Total = \text{completions} + Future \]

Finally, if the value of Total exceeded 20 wells, it was set to 20.

**Original gas-in-place**

For each 1-minute Carter coordinate section, an original gas-in-place was estimated using the formula:

\[ OGIP_{1\text{min}} = Total \times GIP \]

These values were summed over all the sections considered "proven" to determine a total original gas-in-place. Using the standard 300 MMcf per well, total gas in place was calculated to be 13.4 trillion cubic feet (Tcf) of gas. For the log transform of the gas-in-place per well distribution:

<table>
<thead>
<tr>
<th>At 95% confidence</th>
<th>Log10(MMcf)</th>
<th>MMcf/well</th>
<th>Total GIP (Tcf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low estimate</td>
<td>2.52</td>
<td>331.8</td>
<td>14.83</td>
</tr>
<tr>
<td>Mean</td>
<td>2.54</td>
<td>350.7</td>
<td>15.67</td>
</tr>
<tr>
<td>High estimate</td>
<td>2.57</td>
<td>370.8</td>
<td>16.58</td>
</tr>
</tbody>
</table>

**Proven reserves**

Recorded gas production in Kentucky is approximately 5.25 Tcf. If 95% of Kentucky’s production is from eastern Kentucky and 70% of that is from the Devonian shale, then 69% or 3.6 Tcf have been produced from the shale. Subtracting this produced gas from the estimated 13.41 Tcf in place yields 9.81 Tcf proven reserves. Comparably, from the statistical analysis, there are 12.07 Tcf ± approximately 1 Tcf at the 95 percent level of confidence. Note that the range is symmetric about the mean in the log transform of the data, but not when expressed in standard, antilog values. At current production rates (80 billion cubic feet per year total for the state), this represents approximately 150 years of sustained production.
Appendix A: Mathematics of the Triangular Distribution:


For a parameter, given its minimum \( V_{\text{min}} \), most likely \( V_{\text{most}} \), and maximum \( V_{\text{max}} \) values let:

\[
\begin{align*}
x &= V_{\text{most}} - V_{\text{min}} \\
y &= V_{\max} - V_{\text{most}} \\
x' &= \text{any valid value, } V_{\text{min}} \leq x' \leq V_{\max}
\end{align*}
\]

The cumulative frequency \( (CF) \) is calculated for a given \( x' \):

\[
\begin{align*}
\text{For } x' \leq x & \quad CF = \frac{(x')^2}{x(x+y)} \\
\text{For } x' > x & \quad CF = 1 - \frac{x'}{(x+y)} - \frac{x'}{(x+y)}\left[1 - \frac{x'}{(x+y)}\right]^2
\end{align*}
\]

Given a randomly generated cumulative frequency \( (CF) \), the values \( x' \) can be generated to simulate the distribution of the specified parameter.

Define:

- Data range, \( R \)  
  \[ R = x + y \]

- Constant for calculation, \( c \)  
  \[ c = 1 - \frac{x}{R} \]

- Critical value, \( V_c \)  
  \( (Used \ to \ determine \ which \ equation \ to \ apply.) \)  
  \[ V_c = \frac{x}{R} \left(\frac{V_{\text{most}} - V_{\text{min}}}{R}\right)^2 \]
To simulate the distribution, calculate an observed cumulative frequency, \( CF = \) a uniformly distributed random number such that \( 0 \leq CF < 1 \).

For \( CF \leq V_c \)

\[
x' = \sqrt{(CF \times R \times x)} + V_{min}
\]

For \( CF > V_c \)

\[
x' = [1 - \sqrt{(c - CF \times c)}] \times R + V_{min}
\]

Conventional descriptive statistics, histograms, and percentiles can then be determined from these values of \( x' \).
Appendix B. Calculation and method notes

Devonian Black Shale Proven Reserves Estimate

This file is saved in g:\kyshapes\play analysis 2001\aaanotes.txt

1) Implement spreadsheet to compile gas-in-place estimates per well using the triangular distribution.

   Spreadsheet is: m:\my documents\ky reserves estimate.xls

   Calculations are based on GIP = 43560 * p * (1 – Sw) * (520/(460+t)) * (pr/14.73) * 1/.89 * rf

   p  = porosity
   Sw = water saturation
   t  = reservoir temp
   pr = reservoir pressure
   rf = recovery factor

   With reference to values for above from the Gas Atlas, median recoverable GIP for shale wells is 300MMcf.

2) Compile list of plays and use ArcView to make well/play distribution maps. (Note that in this case a well with multiple pays may be assigned to multiple plays.)

   Made a database of pays and total boeq in h:\ogprototype to server.mdb

   boeq = Mcf gas * 0.172

   This was done in conjunction with a request by Robert Andrews in his lineament analysis for Tommy Cate.

   Queries to compile the data set are "Robert ..." and are intended to be executed in order
   Data set created by running Robert 1a) to Robert 1g) is in the table "Robert Pay Info (Statewide)"
   The table Robert Pay Info (numeric) is the same as statewide except the logical values were translated to -1 (true) and 0 (false) for export
   The table Robert Pay Info was exported to R. Andrews and has only those wells east of longitude 83.5 deg.
   Several play translation tables were established
   Play Identifiers is a translation table of play abbreviations and play names
   Play Names associates individual formation codes (and translations) with a play abbreviation
   Plays by Well shows the assignment of a play to each of the pays in a given well (these associations were used by the "Robert" queries

   The information in "Robert Pay Info (numeric)" (table) was imported into ArcView and converted into a shapefile: g:\kyshapes\play analysis\allplays.*

   ArcView project name is: g:\kyshapes\play analysis 2001\plays.apr

   From the allplays shape file, individual play distribution data sets were extracted and individual shape files corresponding to the play abbreviations were compiled.

   List of plays and associated maps are in: g:\kyshapes\play analysis 2001\play summary.pdf

3) Do descriptive statistics of the boeq data, see m:\my documents\boeq.xls

   Data compiled by play (mostly derived from Access queries) include:
   Completions
   Minimum, Average, Median, and Maximum boeq

   Charts and graphs were included in: g:\kyshapes\play analysis 2001\play summary.pdf

4) Devonian Black Shale resource estimates

   For proven reserves, the basic idea is to
   i.   identify areas with successful shale wells
   ii.  estimate the number of new successful wells that will be drilled in these areas
   iii. (existing wells + new wells) * recoverable reserves per well - production = reserves

   a. Use ArcView compile a subsurface distribution map of the black shale based on the carter coordinate grid file.

   Subtract areas with formations older than the shale using the geologic map shape file
Subtract the Jackson Purchase area
Clean up cells outside of KY bound and along Ohio R bound along IN & OH

Resulting shape file is:  g:\kyshapes\play analysis 2001\theme1.*
Theme is labeled "Black Shale Present" in plays.apr

b. Spatial analysis

To count points that occur in a polygon:
i. spatial join attributes of polygons to points
ii. summarize points on polygon names
iii. join polygon table and summary table

Used count technique to count number of shale completions in each CC 1-min sec. See NWells field in themel table.

From tops table, gathered information on drilling depth to top and thickness of the Ohio/Nalb/Chat shale.

Used count technique (selected average function) to compile average thickness of shale where data present. See AvgThick field in themel table.

From kyg27v2 shape file coverage, selected all wells that penetrated the shale and deeper formations.

Used count technique to compile the total number of wells penetrating the shale and deeper formations in each CC 1-min sec. See Ndeeper field in themel table.

NB: the number of completions (NWells) may exceed the total number of wells (Ndeeper) as some wells may have multiple shale completions. Create a new field called Total:

\[ \text{Total} = \text{maximum}(\text{NWells}, \text{Ndeeper}) \]

c. Calculate recoverable gas in place for each CC 1-min sec

Assume spacing is 1000' bt wells, then spacing is about 25 acres (22.956).
Assume a CC 1-min sec is 6060*4850 or 675 (674.72) acres. Thus there are 675/25 or 27 locations in each CC 1-min sec.

At least some of the locations will not be drillable in each section so use 20 locations maximum.

\[ \text{success rate} = \frac{\text{NWells}}{\text{Total}} \]

\[ \text{new wells} = (20 - \text{Total}) \times \text{success rate} \]

\[ \text{proven recoverable reserves} = (\text{NWells} + \text{new wells}) \times 300 \text{ MMcf} \]

Results stored in a field called Reserves in themel table

NB: It is possible to calculate a negative reserves figure when Total>20. An examination of CC 1-min secs with Total>20 shows the areas to be intensively drilled oil-prone areas like Big Sinking, south-central Kentucky, Arthur and Nick. That is, mostly outside of what is generally considered potential shale-gas areas. For those CC 1-min areas with successful shale-gas completions, a negative result will be set to 0 and considered "drilled up."

d. Created g:\kyshapes\play analysis 2001\reserves.mdb and imported themel.dbf table. The sum of the reserves figure is:

12,848,435.243 MMcf or 12.85 Tcf (trillion cubic feet)

e. From the production database (g:\petrodb\production 97.mdb) estimated total production is 5.25 Tcf

If 95% of Kentucky's production is from eastern Kentucky and 70% of that is from the Devonian shale, then 69% or 3.6 Tcf have been produced from the shale.

f. Thus,

12.85 Tcf (original recoverable reserves)
- 3.60 (historic production)
-----
9.25 Tcf proven reserves

Note: in rechecking math, using the formula

\[ \text{proven recoverable reserves} = (\text{NWells} + ((20 - \text{Total}) \times (\text{NWells} / \text{Total}))) \times 300 \]
then summing over the CC 1-min sec where Nwells>0, I can't get the 12.85 Tcf figure to check. As originally in ArcView, I get 12.85. But if I recalculate in ArcView and Access, I get 13.2 (AV) and 13.4 (Access). The difference between AV & Access is the number of significant digits retained in the summation; more digits are retained in Access. So, I'll use the Access figure of 13,409,469.8 MMcf (13.41 Tcf). I suspect that since I did the original calculations piecemeal in AV, the successive truncation of the number of digits in each results systematically reduced the calculated figure.

See g:kyshapes\play analysis 2001\reserves.mdb "calculate reserves per CC1-min and compare with Arcview" (Query)

REVISED:

13.41 Tcf
- 3.60
-----
9.81 Tcf proven reserves (@ 300 MMcf/well)

5) Calculations for proven reserves (log transform of distribution)

@ 331.8 MMcf/well = 14,830,873.6 MMcf = 14.8 Tcf (low end of confidence range)
@ 350.7 MMcf/well = 15,675,670.2 MMcf = 15.7 Tcf Mean
@ 370.8 MMcf/well = 16,575,104.7 MMcf = 16.6 Tcf (high end of confidence range)

14.83 15.67 16.58
- 3.60 - 3.60 - 3.60
----- ----- -----  
11.23 12.07 12.98

So, proven reserves for the Devonian black shale are 12.1 +/- 0.9 Tcf at the 95% level of confidence.