Forecasting Techniques

- Managers require good forecasts of future events.
- Business Analysts may choose from a wide range of forecasting techniques to support decision making.
- Three major categories of forecasting approaches:
  1. Qualitative and judgmental techniques
  2. Statistical time-series models (Quantitative)
  3. Explanatory/causal models (Quantitative)

Qualitative & Judgmental Forecasting

- Qualitative and Judgmental techniques rely on experience and intuition - usually short-term, expert-based: Market research (focus groups, interviews, surveys, product testings) and Delphi method.
- They are necessary when historical data is not available or when predictions are needed far into the future.
- The **historical analogy** approach obtains a forecast through comparative analysis with prior situations.
- The **Delphi method** questions an anonymous panel of experts 2-3 times in order to reach a consensus.

Example 9.1: Predicting the Price of Oil

- Early 1988 - oil price was about $22 a barrel
- Mid-1988 - oil price dropped to $11 a barrel because of oversupply, high production in non-OPEC regions, and lower than normal demand
- In the past, OPEC would raise the price of oil.
- Historical analogy would forecast a higher price.
- However, the price continued to drop even though OPPEC agreed to cut production.
- Historical analogies cannot always account for current realities!

Indicators and Indexes

- **Indicators** are measures that are believed to influence the behavior of a variable we wish to forecast.
- Indicators are often combined quantitatively into an **index**, a single measure that weights multiple indicators, thus providing a measure of overall expectation.
  - Example: Dow Jones Industrial Average (DJIA)

Example 9.2: Economic Indicators

- GDP (Gross Domestic Product) measures the value of all goods and services produced.
  - GDP rises and falls in a cyclic fashion.
- Forecasting GDP is often done using **leading indicators** (series that change before the GDP changes) and **lagging indicators** (series that follow changes in the GDP) indicators.
- Examples

Leading
- formation of business enterprises
- percent change in money supply (M1)

Lagging
- business investment expenditures
- prime rate
- inventories on hand

Example 9.3: Leading Economic Indicators

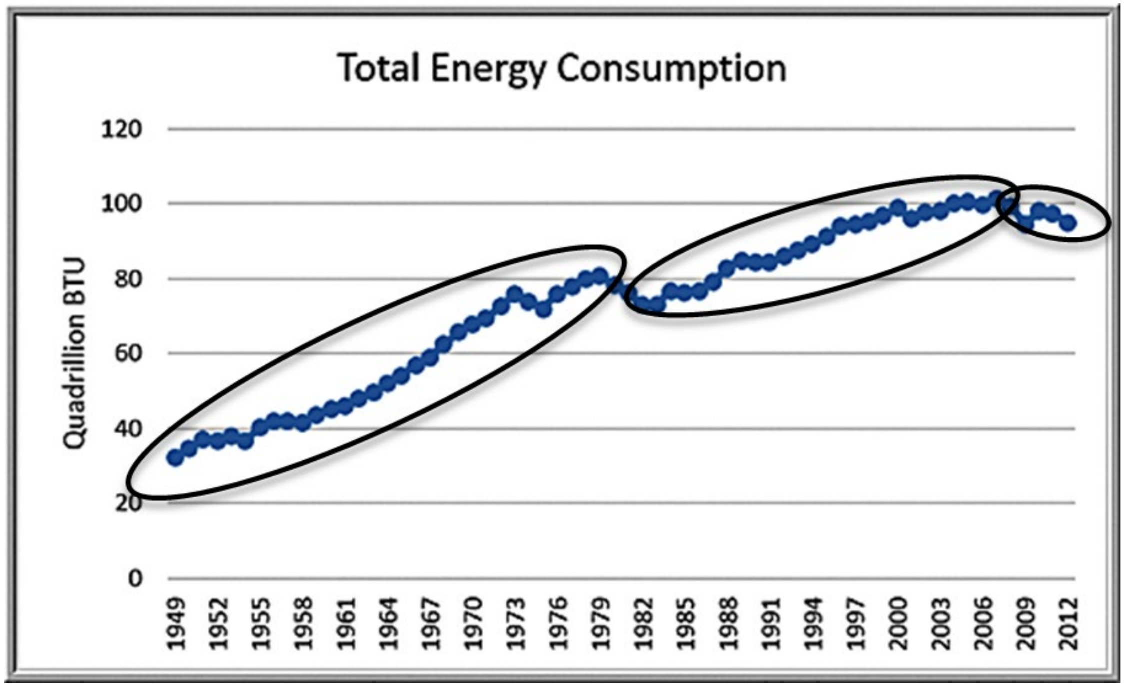
- An Index of Leading Indicators was developed by the Department of Commerce.
- This index is related to the economic performance is available from www.conference-board.org.
- It includes measures such as:
  - average weekly manufacturing hours
  - new orders for consumer goods
  - building permits for private housing
  - S&P 500 stock prices

Statistical Forecasting Models

- **Time Series** - a stream of historical data, such as weekly sales
  - *T* = number of periods, $t = 1, 2, \ldots, T$
- Time series generally have components such as:
  - random behavior
  - trends (upward or downward)
  - seasonal effects
  - cyclical effects
- **Stationary time series** have only random behavior.
- A **trend** is a gradual upward or downward movement of a time series.

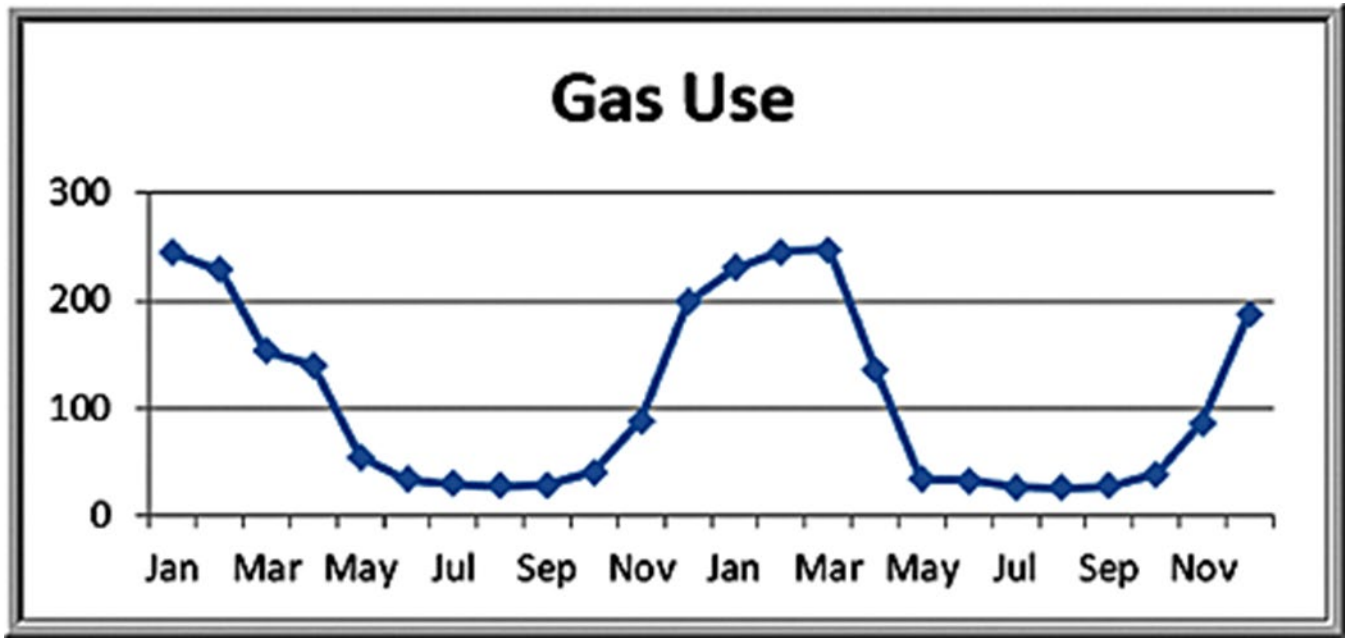
Example 9.4: Identifying Trends in a Time Series

- Total Energy Production & Consumption
  - General upward trend with some short downward trends; the time series is composed of several different short trends.



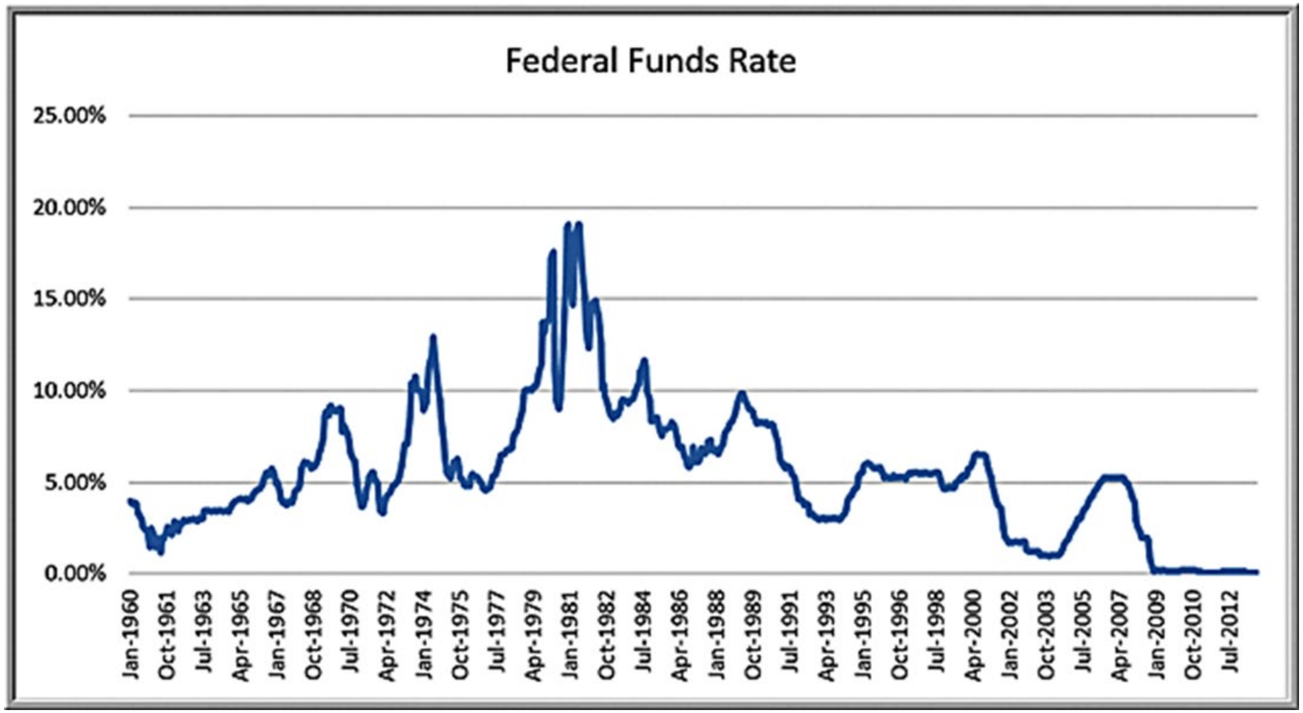
Seasonal Effects

- A **seasonal effect** is one that repeats at fixed intervals of time, typically a year, month, week, or day.



Cyclical Effects

- **Cyclical effects** describe ups and downs over a much longer time frame, such as several years.



Forecasting Models for Stationary Time Series

- Moving average model
- Exponential smoothing model
  - These are useful over short time periods when trend, seasonal, or cyclical effects are not significant.

Moving Average Models

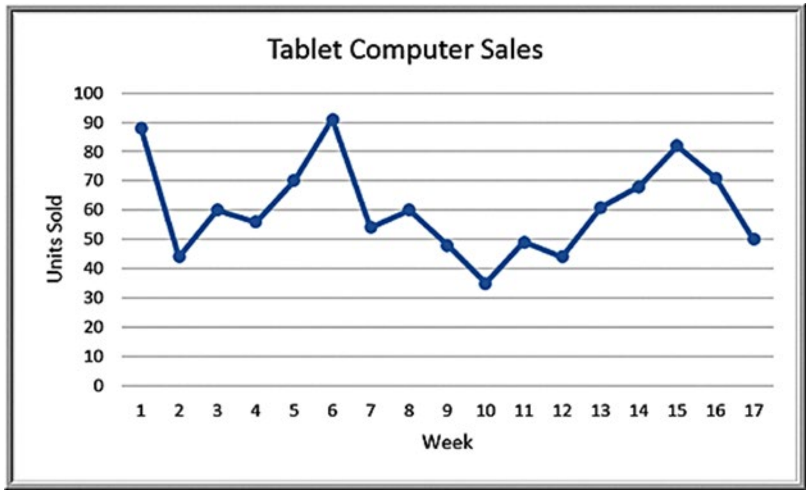
- The **simple moving average** method is a smoothing method based on the idea of averaging random fluctuations in the time series to identify the underlying direction in which the time series is changing.
- The simple moving average forecast for the next period is computed as the average of the most recent *k* observations.

$$F_{t+1} = \frac{A_t + A_{t-1} + \ldots + A_{t-k+1}}{k} \qquad (9.1)$$

  – Larger values of *k* result in smoother forecast models since extreme values have less impact.

Example 9.5: Moving Average Forecasting

- The Tablet Computer Sales file contains the number of units sold over the past 17 weeks.



- Three-period moving average forecast for week 18:

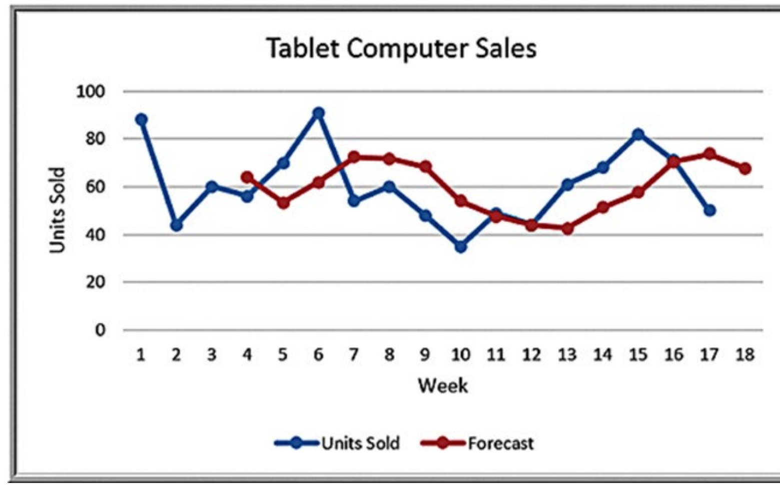
$$F_{18} = \frac{(A_{17} + A_{16} + A_{15})}{3} = \frac{82 + 71 + 50}{3} = 67.67$$

Spreadsheet Implementation of Moving Average Forecasting

	A	B	C	D	E	F
1	Tablet Computer Sales					
2			Moving Average			
3	Week	Units Sold	Forecast			
4	1	88				
5	2	44				
6	3	60				
7	4	56	64.00			
8	5	70	53.33			
9	6	91	62.00			
10	7	54	72.33			
11	8	60	71.67			
12	9	48	68.33			
13	10	35	54.00			
14	11	49	47.67			
15	12	44	44.00			
16	13	61	42.67			
17	14	68	51.33			
18	15	82	57.67			
19	16	71	70.33			
20	17	50	73.67			
21	18		67.67			
22						

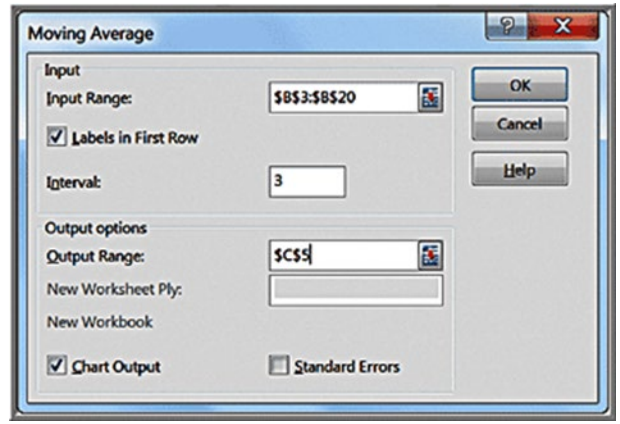
Forecast for week 4
=AVERAGE(B4:B6)

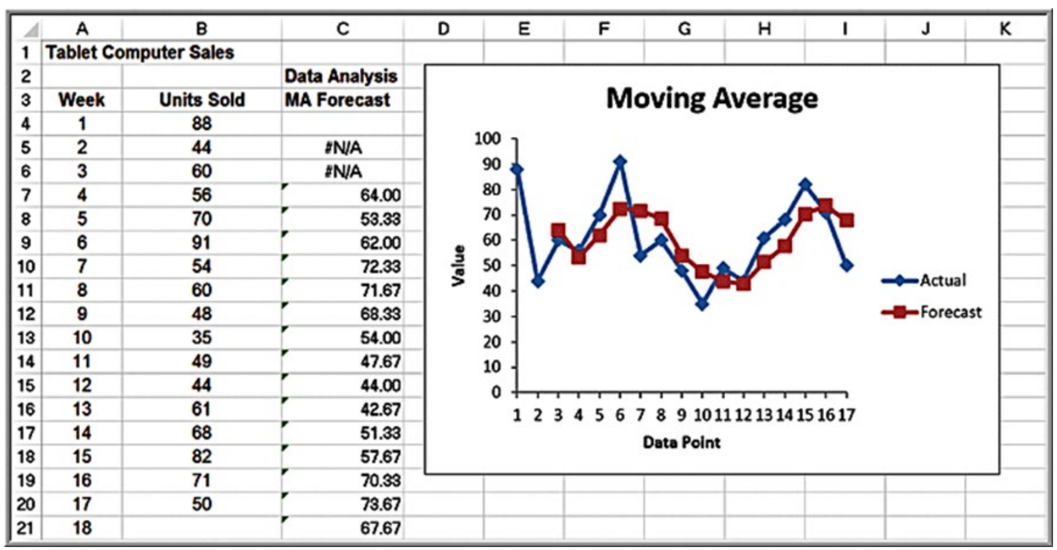
Forecast for week 18
=AVERAGE(B18:B20)



Example 9.6: Using Excel's Moving Average Tool

- Data Analysis options





	A	B	C
1	Tablet Computer Sales		
2			Data Analysis
3	Week	Units Sold	MA Forecast
4	1	88	
5	2	44	#N/A
6	3	60	#N/A
7	4	56	64.00
8	5	70	53.33
9	6	91	62.00
10	7	54	72.33
11	8	60	71.67
12	9	48	68.33
13	10	35	54.00
14	11	49	47.67
15	12	44	44.00
16	13	61	42.67
17	14	68	51.33
18	15	82	57.67
19	16	71	70.33
20	17	50	73.67
21	18		67.67

We do not recommend using the chart or error options because the forecasts generated by this tool are not properly aligned with the data.

Error Metrics and Forecast Accuacry

- Mean absolute deviation (MAD)

$$\text{MAD} = \frac{\sum_{t=1}^{n} \left| A_t - F_t \right|}{n} \qquad (9.2)$$

- Mean square error (MSE)

$$\text{MSE} = \frac{\sum_{t=1}^{n} \left(A_t - F_t \right)^2}{n} \qquad (9.3)$$

- Root mean square error (RMSE)

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} \left(A_t - F_t \right)^2}{n}} \qquad (9.4)$$

- Mean absolute percentage error (MAPE)

$$\text{MAPE} = \frac{\sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|}{n} \times 100 \qquad (9.5)$$

Example 9.7: Using Error Metrics to Compare Moving Average Forecasts

- Tablet Computer Sales data
- 2-, 3-, and 4-period moving average models
- 2-period model has lowest error metric values.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Tablet Computer Sales																
2			k = 2					k = 3					k = 4				
3	Week	Units Sold	Forecast	Error	Absolute	Squared	Absolute	Forecast	Error	Absolute	Squared	Absolute	Forecast	Error	Absolute	Squared	Absolute
4	1	88			Deviation	Error	% Error			Deviation	Error	% Error			Deviation	Error	% Error
5	2	44															
6	3	60	66.00	-6.00	6.00	36.00	10.00										
7	4	56	52.00	4.00	4.00	16.00	7.14	64.00	-8.00	8.00	64.00	14.29					
8	5	70	58.00	12.00	12.00	144.00	17.14	53.33	16.67	16.67	277.78	23.81	62.00	8.00	8.00	64.00	11.43
9	6	91	63.00	28.00	28.00	784.00	30.77	62.00	29.00	29.00	841.00	31.87	57.50	33.50	33.50	1122.25	36.81
10	7	54	80.50	-26.50	26.50	702.25	49.07	72.33	-18.33	18.33	336.11	33.95	69.25	-15.25	15.25	232.56	28.24
11	8	60	72.50	-12.50	12.50	156.25	20.83	71.67	-11.67	11.67	136.11	19.44	67.75	-7.75	7.75	60.06	12.92
12	9	48	57.00	-9.00	9.00	81.00	18.75	68.33	-20.33	20.33	413.44	42.36	68.75	-20.75	20.75	430.56	43.23
13	10	35	54.00	-19.00	19.00	361.00	54.29	54.00	-19.00	19.00	361.00	54.29	63.25	-28.25	28.25	798.06	80.71
14	11	49	41.50	7.50	7.50	56.25	15.31	47.67	1.33	1.33	1.78	2.72	49.25	-0.25	0.25	0.06	0.51
15	12	44	42.00	2.00	2.00	4.00	4.55	44.00	0.00	0.00	0.00	0.00	48.00	-4.00	4.00	16.00	9.09
16	13	61	46.50	14.50	14.50	210.25	23.77	42.67	18.33	18.33	336.11	30.05	44.00	17.00	17.00	289.00	27.87
17	14	68	52.50	15.50	15.50	240.25	22.79	51.33	16.67	16.67	277.78	24.51	47.25	20.75	20.75	430.56	30.51
18	15	82	64.50	17.50	17.50	306.25	21.34	57.67	24.33	24.33	592.11	29.67	55.50	26.50	26.50	702.25	32.32
19	16	71	75.00	-4.00	4.00	16.00	5.63	70.33	0.67	0.67	0.44	0.94	63.75	7.25	7.25	52.56	10.21
20	17	50	76.50	-26.50	26.50	702.25	53.00	73.67	-23.67	23.67	560.11	47.33	70.50	-20.50	20.50	420.25	41.00
21	18		60.50		13.63	254.38	23.63	67.67		14.86	299.84	25.37	67.75		16.13	355.25	28.07
22					MAD	MSE	MAPE			MAD	MSE	MAPE			MAD	MSE	MAPE

Exponential Smoothing Models

- **Simple exponential smoothing** model:

$$F_{t+1} = (1-\alpha)F_t + \alpha A_t$$
$$= F_t + \alpha(A_t - F_t) \qquad (9.6)$$

where F_{t+1} is the forecast for time period $t+1, F_t$ is the forecast for period *t*, A_t is the observed value in period *t*, and *α* is a constant between 0 and 1 called the **smoothing constant**.

- To begin, set F_1 and F_2 equal to the actual observation in period 1, A_1.

Example 9.8: Using Exponential Smoothing to Forecast Tablet Computer Sales

	A	B	C	D	E	F	G	H	I	J	K
1	**Tablet Computer Sales**										
2							**Smoothing Constant**				
3	**Week**	**Units Sold**	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**
4	1	88	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
5	2	44	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
6	3	60	83.60	79.20	74.80	70.40	66.00	61.60	57.20	52.80	48.40
7	4	56	81.24	75.36	70.36	66.24	63.00	60.64	59.16	58.56	58.84
8	5	70	78.72	71.49	66.05	62.14	59.50	57.86	56.95	56.51	56.28
9	6	91	77.84	71.19	67.24	65.29	64.75	65.14	66.08	67.30	68.63
10	7	54	79.16	75.15	74.37	75.57	77.88	80.66	83.53	86.26	88.76
11	8	60	76.64	70.92	68.26	66.94	65.94	64.66	62.86	60.45	57.48
12	9	48	74.98	68.74	65.78	64.17	62.97	61.87	60.86	60.09	59.75
13	10	35	72.28	64.59	60.45	57.70	55.48	53.55	51.86	50.42	49.17
14	11	49	68.55	58.67	52.81	48.62	45.24	42.42	40.06	38.08	36.42
15	12	44	66.60	56.74	51.67	48.77	47.12	46.37	46.32	46.82	47.74
16	13	61	64.34	54.19	49.37	46.86	45.56	44.95	44.70	44.56	44.37
17	14	68	64.00	55.55	52.86	52.52	53.28	54.58	56.11	57.71	59.34
18	15	82	64.40	58.04	57.40	58.71	60.64	62.63	64.43	65.94	67.13
19	16	71	66.16	62.83	64.78	68.03	71.32	74.25	76.73	78.79	80.51
20	17	50	66.65	64.47	66.65	69.22	71.16	72.30	72.72	72.56	71.95
21	18		64.98	61.57	61.65	61.53	60.58	58.92	56.82	54.51	52.20
22		**MAD**	19.33	17.16	16.15	15.36	14.93	14.71	14.72	14.88	15.36
23		**MSE**	496.07	390.84	359.18	346.56	340.77	338.41	339.03	343.32	352.38
24		**MAPE**	38.28%	32.71%	30.12%	28.36%	27.54%	27.09%	27.09%	27.38%	28.23%

Forecast for week 3 when $\alpha = 0.7 : (1-0.7)(88) + (0.7)(44) = 57.2$

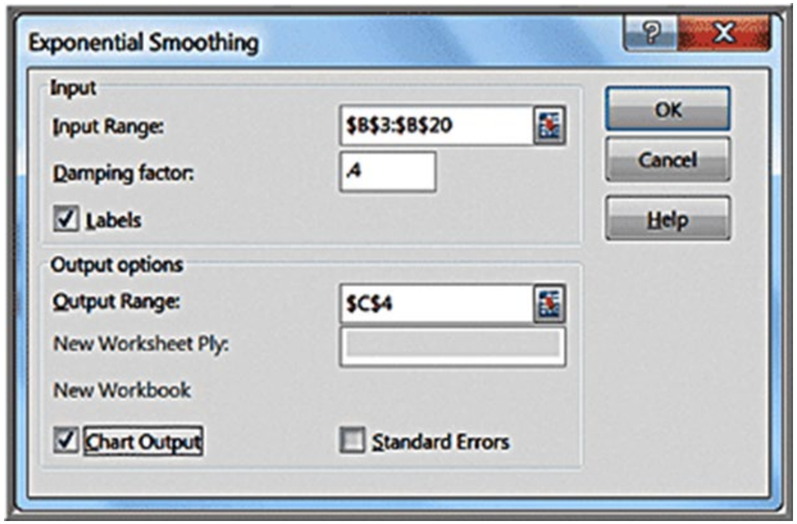
Example 9.9: Finding the Best Exponential Smoothing Model for Tablet Computer Sales

	A	B	C	D	E	F	G	H	I	J	K
1	Tablet Computer Sales										
2						Smoothing Constant					
3	Week	Units Sold	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
4	1	88	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
5	2	44	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
6	3	60	83.60	79.20	74.80	70.40	66.00	61.60	57.20	52.80	48.40
7	4	56	81.24	75.36	70.36	66.24	63.00	60.64	59.16	58.56	58.84
8	5	70	78.72	71.49	66.05	62.14	59.50	57.86	56.95	56.51	56.28
9	6	91	77.84	71.19	67.24	65.29	64.75	65.14	66.08	67.30	68.63
10	7	54	79.16	75.15	74.37	75.57	77.88	80.66	83.53	86.26	88.76
11	8	60	76.64	70.92	68.26	66.94	65.94	64.66	62.86	60.45	57.48
12	9	48	74.98	68.74	65.78	64.17	62.97	61.87	60.86	60.09	59.75
13	10	35	72.28	64.59	60.45	57.70	55.48	53.55	51.86	50.42	49.17
14	11	49	68.55	58.67	52.81	48.62	45.24	42.42	40.06	38.08	36.42
15	12	44	66.60	56.74	51.67	48.77	47.12	46.37	46.32	46.82	47.74
16	13	61	64.34	54.19	49.37	46.86	45.56	44.95	44.70	44.56	44.37
17	14	68	64.00	55.55	52.86	52.52	53.28	54.58	56.11	57.71	59.34
18	15	82	64.40	58.04	57.40	58.71	60.64	62.63	64.43	65.94	67.13
19	16	71	66.16	62.83	64.78	68.03	71.32	74.25	76.73	78.79	80.51
20	17	50	66.65	64.47	66.65	69.22	71.16	72.30	72.72	72.56	71.95
21	18		64.98	61.57	61.65	61.53	60.58	58.92	56.82	54.51	52.20
22		MAD	19.33	17.16	16.15	15.36	14.93	14.71	14.72	14.88	15.36
23		MSE	496.07	390.84	359.18	346.56	340.77	338.41	339.03	343.32	352.38
24		MAPE	38.28%	32.71%	30.12%	28.36%	27.54%	27.09%	27.09%	27.38%	28.23%

The forecast using α =0.6 provides the lowest error metrics.

Example 9.10: Using Excel's Exponential Smoothing Tool (1 of 2)

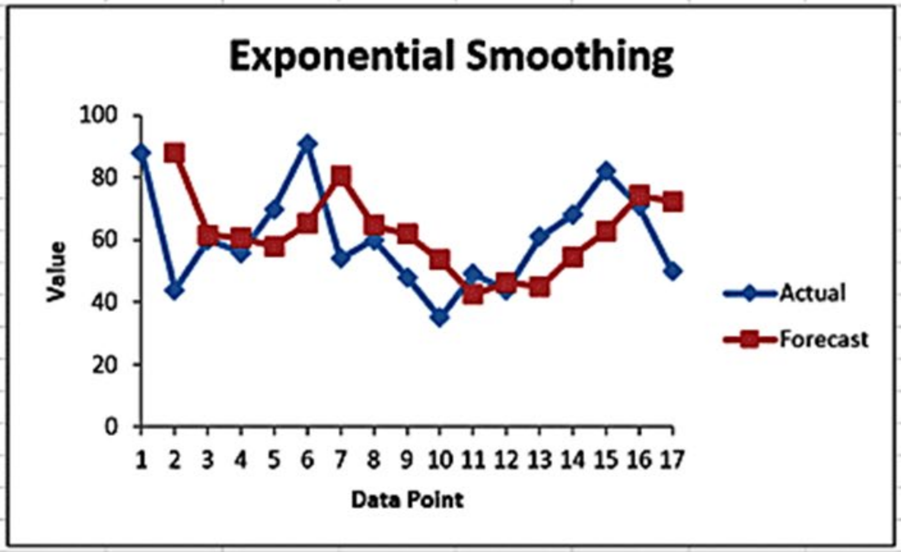
- Select Data Analysis from the Analysis group and then choose Exponential Smoothing.
- Note that Damping factor = 1−α
- The first cell of the Output Range should be adjacent to the first data point.



Example 9.10: Using Excel's Exponential Smoothing Tool (2 of 2)

- Exponential Smoothing tool results

	A	B	C
1	Tablet Computer Sales		
2			Exponential Smoothing
3	Week	Units Sold	Forecast
4	1	88	#N/A
5	2	44	88.00
6	3	60	61.60
7	4	56	60.64
8	5	70	57.86
9	6	91	65.14
10	7	54	80.66
11	8	60	64.66
12	9	48	61.87
13	10	35	53.55
14	11	49	42.42
15	12	44	46.37
16	13	61	44.95
17	14	68	54.58
18	15	82	62.63
19	16	71	74.25
20	17	50	72.30



Forecasting Models for Time Series with a Linear Trend

- **Double moving average** and **double exponential smoothing**
- Based on the linear trend equation

$$F_{t+k} = a_t + b_t k \qquad (9.7)$$

- The forecast for *k* periods into the future is a function of the level a_t and the trend b_t.
- The models differ in their computations of a_t and b_t.

Double Exponential Smoothing

- Estimates of the parameters are obtained from the following equations:

$$a_t = \alpha F_t + (1-\alpha)(a_{t-1} + b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta) b_{t-1} \qquad (9.8)$$

- Initial values are chosen for a_1 as A_1 and b_1 as $A_2 - A_1$.

Equation (9.8) must then be used to compute a_t and b_t

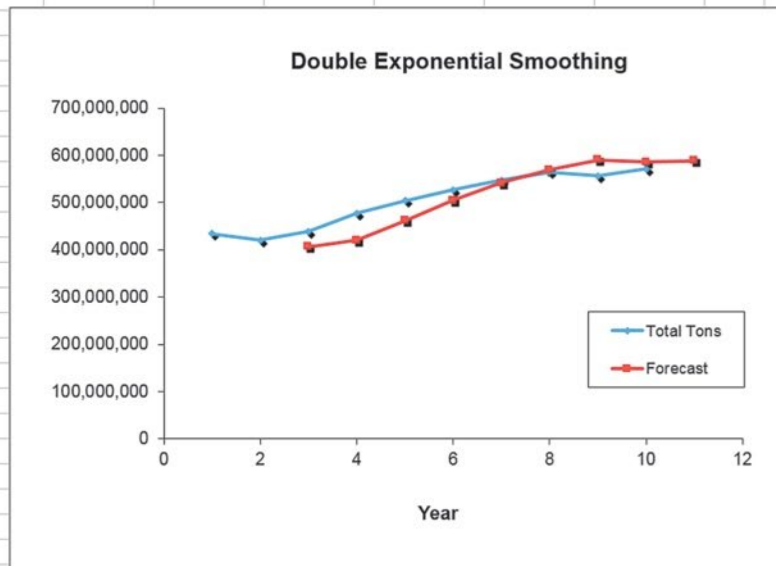
for the entire time series to be able to generate forecasts into the future. The forecast for *k* periods beyond the last period (period *T*) is

$$F_{T+k} = a_T + b_T(k) \qquad (9.9)$$

Example 9.11: Double Exponential Smoothing

- First ten years of data in the Excel file Coal Production
- Choose α = 0.6 and β = 0.4
  - See text for computational details.

Coal Production					
Double Exponential Smoothing					
Alpha	0.6				
Beta	0.4				
					Absolute
Year	**Total Tons**	**Level (a)**	**Trend (b)**	**Forecast**	**Error**
1	434,329,000	434,329,000	-13,906,000		
2	420,423,000	420,423,000	-13,906,000		N/A
3	439,043,000	426,032,600	-6,099,760	406,517,000	32,526,000
4	477,195,000	454,290,136	7,643,158	419,932,840	57,262,160
5	504,182,000	487,282,518	17,782,848	461,933,294	42,248,706
6	526,954,000	518,198,546	23,036,120	505,065,366	21,888,634
7	546,822,000	544,587,066	24,377,080	541,234,666	5,587,334
8	564,882,000	566,514,859	23,397,365	568,964,147	4,082,147
9	556,706,000	569,988,489	15,427,871	589,912,224	33,206,224
10	570,978,000	576,753,344	11,962,665	585,416,361	14,438,361
11				588,716,009	
				MAD	**26,404,946**

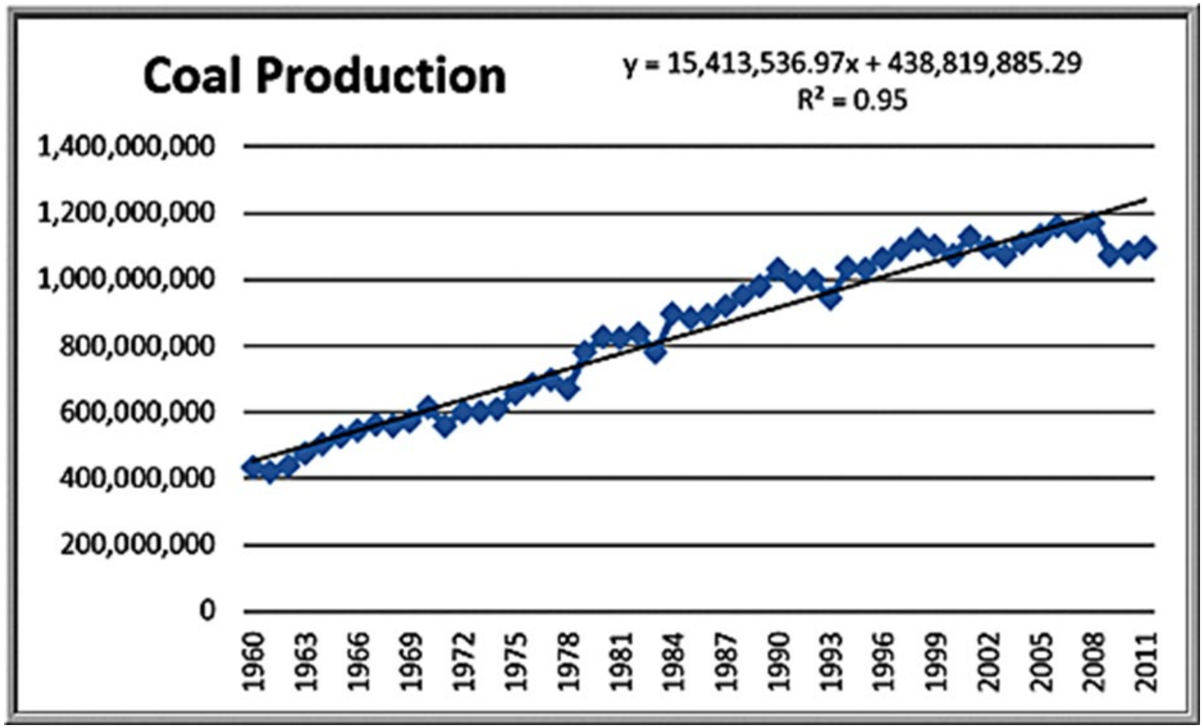


Regression-Based Forecasting for Time Series with a Linear Trend

- Simple linear regression can be applied to forecasting using time as the independent variable.

Example 9.12: Forecasting Using Trendlines

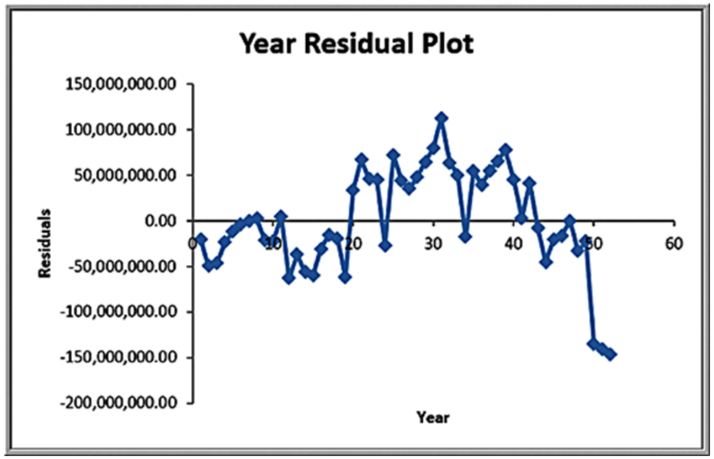
- Coal Production data with a linear trendline



Note that the linear model does not adequately predict the recent drop in production after 2008.

Autocorrelation in Time Series

- When autocorrelation is present, successive observations are correlated with one another; for example, large observations tend to follow other large observations, and small observations also tend to follow one another.
  - In such cases, other approaches, called **autoregressive models**, are more appropriate.

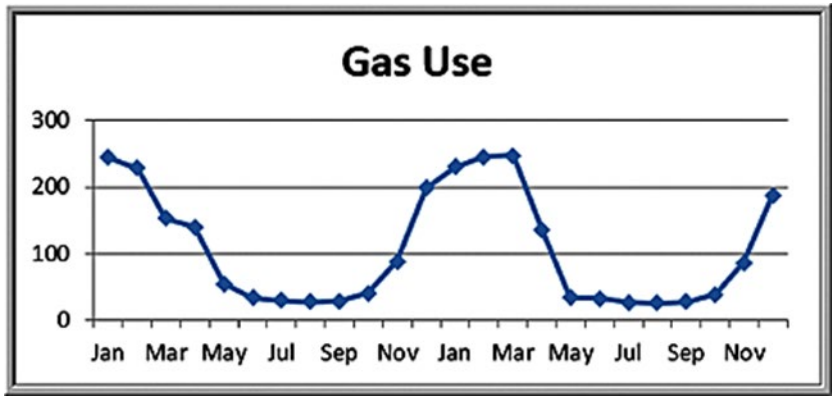


Forecasting Time Series with Seasonality

- When time series exhibit seasonality, different techniques provide better forecasts than the ones we have described:
  - Multiple regression models with categorical variables for the seasonal components
  - Holt-Winters models, similar to exponential smoothing models in that smoothing constants are used to smooth out variations in the level and seasonal patterns over time.

Example 9.13: Regression-Based Forecasting for Natural Gas Usage (1 of 3)

- Gas & Electric Excel file
- Use a seasonal categorical variable with k = 12 levels.
- Construct the regression model using $k - 1$ dummy variables, with January being the reference month.



$$\begin{aligned}\text{gas usage} = {} & \beta_0 + \beta_1 \text{ time} + \beta_2 \text{ February} + \beta_3 \text{ March} \\ & + \beta_4 \text{ April} + \beta_5 \text{ May} + \beta_6 \text{ June} + \beta_7 \text{ July} \\ & + \beta_8 \text{ August} + \beta_9 \text{ September} + \beta_{10} \text{ October} \\ & + \beta_{11} \text{ November} + \beta_{12} \text{ December}\end{aligned}$$

Example 9.13: Regression-Based Forecasting for Natural Gas Usage (2 of 3)

- Data matrix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Gas and Electric Usage													
2														
3	Month	Gas Use	Time	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
4	Jan	244	1	0	0	0	0	0	0	0	0	0	0	0
5	Feb	228	2	1	0	0	0	0	0	0	0	0	0	0
6	Mar	153	3	0	1	0	0	0	0	0	0	0	0	0
7	Apr	140	4	0	0	1	0	0	0	0	0	0	0	0
8	May	55	5	0	0	0	1	0	0	0	0	0	0	0
9	Jun	34	6	0	0	0	0	1	0	0	0	0	0	0
10	Jul	30	7	0	0	0	0	0	1	0	0	0	0	0
11	Aug	28	8	0	0	0	0	0	0	1	0	0	0	0
12	Sep	29	9	0	0	0	0	0	0	0	1	0	0	0
13	Oct	41	10	0	0	0	0	0	0	0	0	1	0	0
14	Nov	88	11	0	0	0	0	0	0	0	0	0	1	0
15	Dec	199	12	0	0	0	0	0	0	0	0	0	0	1
16	Jan	230	13	0	0	0	0	0	0	0	0	0	0	0
17	Feb	245	14	1	0	0	0	0	0	0	0	0	0	0
18	Mar	247	15	0	1	0	0	0	0	0	0	0	0	0
19	Apr	135	16	0	0	1	0	0	0	0	0	0	0	0
20	May	34	17	0	0	0	1	0	0	0	0	0	0	0
21	Jun	33	18	0	0	0	0	1	0	0	0	0	0	0
22	Jul	27	19	0	0	0	0	0	1	0	0	0	0	0
23	Aug	26	20	0	0	0	0	0	0	1	0	0	0	0
24	Sep	28	21	0	0	0	0	0	0	0	1	0	0	0
25	Oct	39	22	0	0	0	0	0	0	0	0	1	0	0
26	Nov	86	23	0	0	0	0	0	0	0	0	0	1	0
27	Dec	188	24	0	0	0	0	0	0	0	0	0	0	1

Example 9.13: Regression-Based Forecasting for Natural Gas Usage (3 of 3)

- Final regression results (time and February were insignificant)

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.985480895							
5	R Square	0.971172595							
6	Adjusted R Square	0.948997667							
7	Standard Error	19.54432831							
8	Observations	24							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*Significance F*			
12	Regression	10	167292.2083	16729.22083	43.79597661	2.33344E-08			
13	Residual	13	4965.75	381.9807692					
14	Total	23	172257.9583						
15									
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
17	Intercept	236.75	9.772164157	24.22697738	3.33921E-12	215.6385228	257.8614772	215.6385228	257.8614772
18	Mar	-36.75	16.92588482	-2.171230656	0.049016211	-73.31615105	-0.183848953	-73.31615105	-0.183848953
19	Apr	-99.25	16.92588482	-5.863799799	5.55744E-05	-135.816151	-62.68384895	-135.816151	-62.68384895
20	May	-192.25	16.92588482	-11.35834268	4.02824E-08	-228.816151	-155.683849	-228.816151	-155.683849
21	Jun	-203.25	16.92588482	-12.00823485	2.07264E-08	-239.816151	-166.683849	-239.816151	-166.683849
22	Jul	-208.25	16.92588482	-12.30364038	1.54767E-08	-244.816151	-171.683849	-244.816151	-171.683849
23	Aug	-209.75	16.92588482	-12.39226204	1.41949E-08	-246.316151	-173.183849	-246.316151	-173.183849
24	Sep	-208.25	16.92588482	-12.30364038	1.54767E-08	-244.816151	-171.683849	-244.816151	-171.683849
25	Oct	-196.75	16.92588482	-11.62420766	3.05791E-08	-233.316151	-160.183849	-233.316151	-160.183849
26	Nov	-149.75	16.92588482	-8.847395666	7.30451E-07	-186.316151	-113.183849	-186.316151	-113.183849
27	Dec	-43.25	16.92588482	-2.555257847	0.023953114	-79.81615105	-6.683848953	-79.81615105	-6.683848953

$$\begin{aligned} \text{gas usage} &= 236.75 - 36.75\ \text{March} - 99.25\ \text{April} \\ &- 192.25\ \text{May} - 203.25\ \text{June} - 208.25\ \text{July} \\ &- 209.75\ \text{August} - 208.25\ \text{September} \\ &- 196.75\ \text{October} - 149.75\ \text{November} \\ &- 43.25\ \text{December} \end{aligned}$$

Holt-Winters Models for Forecasting Time Series with Seasonality and No Trend

- The **Holt-Winters additive seasonality model with no trend** applies to time series with relatively stable seasonality and is based on the equation

$$F_{t+k} = a_t + S_{t-s+k} \qquad (9.10)$$

- The **Holt-Winters multiplicative seasonality model with no trend** applies to time series whose amplitude increases or decreases over time and is

$$F_{t+k} = a_t S_{t-s+k} \qquad (9.11)$$

A chart of the time series should be viewed first to identify the appropriate type of model to use.

Holt-Winters Additive Seasonality Model with No Trend (1 of 2)

- The level and seasonal factors are estimated as

$$\begin{aligned}&\text{Level component}: a_t = \alpha(A_t - S_{t-s}) + (1-\alpha)a_{t-1}\\&\text{Seasonal component}: S_t = \gamma(A_t - a_t) + (1-\gamma)S_{t-s}\end{aligned} \qquad (9.12)$$

- The forecast for the next period is

$$F_{t+1} = a_t + S_{t-s+1}.$$

Holt-Winters Additive Seasonality Model with No Trend (2 of 2)

- Estimate the level and seasonal factors for the first *s* periods (that is, the length of a season)

$$a_t = \frac{1}{s}\sum_{i=1}^{s} A_i, \text{for } t = 1, 2, \ldots, s \qquad (9.13)$$

$$S_t = A_t - a_t, \text{for } t = 1, 2, \ldots, s \qquad (9.14)$$

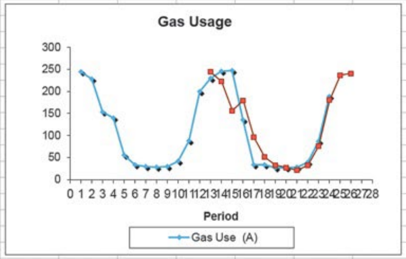
- Then we use the smoothing equations to update a_t and S_t and calculate forecasts.

Example 9.14: Using the Holt-Winters Additive Seasonality Model with No Trend

- Data for gas usage in the Excel file Gas & Electric
- Arbitrarily select $\alpha = 0.4$ and $\gamma = 0.9$.
  - See the text for computational details.

Holt Winters Additive Seasonality Model with No Trend

			α	γ		
			0.4	0.9		
Month	Period (t)	Gas Use (A)	Level (a)	Seasonality (S)	Forecast (F)	MAD
Jan	1	244	105.75	138.25		
Feb	2	228	105.75	122.25		
Mar	3	153	105.75	47.25		
Apr	4	140	105.75	34.25		
May	5	55	105.75	-31.85		
Jun	6	34	105.75	-52.35		
Jul	7	30	105.75	-63.45		
Aug	8	28	105.75	-66.55		
Sep	9	29	105.75	-72.26		
Oct	10	41	105.75	-63.51		
Nov	11	88	105.75	-22.32		
Dec	12	199	105.75	77.27		
Jan	13	230	100.15	130.69	244.00	14.00
Feb	14	245	109.19	134.45	222.40	22.60
Mar	15	247	145.41	96.15	156.44	90.56
Apr	16	135	127.55	10.13	179.66	44.66
May	17	34	102.87	-65.17	95.70	61.70
Jun	18	33	95.86	-61.81	50.52	17.52
Jul	19	27	93.70	-66.37	32.41	5.41
Aug	20	26	93.24	-67.17	27.15	1.15
Sep	21	28	96.05	-68.47	20.98	7.02
Oct	22	39	98.63	-60.02	32.54	6.46
Nov	23	86	102.51	-17.09	76.31	9.69
Dec	24	188	105.80	81.71	179.78	8.22
Jan	25				236.49	
Feb	26				240.25	
					MAD	24.08



Holt-Winters Multiplicative Seasonality Model with No Trend (1 of 2)

- The multiplicative seasonal model has the same basic smoothing structure as the additive seasonal model with some key differences:

$$\begin{aligned} &\text{Level component}: a_t = \alpha(A_t / S_{t-s}) + (1-\alpha)a_{t-1} \\ &\text{Seasonal component}: S_t = \gamma(A_t / a_t) + (1-\gamma)S_{t-s} \qquad (9.15) \end{aligned}$$

- The forecast for the next period is

$$F_{t+1} = a_t S_{t-s+1}.$$

Holt-Winters Multiplicative Seasonality Model with No Trend (2 of 2)

- Initialize the values for the level and seasonal factors:

$$a_t = \frac{1}{s}\sum_{i=1}^{s} A_i, \text{for } t = 1,2,\ldots,s$$

$$S_t = A_t / a_t, \text{for } t = 1,2,\ldots,s \qquad (9.16)$$

- This model can be implemented on a spreadsheet in a similar fashion as the additive model.

Holt-Winters Models for Forecasting Time Series with Seasonality and Trend

- The **Holt-Winters additive model** applies to time series with relatively stable seasonality and is based on the equation

$$F_{t+1} = a_t + b_t + S_{t-s+1} \qquad (9.17)$$

- The **Holt-Winters multiplicative model** applies to time series whose amplitude increases or decreases over time and is

$$F_{t+1} = (a_t + b_t) S_{t-s+1} \qquad (9.18)$$

- A chart of the time series should be viewed first to identify the appropriate type of model to use.

Holt-Winters Additive Seasonality Model with Trend (1 of 2)

- This model is similar to the additive model that incorporates only seasonality, but with the addition of a trend component:

Level component: $a_t = \alpha\left(A_t - S_{t-s}\right) + (1-\alpha)\left(a_{t-1} + b_{t-1}\right)$

Trend component: $b_t = \beta\left(a_t - a_{t-1}\right) + (1-\beta)b_{t-1}$ (9.19)

Seasonal component: $S_t = \gamma\left(A_t - a_t\right) + (1-\gamma)S_{t-s}$

- The forecast for period *t* + 1 is

$$F_{t+1} = a_t + b_t + S_{t-s+1} \qquad (9.20)$$

- The forecast for *k* periods beyond the last period of observed data (period *T*) is

$$F_{T+k} = a_T + b_T k + S_{T-s+k} \qquad (9.21)$$

Holt-Winters Additive Seasonality Model with Trend (2 of 2)

- The initial values for level and seasonal factors are the same as in the additive seasonality model without trend, that is, formulas (9.13) and (9.14).
- The initial values for the trend component are

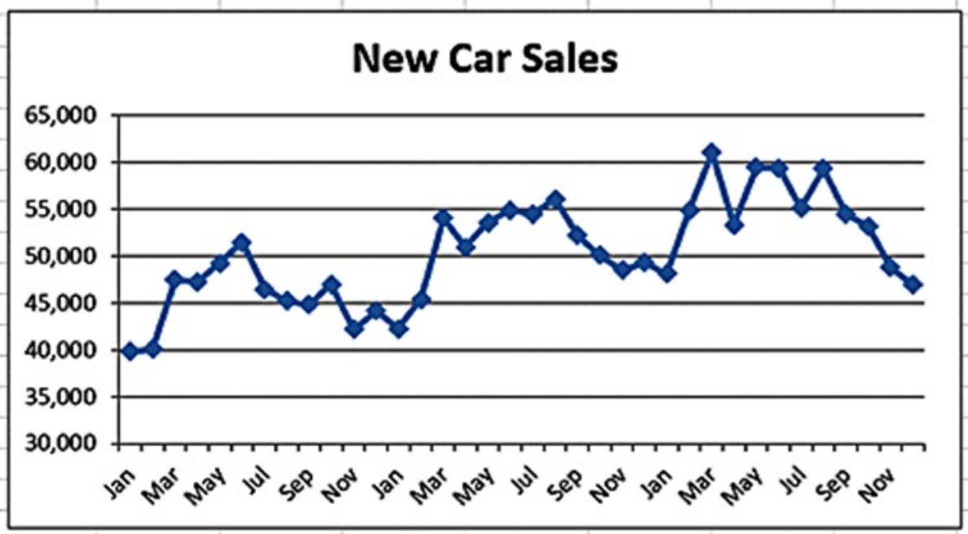
$$b_t = \frac{1}{s}\sum_{i=1}^{s}\frac{\left(A_{s+i} - A_i\right)}{s}, \text{for } t = 1, 2, \ldots, s \qquad (9.22)$$

Example 9.15: Using the Holt-Winters Additive Model for Seasonality and Trend (1 of 2)

- Excel file New Car Sales
- Stable seasonality and an increasing trend

New Car Retail Sales

Year	Month	Units
1	Jan	39,810
1	Feb	40,081
1	Mar	47,440
1	Apr	47,297
1	May	49,211
1	Jun	51,479
1	Jul	46,466
1	Aug	45,208
1	Sep	44,800
1	Oct	46,989
1	Nov	42,161
1	Dec	44,186
2	Jan	42,227
2	Feb	45,422
2	Mar	54,075
2	Apr	50,926
2	May	53,572



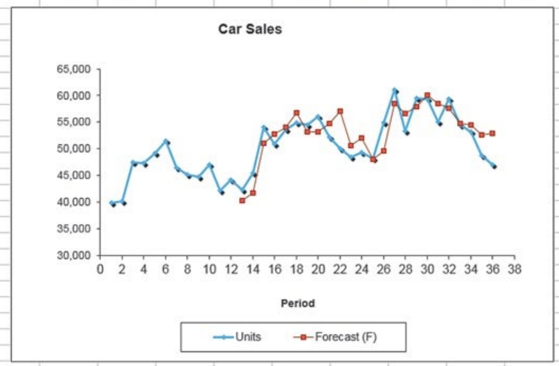
Example 9.15: Using the Holt-Winters Additive Model for Seasonality and Trend (2 of 2)

- Arbitrarily select $\alpha = 0.3, \beta = 0.2$, and $\gamma = 0.9$.
  - See text for computational details.

Holt Winters Additive Model with Seasonality and Trend

α	β	γ
0.3	0.2	0.9

Month	Period (t)	Units	Level (a)	Trend (b)	Seasonality (S)	Forecast (F)
Jan	1	39,810	45427.33	462.48	-5617.33	
Feb	2	40,081	45427.33	462.48	-5346.33	
Mar	3	47,440	45427.33	462.48	2012.67	
Apr	4	47,297	45427.33	462.48	1869.67	
May	5	49,211	45427.33	462.48	2843.57	
Jun	6	51,479	45427.33	462.48	4911.87	
Jul	7	46,466	45427.33	462.48	1136.07	
Aug	8	45,208	45427.33	462.48	-10.43	
Sep	9	44,800	45427.33	462.48	-280.24	
Oct	10	46,989	45427.33	462.48	1896.69	
Nov	11	42,161	45427.33	462.48	-2826.09	
Dec	12	44,186	45427.33	462.48	-1118.24	
Jan	13	42,227	46476.17	579.75	-4385.99	40272.48
Feb	14	45,422	48169.64	802.50	-3007.51	41709.59
Mar	15	54,075	49899.20	987.91	3959.49	50984.81
Apr	16	50,926	50337.87	878.06	716.28	52756.77
May	17	53,572	51069.68	848.81	2536.44	54059.50
Jun	18	54,920	51345.39	734.19	3708.34	56830.36
Jul	19	54,449	52449.58	808.19	1913.08	53215.64
Aug	20	56,079	54107.27	978.09	1773.51	53247.34
Sep	21	52,177	54296.93	820.40	-1935.96	54805.12
Oct	22	50,087	53039.22	404.78	-2467.33	57014.02
Nov	23	48,513	52812.53	278.49	-4152.19	50617.91
Dec	24	49,278	52282.59	116.80	-2815.95	51972.78
Jan	25	48,134	52435.57	124.04	-4310.01	48013.40
Feb	26	54,887	54160.08	444.13	353.48	49552.09
Mar	27	61,064	55354.30	594.15	5534.68	58563.70
Apr	28	53,350	54954.03	395.27	-1372.00	56664.73
May	29	59,467	55823.67	490.14	3532.64	57885.74
Jun	30	59,370	56118.17	451.01	3297.48	60022.15
Jul	31	55,088	55550.90	247.36	-225.30	58482.26
Aug	32	59,349	56331.43	353.99	2893.17	57571.77
Sep	33	54,472	56602.18	337.34	-2110.76	54749.46
Oct	34	53,164	56547.07	258.85	-3291.49	54472.19
Nov	35	48,793	55647.70	27.21	-6584.45	52653.73
Dec	36	46,956	53904.02	-326.97	-6534.81	52858.95
Jan	37					49267.04



Holt-Winters Multiplicative Seasonality Model with Trend (1 of 2)

- The Holt-Winters multiplicative model is similar to the additive model for seasonality, but with a trend component:

$$
\begin{aligned}
\text{Level component: } a_t &= \alpha\left(A_t / S_{t-s}\right) + (1-\alpha)\left(a_{t-1} + b_{t-1}\right) \\
\text{Trend component: } b_t &= \beta\left(a_t - a_{t-1}\right) + (1-\beta)\, b_{t-1} \qquad (9.23) \\
\text{Seasonal component: } S_t &= \gamma\left(A_t / a_t\right) + (1-\gamma)\, S_{t-s}
\end{aligned}
$$

Holt-Winters Multiplicative Seasonality Model with Trend (2 of 2)

- The forecast for period t + 1 is

$$F_{t+1} = (a_t + b_t)S_{t-s+1} \qquad (9.24)$$

- The forecast for k periods beyond the last period of observed data (period T) is

$$F_{T+k} = (a_T + b_T k)S_{T-s+k} \qquad (9.25)$$

- Initialization is performed in the same way as for the multiplicative model without trend. The model can be implemented in a similar fashion on a spreadsheet as the additive model.

Selecting Appropriate Time-Series-Based Forecasting Method

	No Seasonality	Seasonality
No trend	Simple moving average or simple exponential smoothing	Holt-Winters additive or multiplicative seasonality models without trend or multiple regression
Trend	Double exponential smoothing	Holt-Winters additive or multiplicative seasonality models trend

Regression Forecasting with Causal Variables

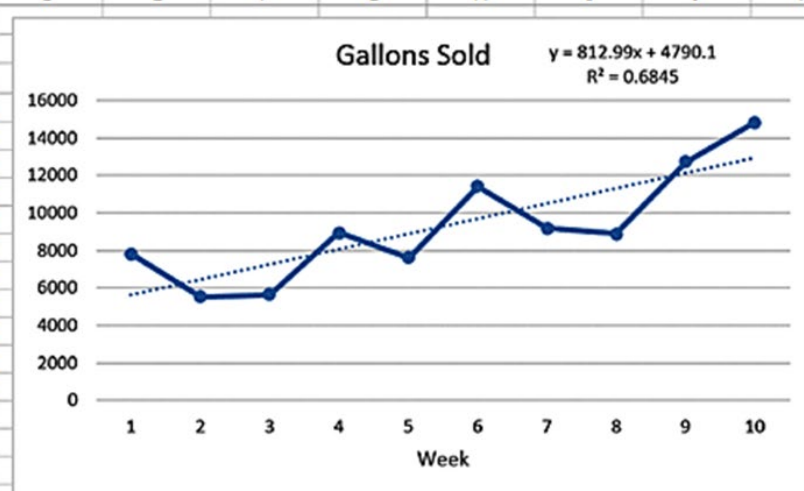
- In many forecasting applications, other independent variables besides time, such as economic indexes or demographic factors, may influence the time series.
- Explanatory/causal models, often called **econometric models**, seek to identify factors that explain statistically the patterns observed in the variable being forecast, usually with regression analysis.

Example 9.16: Forecasting Gasoline Sales Using Simple Linear Regression

- Excel file Gasoline Sales
- Simple trendline using week as the independent variable

Gasoline Sales

Gallons Sold	Week	Price/Gallon
7815	1	$3.95
5541	2	$4.20
5650	3	$4.12
8949	4	$3.98
7600	5	$4.01
11430	6	$3.92
9190	7	$4.03
8889	8	$3.98
12721	9	$3.92
14830	10	$3.90
	11	$3.80



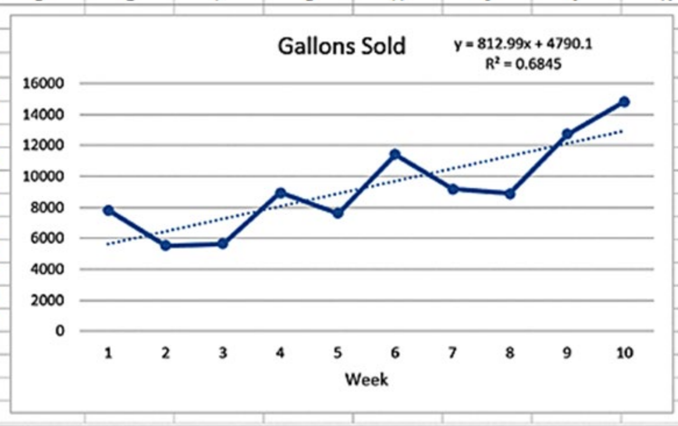
Predicted sales for week 11 = $812.99(11) + 4790.1 = 13,733$ gallons

Example 9.17: Incorporating Causal Variables in a Regression-Based Forecasting Model (1 of 2)

- The average price per gallon changes each week, and this may influence consumer sales. Average price per gallon is a causal variable.
- Develop a multiple linear regression model to predict gasoline sales using both time and price per gallon.

Gasoline Sales

Gallons Sold	Week	Price/Gallon
7815	1	$3.95
5541	2	$4.20
5650	3	$4.12
8949	4	$3.98
7600	5	$4.01
11430	6	$3.92
9190	7	$4.03
8889	8	$3.98
12721	9	$3.92
14830	10	$3.90
	11	$3.80



Example 9.17: Incorporating Causal Variables in a Regression-Based Forecasting Model (2 of 2)

- Multiple regression model $\text{sales} = \beta_0 + \beta_1 \text{ week} + \beta_2 \text{ price/gallon}$

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.930528528					
5	R Square	0.865883342					
6	Adjusted R Square	0.827564297					
7	Standard Error	1235.400329					
8	Observations	10					
9							
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	2	68974748.7	34487374.35	22.59668368	0.000883465	
13	Residual	7	10683497.8	1526213.972			
14	Total	9	79658246.5				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	72333.08447	21969.92267	3.292368642	0.013259225	20382.47252	124283.6964
18	Week	508.6681395	168.1770861	3.024598364	0.019260863	110.9925232	906.3437559
19	Price/Gallon	-16463.19901	5351.082403	-3.076611005	0.017900405	-29116.49823	-3809.899786

Predicted sales for week 11

$$= 72{,}333 + 508.7(11) - 16{,}463(3.80) = 15{,}368 \text{ gallons}$$

The Practice of Forecasting

- Judgmental and qualitative methods are used for forecasting sales of product lines and broad company and industry forecasts.
- Simple time-series models are used for short- and medium-range forecasts.
- Regression methods are typically used for long-term forecasts.