OPEN-ENDED QUESTIONS FOR MATHEMATICS

DEVELOPED BY DR. RON PELFREY, MATHEMATICS CONSULTANT AND
PROVIDED AS A SERVICE OF THE ARSI RESOURCE COLLABORATIVE UNIVERSITY OF KENTUCKY

REVISED APRIL 2000 TO ALIGN TO CORE CONTENT VERSION 3.0
TABLE OF CONTENTS

GRADE 4 OPEN-ENDED QUESTIONS.................................................................4
GRADE 4 SOLUTIONS .................................................................................11
GRADE 5 OPEN-ENDED QUESTIONS WITH SOLUTIONS .........................18
GRADE 8 OPEN-ENDED QUESTIONS..........................................................37
GRADE 8 SOLUTIONS .................................................................................56
ALGEBRA I / PROBABILITY / STATISTICS OPEN-RESPONSE QUESTIONS ...68
ALGEBRA I / PROBABILITY / STATISTICS SOLUTIONS ..........................72
GEOMETRY OPEN-RESPONSE QUESTIONS ............................................77
GEOMETRY SOLUTIONS ............................................................................81
This packet contains open-ended questions for grades 4, 5, and 8 as well as open-response questions for Algebra I / Probability / Statistics and Geometry. The questions were developed with two separate intentions.

Before stating these intentions, let’s examine the differences – as used in this packet – between “open-ended” and “open-response.” In this set of materials, open-ended refers to a question or problem which has more than one correct answer and more than one strategy to obtain this answer. Open-response refers to a question or problem that may only have one correct answer or one strategy to obtain the answer. In both open-ended and open-response mathematics problems, students are expected to explain or justify their answers and/or strategies.

Now for the intentions for the use of these questions. The questions identified for grades 4, 5, and 8 should be used as classroom practice questions. Students can either work with them as members of cooperative groups or the teacher can use the questions for demonstration purposes to illustrate proper use of problem solving strategies to solve problems – as practice either for CATS or for other problem solving situations that students may encounter. The problems are not intended to be ones that can be solved quickly or without thought. However, the challenge provided by these questions should elicit classroom discussion about strategies that may or may not be obvious to the average student. Each of the questions is correlated to the Core Content for Assessment for Grade 5 (the grade 4 and grade 5 questions) or for Grade 8 (the grade 8 questions). If a teacher receiving a copy of these questions does not have the Core Content for Assessment coding page, she/he may contact either the ARSI Teacher Partner in his/her district, the ARSI office (888-257-4836), or the ARSI website of the University of Kentucky resource collaborative at [http://www.uky.edu/OtherOrgs/ARSI/curriculum.html](http://www.uky.edu/OtherOrgs/ARSI/curriculum.html) then click on Assessments.

The high school questions were developed as part of professional development provided to mathematics teachers on how to adapt textbook or other problem sources into open-ended questions. As presently configured, many of these questions can be used in classrooms for assessment purposes. However, the teacher should consider modifying the problems to provide additional practice to their students on how to answer open-ended questions. Assistance in helping teachers in this modification can be found on the Kentucky Department of Education website at [http://www.kde.state.ky.us/](http://www.kde.state.ky.us/) or through professional development provided by ARSI or the Regional Service Center support staff in mathematics.

If you have any questions about the use of these materials, please contact the ARSI Resource Collaborative at the University of Kentucky (888-257-4836).


GRADE 5 OPEN-ENDED QUESTIONS WITH SOLUTIONS

1. My thousands digit is three times my hundreds digit. The sum of my tens digit and one digit is one less than my thousands digit. My hundreds digit is two less than my tens digit. What 4-digit number am I?

   Ans. Could be either 9353 or 6241

2. a) The digit in the tens thousands place is less than the digit in the hundred thousands place. The thousands place digit is larger than the digit in the millions place. The digit in the ones place is at least twice as large as the digit in the tens place. Which park could I be?

   Park | Number of Acres
   ----|------------------
   Danali | 4,716,726
   Gates of the Arctic | 7,523,888
   Katmai | 3,716,000
   Kobuk Valley | 1,750,421
   Lake Clark | 2,636,839
   Glacier Bay | 3,225,284

   b) Give another clue that would produce only one park as the solution.

   Ans. a) Danali or Lake Clark
   b) Varies

3. a) On a hiking trip, Bob and his two friends hiked 15 miles the first day, 10 miles the second, and 7 miles the third. If they walked a total of 60 miles over the 5-day trip and did not walk more than 18 miles on any one day, how many miles could they have walked each of the last two days?

   b) How many miles did they average per day?

   c) If they could hike at a top speed of 6 miles per hour on flat terrain, but had to slow down when climbing hills, could they have made the trip in a single 10-hour day of sunshine? Support your answer.

   Ans. a) 18 one day and 10 the other, or
   17 one day and 11 the other, or
   14 one day and 14 the other
c) No. Varies, but should include fact that 6 mph for 10 hours would give 60 miles, but would assume that the entire trip was over flat terrain - unlikely.

4. Methods of estimating sums include front-end, clustering, finding a range, and rounding. Estimate each of the following, indicate which method you used for each sum, and explain why you used that method.

   a) 456 + 453
   b) 906 + 214
   c) 43 + 47 + 41 + 42 + 44
   d) 127 + 149 + 182

5. a) Use each of the digits 2 through 8 once to write a 4-digit number plus a 3-digit number that gives the greatest possible sum and support your case that it will be the largest.

   b) Use the same digits to find the sum of a 4-digit number and a 3-digit number that gives the smallest sum.

   c) Generalize rules for producing the greatest and least sums for a 4-digit plus a 3-digit number. How would the rules differ, if any, when adding a 4-digit number to another 4-digit number?

   Ans. a) 8753  b) 2357  c) Varies
   +  642     +  468
   9395 2825

6. Jane used front-end digits to estimate a difference. The minuend was greater than 800 but using front-end its estimate was 800. The estimated difference was 300. (a) What is the least possible number the actual subtrahend could have been? (b) What is the greatest possible number for the subtrahend?

   Ans. a) 801  b) 899
   - (402) - (598)
   399 301

7. Suppose you estimate a difference between two 3-digit numbers using front-end digits. Give an example in which:

   a) the estimate is less than the actual difference
   b) the estimate is greater than the actual difference

   Ans. Varies, for example
   a) 425 - 215 (actual = 210, estimate = 200)
   b) 425 - 275 (actual = 150, estimate = 200)
8. a) Using each of the digits 0 through 7 once to fill in the spaces below so that the difference is 8 in the hundreds place, 8 in the tens place, and 9 in the ones place.

\[
\begin{array}{c}
\hline
\hline
\hline
\hline
\hline
8 & 8 & 9
\hline
\end{array}
\]

b) Is there more than one solution to this problem? Explain briefly.

c) Is there more than one solution in which the digit in the thousands place of the difference is 0?

Ans. a) 7531 or 1246
- 642
- 357

b) Yes, explanation varies.

c) No.

9. Marlene has a board game in which it is possible to land on spaces worth 1, 3, 5, or 7 points each.

a) Write an equation to show how it is possible to score a total of 10 points in exactly 4 moves.

b) Are all scores between 5 and 35 possible with exactly 5 moves? If not, which scores are possible?

Ans. a) 3(3) + 1 = n
b) No, only the odd numbers are possible.

10. The math symbols in these equations are missing. Make each equation true by adding two or more symbols (+, -, x, /, or =) to each. For example, Problem: 4 2 2 Solutions: 4 = 2 + 2 or 4 - 2 = 2

a) 8 8 4 4
b) 18 9 2 36
c) 2 14 20 4 1

Ans. a) 8 / 8 = 4 / 4
b) 18 + 9 x 2 = 36
c) 2 + 14 = 20 - 4 x 1
11. There were 42 band members. When the chorus joined them for a performance, there were 77 performers. Which equation below would be best to use to solve this problem? Explain how you made this decision.

\[ a) \quad 77 = 42 - n \quad \text{c) } \quad 77 + 42 = n \]
\[ b) \quad 77 = 42 + n \quad \text{d) } \quad 77 + n = 42 \]

Ans. (a). Explanation varies.

12. a) Is the least common multiple of an even and an odd number, even, or odd? Give an example to support your answer.

b) If one number is a divisor of a second number, what is the least common multiple? Give an example.

c) If two numbers do not have any common divisor, what is the least common multiple? Support your answer with an example.


"Welcome to the World of the Orient," spoke the genie to Aladdin. "You see before you diamonds, rubies, and emeralds. Two diamonds are worth as much as three rubies. Five rubies are worth as much as nine emeralds. Make a pile of diamonds, and then make a pile of emeralds. If you can do this so that the two piles have exactly the same worth, you may keep them all!"

How many diamonds should be in the diamond pile, and how many emeralds should be in the emerald pile? Explain your answer.

Ans. Any solution which is a multiple of 10 diamonds and 27 emeralds.

14. A parking lot permits either cars or motorcycles. All together the vehicles parked in a particular day have 60 wheels. Use your reasoning and problem solving skills to find how many cars and how many motorcycles there could be in the parking lot this day.

<table>
<thead>
<tr>
<th>Cars</th>
<th>Wheels</th>
<th>Motorcycles</th>
<th>Wheels</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>28</td>
<td>56</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>26</td>
<td>52</td>
<td>60</td>
</tr>
</tbody>
</table>

Ans. | Cars | Wheels | Motorcycles | Wheels | Total |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>
15. The Carnival Committee is having their annual meeting to plan for this year's carnival. During this meeting, 8 people will meet to decide upon the location, while another 8 people will meet to discuss which booths will be present this year. You have been selected to set up for this meeting. You have been given 7 square tables and 16 chairs. The criteria for seating arrangements include: the two groups of 8 people each want to be separated and each person will require the use of one side of the table. How will you arrange the tables and chairs for this meeting?

Ans. Possible answers include:

```
X X       X X X
X□□X      X□□□X
X□□X      X X X
 X X
```

16. You have to use mental arithmetic to multiply 15 x 18. Use (a) the distributive property; (b) a sketch; and, (c) grid paper to explain how the problem was worked.

Ans. a) 15x18 = 15x (20-2) = (15x20)-(15x2) = 300 - 30 = 270
     b) & c)
17. Cinemark Theater has three categories for ticket prices. Adult tickets sell for $6.50 after 3pm or $3.25 for a matinee (before 3pm) and Under 12 tickets sell for $3 at any time. The theater has a maximum capacity of 408 seats. What is the greatest amount that could be collected at:

a) an 8pm showing?
b) a 1pm showing?
c) Explain how much you think would be collected at a matinee showing of a children's movie.

Ans. a) $2,652  
b) $1,326  
c) Varies

18. When you multiply large numbers, is there any way to estimate how many digits an answer will have? Look at the following examples and see if you can find a pattern that will help you to answer this question. Describe the pattern and then predict how many digits will be in (f).

a) 10 x 100 = 1,000  
b) 100 x 100 = 10,000  
c) 100 x 1,000 = 100,000  
d) 104 x 4,211 = 437,944  
e) 989 x 8,657 = 8,561,773  
f) 405 x 4,508 = ?

Ans. Yes, you can predict the number of digits by finding one less than the total number of places in both numbers being multiplied as well as a front-end estimate of the product of both numbers. Thus (f) would have 7 digits since (3 digits + 4 digits) - 1 = 6 and 4 x 4 = 16 or one additional decimal place.

19. You can use color tiles to build rectangles. If you have 16 tiles, what is the perimeter of the rectangle that can be built? What other perimeters are possible if you can build shapes other than rectangles, but the shapes must all be made with tiles that share sides (see example)?

Example:

Yes  No  No
20. a) If you double the length of a rectangle and leave the width the same, how does the area change? the perimeter?

b) If you double both the length and width of a rectangle, how does the area change? the perimeter?

Ans. a) Area doubles and perimeter increases by an amount that is twice the original length.

b) Area quadruples and the perimeter increases by an amount that is twice the original length and twice the original width.

21. Explain how to find the area of the following shape in at least two ways.

![Area diagram](image)

Area = 34 square units

Ans. a) By counting squares and half-squares

b) By subdividing into rectangles and triangles and finding area of each by counting length/width/height and using formulas.

22. Explain in at least two ways whether there is a remainder or not when:

a) you divide an even number by an odd number

b) you divide an even number by an even number

c) you divide an odd number by an even number

d) you divide an odd number by an odd number
Ans. Using grids, manipulatives, numerical examples, etc.
   a) remainder
   b) no remainder
   c) remainder
   d) no remainder if the divisor is a factor of the
      dividend, otherwise there is a remainder

(E-4.2.1) Problem Solving
Guess & Check

23. Place the numbers 1, 2, 3, 4, 5, and 6 in the spaces so that each side will give the same sum.

\[ \begin{array}{ccc}
   & 1 & \\
   4 & 6 & \text{or} \\
   5 & 2 & 3
\end{array} \]

Ans. Using grids, manipulatives, numerical examples, etc.
   a) remainder
   b) no remainder
   c) remainder
   d) no remainder if the divisor is a factor of the
      dividend, otherwise there is a remainder

(E-4.1.2, E-4.2.2) Missing Factors

24. Problems such as 3 \( n \) = 84 can be solved by several methods, including guess-and-check, fact families, and a table of multiples. Solve this problem and explain how you solved it using at least two of these methods.

Ans. \( n = 28 \) Fact Family: \( 84 \div 3 = 28 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiples of 3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>75</td>
<td>78</td>
<td>81</td>
<td>84</td>
</tr>
</tbody>
</table>

(E-4.2.2) Problem Solving
Logical Reasoning

25. There are six new babies in the nursery lying in cribs one through five. Ingrid is working today and has no idea what any of the baby's names are. When the new parents come to pick up their babies she needs to make sure to give them the right ones. She knows that the baby's names are Kelsie, Kevin, Klyde, James, and Janie. She has found a few clues to help her name each baby.

Kelsie and Janie are the only girl's names.
Baby 1 has a sign in its crib that has a "J" on it.
Babies 2 and 5 are twins and their names start with the same letter.
Baby 1 is a girl.
The twins' names have the same number of letters.
Baby 3 is a boy.
Kelsie and Klyde are lying next to each other.

Ans. Baby 1 = Janie, Baby 2 = Kevin, Baby 3 = James, Baby 4 = Kelsie, and Baby 5 = Klyde.
(E-4.2.2) Problem Solving  Logical Reasoning  
26. Four fifth grade teachers must be assigned to rooms for this school year. The following requirements must be met for the room assignments:

- Mrs. A has to be in room #2 or #3
- Mrs. B can't be in room #2 or #4
- Mrs. C has to be next to Mrs. B
- Mrs. D can't be next to Mrs. C

Ans. Possible solutions include Room 1-B; Room 2-C; Room 3-A; Room 4-D, or Room 1-D; Room 2-A; Room 3-B; Room 4-C, etc.

(E-4.2.1) Problem Solving Find a Pattern  
27. There are less than 15 houses on one side of a street that are numbered 2, 4, 6, etc. Mrs. Murphy lives in one of these houses. The numbers of all the houses numbered below hers have the same sum as all those numbered above hers.

a) How many houses are there on her side of the street?
b) What is her house number?

Ans. a) 8, since 2+4+6+8+10 = 30 and 14+16 = 30
     b) Mrs. Murphy's house number is 12

(E-4.2.1) Problem Solving Find a Pattern  
28. Consider the following array of dots.

\[ \begin{array}{ccccccc}
  . & . & . & . & . \\
  . & . & . & . & . \\
  . & . & . & . & . \\
  . & . & . & . & . \\
  . & . & . & . & . \\
\end{array} \]

a) How many squares can you form by connecting these dots?
b) How many contain the center dot of the array in their interior?
c) Write an explanation of how you found your answers and describe any patterns you found.

Ans. a) length of side 1x1 = 16
     2x2 = 9
     3x3 = 4
     4x4 = 1
     Total = 30
     b) 0 + 1 + 4 + 1 = 6
     c) Varies
(E-4.1.3) Graphing Points

29. Imagine you are talking to a student in your class on the telephone who has been absent from school for an illness. Part of the homework that you are trying to explain to him requires him to draw some figures. The other student cannot see the figures because he hasn't received the worksheet. Write a set of directions that you would use to help the other student to draw the figures exactly as shown below.

a)  

\[ \begin{array}{c}
\text{a figure with coordinates.}\n\end{array} \]

b)  

\[ \begin{array}{c}
\text{a figure with coordinates.}\n\end{array} \]

Ans. Varies, but may include coordinates, geometric terms, etc.

(M-1.2.1) Multiplying Fractions

30. Kids Bake 'em Cookies (yields 24 cookies)

- 1 and 1/4 cup flour
- 1/8 tsp. baking soda
- 1/8 tsp. salt
- 1/2 cup salted butter, softened
- 1/2 cup honey
- 1 cup chocolate chips
- 1/4 cup white sugar

Mix flour, soda, and salt in a bowl. Mix butter, sugar, and honey in a separate bowl. Add the flour mixture and chocolate chips to this bowl. Bake at 300° for 18-20 minutes.

It is Saturday afternoon and you have decided to bake some cookies. The recipe that you have makes 24 cookies, but you only want to make 12. Figure out how much of each ingredient is needed for preparing the recipe for only half of the servings. Explain how you arrived at your answers and then rewrite the recipe for that number of servings so that you and your family members can use the adjusted recipe in the future.

Ans. 3/4 cup flour
- 1/16 tsp. baking soda
- 1/16 tsp. salt
- 1/4 cup salted butter, softened
- 1/4 cup honey
- 1/2 cup chocolate chips
- 1/8 cup white sugar
31. Sharon, planning a party, bought some compact discs, pizzas, and helium-filled balloons. The CDs cost $15 each, the pizzas cost $10 each, and the balloons cost $5 each. If she spent a total of $100, how many CDs, pizzas, and balloons did she buy?

Ans. The assumption is she bought at least one of each item. The answers vary, for example:

<table>
<thead>
<tr>
<th>CD</th>
<th>Pizza</th>
<th>Balloon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

32. A demonstration model of a green pattern block (triangle) has side length of 6cm. It is rolled to the right a number of times. If the triangle stops so that the letter "T" is again in the upright position, what possible distances could it have rolled?

Ans. Multiple of 18 distance, i.e., 18, 36, 48, …cm.


Ans. Varies, but will always be the first number selected followed by the second number selected. For example, if the first number chosen is 15 and the other number is 25, then the final result is 1525.

34. Find a word worth $0.60 when A=$0.01, B=$0.02, C=$0.03, etc.

Ans. Varies, e.g., SMUG = 0.19 + 0.13 + 0.21 + 0.07 = $0.60
35. Look at the pattern on the right.

a) How many \( \square \)s will be needed for the 16\(^{th} \) design? 1
b) Describe the number pattern.
c) Can a design be made using exactly 79 \( \square \)s? 2
   Why or why not?
d) How many \( \square \)s are needed for the 67\(^{th} \) design? 3
e) Which design can be made with 84 \( \square \)s? 3
f) The outstanding dimensions of the rectangles created in designs 1-5 are listed below. Fill in the missing dimensions.

<table>
<thead>
<tr>
<th>Design</th>
<th>Base x Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 x 1</td>
</tr>
<tr>
<td>2</td>
<td>2 x 2</td>
</tr>
<tr>
<td>3</td>
<td>2 x 3</td>
</tr>
<tr>
<td>4</td>
<td>2 x 4</td>
</tr>
<tr>
<td>5</td>
<td>2 x 5</td>
</tr>
</tbody>
</table>

Ans. a) 32. If \( n = 16 \), then \( 2n = 32 \) \( \square \)s
b) Add 2 to each successive number, resulting in the even numbers or the multiples of 2.
c) No. Only even numbers of blocks are used and 79 is odd.
d) 134. If \( n = 67 \), then \( 2n = 134 \) \( \square \)s.
e) 42\(^{nd} \). If \( 2n = 84 \) \( \square \)s, then \( n = 42 \).
f) \( 2 \times 8; 2 \times 10; 2 \times 20; 2 \times n \).

36. Pattern blocks have been used to construct the pattern below.

a) How many hexagons will be needed for the 34\(^{th} \) design? How many triangles?
b) Describe the number pattern formed along the hexagons, the triangles, and the total.
c) Which design can be built using a total of 97 blocks?
d) Study the block designs and explain one way the arrangements show the generalization for the total number of blocks.
e) How many hexagons and how many triangles will be needed for the 87\textsuperscript{th} design?

f) By how much do the numbers grow when looking at the totals? How does that constant difference relate to a generalization for design \( n \)?

Ans. a) 34, 35. For the 34\textsuperscript{th} design, \( n = 34 \) & \( n + 1 = 35 \).

b) Hexagons: The numbers are consecutive (counting) numbers, starting with 1. Triangles: The numbers are consecutive numbers, starting with 2. Total: The numbers increase by 2, starting with 3 (the odd numbers).

c) 48\textsuperscript{th}. If \( 2n + 1 = 97 \), then \( n = 48 \).

d) There are always \( n \) hexagons in each row and \( n + 1 \) triangles in each row for a total of \( n + (n + 1) = 2n + 1 \) pattern blocks.

e) 87, 88. For design 87, \( n = 87 \) and \( n + 1 = 88 \).

f) The numbers grow by 2; 2 is the number in front of \( n \) in the generalization \( 2n + 1 \).

(M-1.2.1)
Division of Whole Numbers (Dividing Money)

37. Look at the dessert portion of the menu below. Three friends, Chris, Ward, and Phillip have decided that they will each buy a dessert and share the cost evenly. What is the least that each will pay if each orders a different dessert? The most?

<table>
<thead>
<tr>
<th>Peaches 'n' More</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desserts</td>
</tr>
<tr>
<td>Apple Pie</td>
</tr>
<tr>
<td>Peach Cobbler</td>
</tr>
<tr>
<td>Ice Cream</td>
</tr>
<tr>
<td>Frozen Yogurt</td>
</tr>
<tr>
<td>Pecan Pie</td>
</tr>
</tbody>
</table>

Ans. \( \frac{1.89 + 0.96 + 0.84}{3} = $1.23 \)
\( \frac{2.10 + 2.04 + 1.89}{3} = $2.01 \)

(E-1.2.2)
Divide Whole Numbers

38. List the first three whole numbers that will have a remainder of 3 when you divide by 7. Describe the pattern and explain how you found these numbers and how you would solve any similar problem.

Ans. 10, 17, 24. Explanation varies, but should mention that the pattern in the solution differs by the divisor and that it is found by adding the remainder to each of the multiples.
39. Woe is me! My calculator does not have a 3 key that works! How can I use this broken calculator to do this problem? Explain your reasoning carefully and clearly.

\[
\begin{align*}
23 \\
x \\ 45
\end{align*}
\]

Ans. Varies, for example:
- Using distributive property -- \((45 \times 25) - (45 \times 2)\) or,
- \((45 \times 22) + 45\)

40. Adam, Betty, Charles, and Darlene were using their calculators to work on their math homework. The first problem they had to do was the following:

\[
3 + 4 \times 6 / 2 - 5
\]

Adam used his Casio and got an answer of 16. Betty used a Math Explorer and got an answer of 10. Charles then used his Hewlett Packard and also got an answer of 10. Finally, Darlene used her Sharp calculator and the display showed an answer of 16.

They knew they pushed the right keys and they thought that each calculator worked the problem correctly, but the displays were different. How did each calculator solve the problem? How could you enter the problem on the Casio so that the answer would be 10?

Ans. The Casio and Sharp calculators worked the problem from left to right (arithmetic logic). The Math Explorer and Hewlett Packard calculators used the order of operations (algebraic logic) and did the multiplication and division first. Key entries on the Casio could vary, but would stress that the multiplication and division would have to be done first.
41. You have a three-digit dividend and a one-digit divisor. Can you predict how many digits will be in the quotient? Give examples to support your answer.

Ans. The quotient can have either 2 or 3 digits. It will always have 3 digits if the digit in the hundreds' place in the dividend is divisible by the divisor, e.g., 569/3 = 189 with a remainder where 5 is divisible by 3. If the digit in the hundreds' place in the dividend is not divisible by the divisor, then the quotient will have 2 digits, for example, 589/6 = 98 with a remainder where 5 is not divisible by 6.

42. A jewelry store sold 108 pieces of jewelry in 13 days. How many pieces of jewelry did the store sell each week?

Ans. If 13 days is over a 2-week period, then the store sold 54 pieces of jewelry per week. If 13 days is over a 3-week period, then the store sold 36 pieces per week.

43. A famous mathematician, Christian Goldbach, stated that every even number, except 2, is equal to the sum of two prime numbers. Goldbach's Conjecture has never been proven. Test his conjecture for the even numbers between 50 and 59.

Ans. Varies, for example: 52 = 5,47; 54 = 7,47; 56 = 3,53

44. Examine these number patterns:

\[ 15 = 7 + 8 \quad 15 = 4 + 5 + 6 \quad 15 = 1 + 2 + 3 + 4 + 5 \]
\[ 9 = 4 + 5 \quad 9 = 2 + 3 + 4 \]

a) What kind of numbers can be written as a sum of 2, or 3, or 4 consecutive numbers? Give examples to verify your response.

b) What kind of numbers cannot be written as a sum of either 2, 3, or 4 consecutive numbers? Give examples of such numbers.

c) Describe the patterns you noticed.
Ans.  

a) All odd numbers can be written as the sum of 2 consecutive numbers. Numbers which are multiples of 3 can be written as the sum of 3 consecutive numbers. Numbers that can be written as the sum of 4 consecutive numbers are those in which twice the product of the sum of the middle two numbers is equal to the sum of all four numbers. Example (4): 

\[ 26 = 5 + 6 + 7 + 8, \text{ or } 2(6 + 7). \]

b) Even numbers cannot be written as the sum of two consecutive numbers. Numbers which are not multiples of 3 cannot be written as the sum of 3 consecutive numbers. No odd numbers can be written as the sum of 4 consecutive numbers, not can an even number be written as the sum of 4 consecutive numbers if it is divisible by 4. Example (4): 38 is an even number, but not divisible by 4. Dividing it by 2 gives 19 and the two consecutive numbers that add to 19 are 9 and 10. Thus the two middle numbers are 9 and 10 and the remaining numbers are 8 and 11, i.e., 

\[ 8 + 9 + 10 + 11 = 38. \]

40 is divisible by 4 and thus is cannot be written as the sum of 4 consecutive numbers.

c) Various patterns can be observed such as those discussed above.

(E-4.3.2)  
Sequences  

45. a) Cover the figure below using 10 pattern blocks of 4 different colors.

b) Describe the strategy you used to solve the problem.

(\text{Note for teacher: to make this pattern, trace around 3 yellow hexagon pattern blocks})

Ans. Varies, but will likely include beginning with some blocks that cover the shape and then substituting pieces one by one until the right number of total pieces is found.
46. Vanessa is reading a book for a book report. The book is 132 pages long. Vanessa has already read 87 pages. She normally reads 12 pages a night. At this rate, how many more nights will it take her to finish the book? How many pages will Vanessa read the last night? Show how you solved this problem in at least two ways.

Ans. 4 nights, including reading 9 pages the last night.
Strategies could include showing arithmetic steps, making a table/chart, making a pictograph/bar graph, etc.

47. In last night's homework, Mrs. Jones asked that the students use manipulatives to find \( \frac{2}{3} + \frac{4}{5} \). Johnny used fraction pieces and got a sum of \( 1\frac{5}{12} \). Kay used counters and got a sum of \( \frac{9}{7} \). Could both of them have been right? Draw pictures to illustrate how each may have got their answer. What could the teacher have said to make it more clear how to do the problem?

Ans. Fraction pieces:

```
+-----------------------------+
|   2/3                      |
+-----------------------------+
|     +                      |
|     +                      |
|     +                      |
|     +                      |
|     +                      |
```

Counters:

```
+-----------------------------+
|   2/3                      |
+-----------------------------+
| +                           |
| +                           |
| +                           |
| +                           |
|     +                      |
```

The teacher should have been more specific in the instructions, e.g., state that the fractions were part of a whole (fraction pieces) or parts of a set (counters).

48. Often there can be more than one way to estimate a quotient. Give several methods that you can use for the estimate for the quotient for \( 486 \div 72 \). Compare each of your methods and choose the one you think you would use.

Ans. Varies, but should include not only rounding, but also at least one other method, such as front-end, compatible numbers, or truncating.
49. Michael began working on his division homework and his work on the first problem looked like this:

\[
\begin{array}{c}
\text{26)182} \\
\hline
\text{208} \\
\end{array}
\]

Explain where Michael got the 208. What does 208 tell you about the quotient, 8? What number should Michael try next? Explain why you selected this number.

Ans. Varies, but should state something on the order of: "Michael estimated that 182 could be divided by 26 8 times, and when he tried it 8 \times 26 was 208. This number is too large so he should try a smaller number. I would try 7 because it is the next closest number (or I know that 6 \times 7 = 42 and the dividend 182 also ends in a 2)."

50. a) Beth picked up a solid figure and described one of its characteristics by saying that it had a rectangular face. Identify the shape she may have picked up.

b) Gwen then picked up a solid and said it had a flat face. Identify the shape she may have picked up.

Ans. a) A rectangular prism, a triangular prism, or a cube.

b) A rectangular prism, a triangular prism, a cube, a square pyramid, a rectangular pyramid, a triangular pyramid, a cone, or a cylinder.

51. a) How many cubes of different sizes could you make if you have 16 multilink cubes?

b) How many rectangular prisms could you make with 8 multilink cubes? Describe any that might be the same if you changed their orientation, that is any that would be the same if you moved them around.

Ans. a) 4 cubes: 1x1, 2x2, 3x3, and 4x4

b) 12 different rectangular prisms: 1x1x1, 1x1x2, 1x1x3, 1x1x4, 1x1x5, 1x1x6, 1x1x7, 1x1x8, 2x2x1, 2x3x1, 2x4x1, 2x2x2. Any others are different orientations of these 12.
52. Tabitha's mother asked her to buy a quart of orange juice for a recipe her mother was preparing. She knew that the cost of a pint of orange juice was $0.79. How much would the price for a quart be to cost less? Explain how you figured this out.

Ans. Since there are 2 pints to a quart, the quart of O.J. would have to be less than twice $0.79, or less than $1.58.