Alternative Connectives

Standard propositional logic is “expressively complete”. By this is meant that for all possible truth-conditions the truth-values of the complexes can be expressed by our logical connectives.

A *truth-function* is a function that takes a truth-value as an input and produces an output. These functions can take 1 or many arguments, which is also to say that truth-functions can be 1-place or $n$-place.
So, consider the truth-functions $f$, $g$, $h$.

\[ f(1) = 0 \]
\[ f(0) = 1 \]

This is the truth-function for negation.

\[ g(1, 1) = 1 \]
\[ g(1, 0) = 0 \]
\[ g(0, 1) = 0 \]
\[ g(0, 0) = 0 \]

This is the truth-function for conjunction.
What about this?

\[ h(1, 1) = 0 \]
\[ h(1, 0) = 1 \]
\[ h(0, 1) = 1 \]
\[ h(0, 0) = 1 \]

This represents the relation “not-both” and is usually given the connective “\(\mid\)” — the “Sheffer stroke”.
We will call some set of connectives “adequate” iff the members of the set can express all possible truth-functions, that is, iff all truth-functions can be symbolized by using sentences containing no connectives not in that set.

We have already seen in Chapter 2 that \{\sim, \rightarrow\} is adequate. The interesting feature of the Sheffer stroke is that it alone is expressively adequate.
Beyond Propositional Logic

Polish Notation

In Polish Notation, the connectives are placed \textit{before} the wffs. The virtue of this sentence is that its grammar is simpler, for it has no need for parentheses.
Sider’s examples: \((P \land Q) \rightarrow R\) and \(P \land (Q \rightarrow R)\) become \(\rightarrow \land PQR\) and \(\land P \rightarrow QR\).

If you \textit{actually} look at a text in this tradition, you’ll find something slightly different. “C” stands for “consequence”, i.e., implication (\(\rightarrow\)) and “N” for negation (\(\sim\)).
So, consider the following from Tarski and Łukasiewicz's *Investigations into the Sentential Calculus*. The claim there is that there are three axioms:

\[ 'CCpqCCqrCpr' \\
     'CCNppp' \\
     'CpCNpq' \]

And we are told that these should replace the six Fregean axioms:

\[ 'CpCpq' \\
     'CCpCqrCCpqCpr' \\
     'CCpCqrCqCpr' \\
     'CCpqCNqNp' \\
     'CNNpp' \\
     'CpNNp' \]
Now the fun stuff begins. As Sider says, one of the best reasons to be interested in these nonclassical logics is that they spring from the idea that classical logic itself could be wrong.

Intuitionists, for example, deny the law of excluded middle $\neg \phi V \sim \phi \Downarrow$. Paraconsistent logicians deny ex falso quodlibet $\neg \phi \rightarrow (\sim \phi \rightarrow \psi) \downarrow$ (or $\neg (\phi \land \sim \phi) \rightarrow \psi \downarrow$). Some logicians argue that there are true contradictions.
Beyond Propositional Logic

Three-valued Logic

Thus far, we have been concerned with “two-valued logic” — for we have assumed that there are only two truth values the True (1) and the False (0).

Why should we have a third truth value?

Sider gives several examples of statements that do not obviously have truth values.

1. There will be a sea battle tomorrow.
2. Ted stopped beating his dog.
3. Sherlock Holmes has a mole on his left leg.
The point is that statements that refer to events in the future might not be thought to have a truth value. If they did, would that not amount to determinism?

Statements involving *failed presumption* are also suspect for neither the truth nor the falsity of such a claim seems to be something one would want to assert.

And, finally, what about statements that refer to non-existent or fictional beings? In the case of Sherlock Holmes, we might be happy to say that “Sherlock Holmes’ brother’s name was Mycroft” is True and that “Sherlock Holmes lived at 223c Baker Street” is False — Conan Doyle created a world to justify those claims. But Conan Doyle *never* wrote about a mole on Holmes’ leg.