14

Variable Product and Input Prices

This chapter relaxes the fixed input and product price assumptions of the purely competitive model and derives the marginal conditions for profit maximization, allowing for the possibility of variable input and product prices. The possibility exists that input or product prices may vary according to how much product is produced or input is used. For a single farmer to affect the price of a product, he or she must control a significant share of the output for the product. The farmer may be able to buy inputs in volume at discount, thus affecting the constant input price assumption.

Key terms and definitions:

Price Variation
Downward Sloping Demand Curve
Volume Discounts
Price Flexibility
Function-of-a-Function Rule
Composite Function Rule
Total Value of the Product (*TVP*)
General Profit Maximization Conditions

14.1 Relaxing the Assumptions of Pure Competition

Until now, two key assumptions of the purely competitive model have been carefully followed. These assumptions were (1) that the farmer can produce and sell as much output as desired at the going, fixed market price, and (2) the the farmer can purchase as much of any input as needed at the going market price. But what if one or both of these assumptions about the real world no longer hold? There are several possible instances in which one or both of these assumptions might not hold.

It is not easy to see how the individual North Dakota wheat producer, by his or her output decision, could possibly influence the market price for wheat, but what about the broiler producer large enough to produce 10 percent of the available broilers for sale in U.S. markets? Surely his or her output decisions could have an influence on broiler prices in the United States. What about a single producer who dominates a small market such as the parsley market? His or her decision not to produce would have an impact on the price of parsley. Control of the market price for an agricultural commodity requires a degree of size on the part of the individual firm.

For certain agricultural commodities, such as broilers, the firm must be rather large in terms of the dollar volume of sales relative to total production of the agricultural commodity to have an impact on prices. For other commodities, such as parsley, where the total market is small, all that is required is that the firm control a significant share of the total output. The percent of the total market that a single farm firm must control in order to have an impact on the price of the commodity varies from commodity to commodity. For a product with a highly inelastic demand curve by consumers in the aggregate, control of but 1 percent of the total output may be sufficient for the individual firm to exert an influence on the market price.

There are two possible rationale for variation in input prices in response to changes in the demand for an input by a farmer. A farmer might be so large as to be the dominant buyer of a particular input in the local market. The farm is large enough such that additional units of the input cannot be purchased without incurring a higher price. It is difficult to see how a market for feed grain or fertilizer could be dominated by a single producer such that the price of feed grains or fertilizer for all producers would be influenced. More likely, market domination in the purchase of inputs might occur for a highly specialized input required solely by the producers of the single commodity which the farm firm dominates, and in a situation for which there may be but a few producers of the input.

The second rationale for variation in input prices is as a result of quantity or volume discounts by input suppliers. Fertilizer purchased by the ton is often cheaper than fertilizer purchased by the pound in a bag, but the crop does not care if the fertilizer was bagged or not. What is required here is that the farm merely be of sufficient size such that the quantities of inputs required to take advantage of the volume discount can be used.

14.2 Variation in Output Prices from the Output Side

If output prices vary with the output level for the farm, the farm must have a degree of monopoly power over the market. The farm need not be the sole producer of the commodity in order to have monopoly power. All that is required is that the output level by the farmer be sufficiently large such that if the level of output from the farm is changed, the market price level will also change.

The example used here relies on some of the characteristics of the model of pure monopoly that are a usual part of introductory economics courses. An important characteristic of a model of a monopoly is a down ! sloping demand curve for the product. A down ! sloping demand curve, in turn, results in marginal revenue no longer the same as the price of the product. The producer can sell additional units of output only by accepting a lower price for each incremental unit.

In the model of pure competition, with fixed output prices, total revenue is price times output ($TR = py$). Thus the total revenue function under pure competition is a line with a constant positive slope p. Now suppose that price is a function of output. Or $p = p(y)$ (this notation is read p equals p of y, not p equals p times y). Then total revenue is defined as

†14.1 $$TR = p(y)y$$

Marginal revenue can be obtained by differentiating total revenue with respect to output using the composite function rule

†14.2 $$MR = dTR/dy = p\ dy/dy + y\ dp/dy$$

†14.3 $$MR = p + y\ dp/dy$$

The derivative dp/dy represents the slope of the demand function by consumers for y. The new marginal revenue is equal to marginal revenue under constant product prices plus an expression that explicitly takes into account the slope of the demand function for the output.

Now divide and multiply MR by the output price p

†14.4 $$MR = p[1 + (y/p)(dp/dy)]$$

The expression $(dy/dp)(p/y)$ is the elasticity of demand for the output y or E_d. Marginal revenue under variable output prices is

†14.5 $$MR = p(1 + 1/E_d)$$

The term *price flexibility* is sometimes used as the expression for 1 over an elasticity of demand. A price flexibility represents the percentage change in output price divided by a percentage change in quantity.

The elasticity of demand will be negative if the demand function is downward sloping. As the elasticity of demand for y becomes larger and larger in absolute value (approaching negative infinity), 1 over the elasticity of demand becomes smaller and smaller. At the limit, when the elasticity of demand becomes infinite, marginal revenue is the price of the product and the pure competition assumption is met. If the industry contains monopoly elements, the demand curve will slope downward to a degree and the price of the product will not be equal to marginal revenue.

In other words, if the elasticity of demand lies between zero and !4, marginal revenue will not be the same as the price of the product. If the elasticity of demand falls in the range (!4 < E_d < !1), marginal revenue will be positive, but less than the product price. If the elasticity of demand falls in the range (!1 < E_d < 0), marginal revenue will be negative. If the elasticity of demand equals !1, then marginal revenue is zero. The gain in revenue from an increase in the physical quantity of output is just offset by the reduction in revenue attributable to the decrease in the product price.

When marginal revenue is positive, total revenue increases as output is increased. Total revenue is increasing if the elasticity of demand for the product is between !4 and !1. When marginal revenue is zero, total revenue is constant or perhaps at its maximum. Total revenue is constant when the elasticity of demand for the product exactly !1 (sometimes called *unitary* *elasticity*). When marginal revenue is negative, total revenue is declining as output is increased. The decrease in revenue from the price reduction more than offsets the increase in revenue from the additional physical quantity of output. Total revenue is decreasing when elasticities of demand for the product lie between 0 and -1 (Figure 14.1).

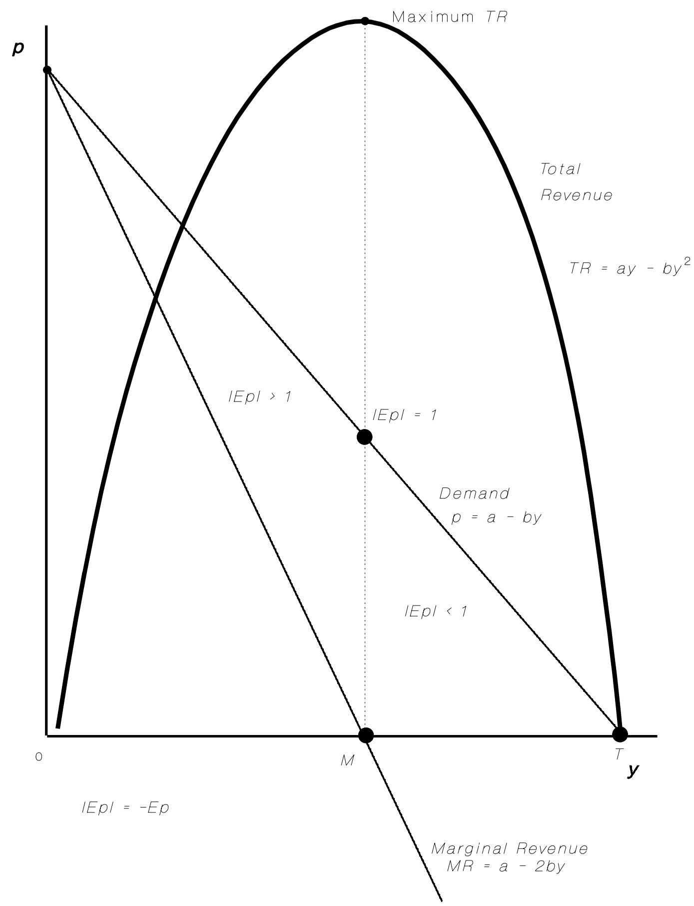

Figure 14.1 Total Revenue, Marginal Revenue, and the Elasticity of Demand

Now suppose that the demand function for the output is

†14.6 $$p = a - by$$

where a and b are constants. Total revenue is

†14.7 $$TR = py = (a - by)y = ay - by^2$$

Marginal revenue is

†14.8 $$MR = dTR/dy = a - 2by$$

Marginal revenue descends at a rate twice as fast as the demand curve. Hence, for a linear demand function, marginal revenue cuts the horizontal axis at a point exactly one half the distance from the origin to where the demand function cuts the horizontal axis (Figure 14.1).

The slope of the demand function for y is

†14.9 $$dp/dy = -b$$

The price flexibility of demand for y is

†14.10 $$(dp/dy)y/p = -b(y/p)$$

The elasticity of demand (E_d) for y is

†14.11 $$E_d = (-1/b)p/y$$

For a linear demand function, the elasticity of demand will vary along the demand function. The elasticity of demand at a particular point on the demand function can be determined if the corresponding p and y is known.

Marginal revenue is

†14.12 $$MR = p[1 + (-b)(y/p)]$$
$$= p - (bp)(y/p)$$

14.3 Variation in Output Prices from the Input Side

The farmer controls the level of output that is produced by adjusting the quantity of input that is used. A change in the amount of input that is used will, in turn, affect the amount of output produced. If the market price changes as a result of a change in output, the change in the amount of input that is used can also indirectly affect output prices.

Suppose that the production function is given by

†14.13 $$y = y(x)$$

Equation †14.13 should be read y equals y of x, not y equals y times x. The function $y(x)$ is the same old production function as $f(x)$, but the new notation will simplify the economic interpretation of some of the derivatives.

The product price is again given by

†14.14 $$p = p(y)$$

The price of the product is a function of y, or some p of y, not p times y.

Equations †14.13 and †14.14 , when taken together mean that

†14.15 $$p = p[y(x)]$$

Output price is equal to p of y of x, not p times y times x. In this example the output price is determined by the quantity of output that is produced. The quantity of output that is produced in turn is a function of the quantity of input that is used. This model suggests that the price of the output is indirectly determined by the quantity of the input that is used.

The equation

†14.16 $p = p[y(x)]$

is known as a *function of a function*. Such an equation can be differentiated using the simple *function-of-a-function* rule, whichstates that the function should be differentiated from the outside in and the result multiplied together

†14.17 $dp/dx = (dp/dy)(dy/dx)$

The change in the product price with respect to the change in the quantity of the input used is the product of two slopes. The first (dp/dy) is the slope of the demand function and represents the rate of change in product price as a result of a change in output. The second slope (dy/dx) is our old friend MPP_x and indicates how fast output changes in response to an increase in the use of the input x. The derivatives dp/dy and dy/dx might be constants but they need not be constant. If dp/dy is constant and negative, then the demand function has a constant negative slope. If dy/dx is constant, *MPP* is constant.

The rule is readily extended for a production function with more than one input. Recognize that a change in the use of x_1 also affects the use of x_2. The partial notation for MPP_{x_1} and MPP_{x_2} is used, and the products are summed for each input

†14.18 $y = y(x_1, x_2)$

†14.19 $p = p(y)$

†14.20 $p = p[y(x_1, x_2)]$

†14.21 $dp/dx_1 = (dp/dy)(\partial y/\partial x_1) + (dp/dy)(\partial y/\partial x_2)(dx_2/dx_1)$

†14.22 $dp/dx_2 = (dp/dy)(\partial y/\partial x_2) + (dp/dy)(\partial y/\partial x_1)(dx_1/dx_2)$

The expressions to the far right of the equalities in equations †14.21 and †14.22 link explicitly the use of x_1 to x_2 and the use of x_2 to x_1. If the slope of the demand function (dp/dy) is nonzero and MPP_{x_1} and MPP_{x_2} are nonzero, the expressions on the far right will be zero only if a change in the use of one of the two inputs is not accompanied by a change in the use of the other input. This would be highly unlikely.

Suppose again the single-input production function

†14.23 $y = y(x)$

The total value of the product (*TVP*) is given by

†14.24 $TVP = p[y(x)][y(x)]$

Figure 14.2 illustrates the relationship between total value of the product under fixed product prices and under variable product prices. There is no assurance that when output increases, total value of the product will also increase if product prices decline in the face on the increase in the output level. Diagram A illustrates a case where the new *TVP* actually declines. In diagram B, the new *TVP* remains constant. In diagram C, both the old and new *TVP* increase, but the *TVP* with decreasing product prices at a slower rate than *TVP* with constant product prices.

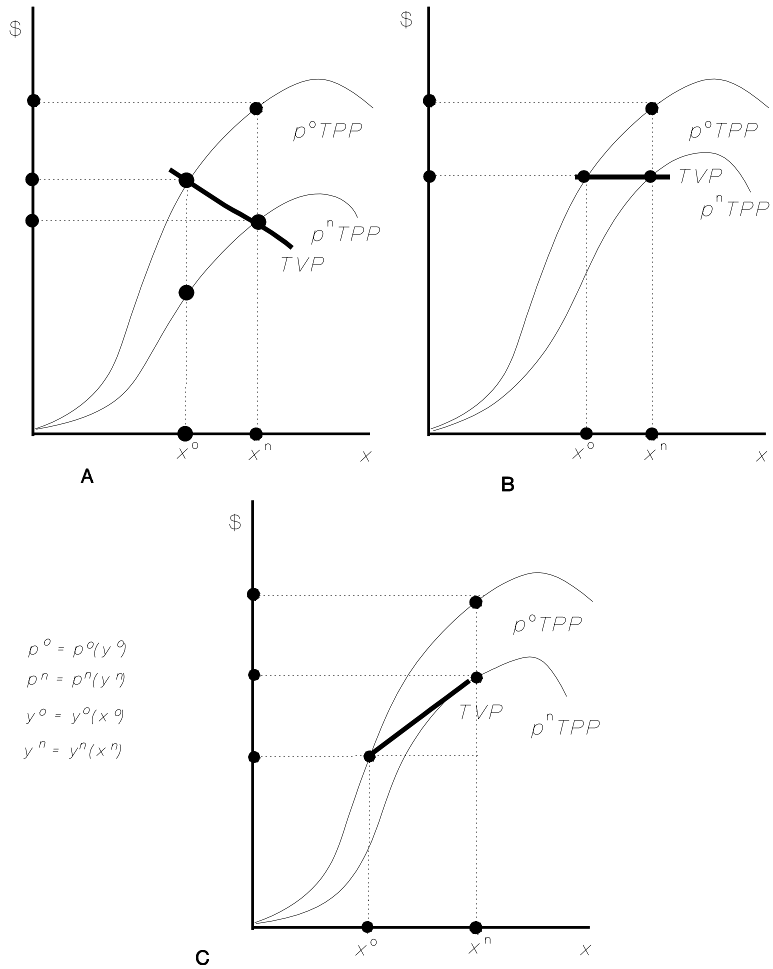

Figure 14.2 Possible *TVP* Functions Under Variable Product Prices

To calculate the marginal value of the product (*MVP*), both the composite function rule and the function of a function rule are needed

†14.25 $$MVP = dTVP/dx = p\ dy/dx + y[(dp/dy)(dy/dx)]$$

$$= dy/dx(p + y\ dp/dy)$$

$$= MPP_x\,[p + y(\text{the slope of the demand function})]$$

$$= pMPP_x\,(1 + 1/E_d)$$

$$= VMP_x\,(1 + 1/E_d)$$

$$= VMP_x + VMP_x/E_d$$

The marginal value of the product (*MVP*) under variable product prices equals the old value of the marginal product (VMP_x) under constant market prices plus the value of the marginal product under constant market prices divided by the price elasticity of demand. Since the price elasticity of demand for the output is usually negative, *MVP* will usually be less than *VMP* under constant product prices. Moreover, the slope of *MVP* will usually be more strongly negative than the slope of *VMP*. The slope of *VMP* is influenced only by the declining marginal product of the underlying production function, The slope of *MVP* is influenced both by the declining *MPP* of the production function and the decrease in price associated with the downward-sloping demand function.

14.4 Variable Input Prices

Two possibilities arise. Increasing input prices in response to increased demand could occur if the individual producer were large enough to influence the market. This would imply $dv/dx > 0$, where v is the price of x. The other possibility is $dv/dx < 0$. This would imply quantity discounts, which result in a lower price per unit of input in response to an increase in demand for the input. If $dv/dx = 0$, input prices are constant and the pure competition assumption with regard to input prices is met.

The variable input price could be defined as

†14.26 $$v = v(x)$$

The input price v is a function of x and equal to v of x (not v times x).

The total factor cost (*TFC*) is

†14.27 $$TFC = v(x)x$$

The total factor cost is v of x times x.

Marginal factor cost (*MFC*) can be found with the aid of the composite function rule

†14.28 $$MFC = dTFC/dx = v\, dx/dx + x\, dv/dx$$

$$= v + x\, dv/dx$$

$$= v[1 + (x/v)(dv/dx)]$$

$$= v(1 + 1/E_x)$$

where

$$E_x = (dx/dv)(v/x)$$

The elasticity E_x may be positive, negative, or zero. If v is constant, then $dx/dv = 0$, the assumption of pure competition is met and MFC = v. If dx/dv is positive, the farmer can obtain additional units of x but only at an increasing price. This condition is consistent with the dominant input buyer case. If dx/dv is negative, additional units of the input can be purchased at a decreasing incremental cost per unit. This condition is broadly consistent with the quantity or volume discounts case.

14.5 A General Profit Maximization Statement

The general conditions for profit-maximization on the input side can be derived. These conditions allow for variable product and input prices, but include fixed product and input prices as a special case.

Suppose that revenue (R) is a function of output (y)

†14.29 $R = r(y)$

Output is a function of input use

†14.30 $y = y(x)$

Cost (C) is a function of input use

†14.31 $C = c(x)$

Profit (A) is revenue minus cost

†14.32 $A = r[y(x)] \; ! \; c(x)$

The first-order conditions for the maximization of profit in equation †14.32 require that

†14.33 $dA/dx = (dr/dy)(dy/dx) \; ! \; dc/dx = 0$

†14.34 $MVP \; ! \; MFC = 0$

†14.35 $MVP = MFC$

The first-order or necessary conditions require that the marginal value of the product (*MVP*) equal marginal resource cost (*MFC*), and this occurs at the point where the profit function has a slope of zero. This rule must be followed irrespective of whether or not the input prices are fixed or variable.

The second-order conditions require that

†14.36 $d^2A/dx^2 = dMVP/dx \; ! \, dMFC/dx < 0$

†14.37 $dMVP/dx < dMFC/dx$

The slope of *MVP* must be steeper or more negative than the slope of *MFC*. If *MVP* is downward sloping and *MFC* is constant or sloping upward, the second-order condition is always satisfied at the intersection. A downward-sloping demand curve for the output leads to an *MVP* function with a more strongly negative slope than would be the case under constant output prices. But if *MVP* is downward sloping and so is *MFC*, the *MVP* function must cut the *MFC* function from above. This condition is ordinarily met, but if the farmer were receiving huge discounts for volume purchase of x, it might be possible for the slope of *MFC* to be more strongly negative than the slope of *MVP*, and profits would not be maximum.

In the two-input case, the production function is defined as

†14.38 $y = y(x_1, x_2)$

The revenue function is

†14.39 $R = r(y)$

Thus

†14.40 $R = r[y(x_1, x_2)]$

The cost function is

†14.41 $$C = c(x_1, x_2)$$

Profit is

†14.42 $$\pi = r[y(x_1, x_2)] - c(x_1, x_2)$$

First-order (necessary) conditions for maximum profit in equation †14.42 require that

†14.43 $$\pi = (dr/dy)\partial y/\partial x_1 - \partial c/\partial x_1 = 0$$

†14.44 $$\pi = (dr/dy)\partial y/\partial x_2 - \partial c/\partial x_2 = 0$$

The slope of the profit function equals zero with respect to both inputs. Moreover,

†14.45 $$MVP_{x_1} = MFC_{x_1}$$

†14.46 $$MVP_{x_2} = MFC_{x_2}$$

and

†14.47 $$MVP_{x_1}/MFC_{x_1} = MVP_{x_2}/MFC_{x_2} = 1$$

Second order conditions require that

†14.48

$$\{\partial[(dr/dy)(\partial y/\partial x_1)]/\partial x_1 - \partial(\partial c/\partial x_1)/\partial x_1\}\ \{\partial[(dr/dy)(\partial y/\partial x_2)]/\partial x_2 - \partial(\partial c/\partial x_2)/\partial x_2\} > \{\partial[(dr/dy)(\partial y/\partial x_1)]/\partial x_2 - \partial(\partial c/\partial x_1)/\partial x_2\}\ \{\partial[(dr/dy)(\partial y/\partial x_2)]/\partial x_1 - \partial(\partial c/\partial x_2)/\partial x_1\}$$

One implication of these second-order conditions is that if profit maximization is to take place, the *MVP* curve must intersect the *MFC* curve from above. This condition holds irrespective of whether the *MVP* curve or the *MFC* curve has a positive or a negative slope. These first- and second-order conditions could be extended to any number of inputs.

14.6 Concluding Comments

This chapter has provided a set of general profit maximization conditions that are no longer linked to the pure competition assumption of constant input and product prices. These conditions allow for the possibility of downward-sloping demand curves for the product and volume discounts for input purchases. However, the marginal rules developed in Chapter 7 have not been significantly altered. The value of the incremental unit of the input in terms of its worth in the production process is still equated to the cost of the incremental unit. This rule applies irrespective of whether product and factor prices are allowed to vary.

Problems and Exercises

1. For the following, indicate if a point of profit maximization exists. Explain your answer for each case.

a. *VMP* cuts *MFC* from above.
b. *VMP* cuts *MFC* from below.
c. *VMP* and *MFC* are parallel.
d. *VMP* and *MFC* diverge.
e. *VMP* = \$3 everywhere; *MFC* = \$3 everywhere.
f. *MFC* and *VMP* intersect, but *MFC* has a more strongly negative slope than *VMP*.

2. Assume the following values. In each case find marginal revenue.

a. Total revenue (TR) = \$3y
b. $y = 50 - 2p$
c. $p = 10 - y$
d. $p = (10 - y)^{0.5}$

3. Find the relationship between *VMP* and *MVP* for the following elasticities of demand for product y.

a. -0.001
b. -0.2
c. -1
d. -5
e. -1000

4. Suppose that the revenue (R) and cost (C) functions are given by

$$R = 6y^{0.5}$$

$$C = 3y^2$$

Find the first- and second-order conditions for profit maximization.