15

Production of More Than One Product

This chapter introduces the product-product model, in which a single input is used in the production of two products. The basic production possibilities model familiar to students in introductory microeconomics courses is reviewed. The linkages between the production possibilities curve and the product transformation curve for the product-product model are developed. The rate of product transformation represents the slope of the product transformation function. Examples of competitive, complementary supplementary, and joint enterprises are given. Product transformation functions are derived from single-input production functions. An elasticity of substitution on the product side is defined.

Key terms and definitions:

Production Possibilities Curve
Concave to the Origin
Bowed Outward
Product-Product Model
Product Transformation Function
Total Differential
Rate of Product Transformation
Competitive Products
Complementary
Supplementary Products
Joint Products
Elasticity of Substitution on the Product Side

15.1 Production Possibilities for a Society

The concept of a production possibilities curve is familiar to students in introductory economics courses. A production possibilities curve represents the range of options open to a society given the resources that are available to the society. The appearance of a production possibilities curve differs from an isoquant in important ways. For example, alternative outputs, not inputs, appear on the axes.

The production possibilities curve represents the amount of each output that can be produced given that the available resources or inputs are taken as fixed and given. The production possibilities curve is usually drawn bowed outward, or concave to the origin of the graph, rather than convex to the origin of the graph. Figure 15.1 illustrates the classical production possibilities curve.

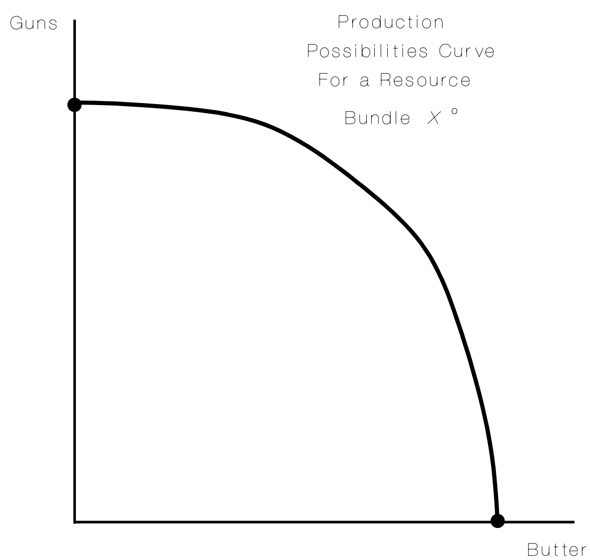


Figure 15.1 A Classic Production Possibilities Curve

The classical example of a production possibilities curve for a society has but two goods, butter and guns. Butter represents consumer goods that a society might be able to produce with its resources. Guns represent military weapons. A society might choose any point on its production possibilities curve.

The position of the Soviet Union would be near the guns axis on its production possibilities curve. The United States has chosen to produce some guns and some butter, with a somewhat greater emphasis on butter than guns. The United States would be nearer the butter axis of its curve than would the Soviet Union. A society such as Japan, which invests nearly everything in goods for consumers and virtually nothing on defense, would be found very near the butter axis of its curve.

No two societies have the exact same set of resources available for the production of butter and guns. Therefore, no two societies would have the same production possibilities curve. A society could choose to produce at a point interior to its production possibilities curve, but this would mean that some of the resources available to the society would be wasted. A society could not operate on a point outside its production possibilities curve in that this would require more resources than are available to the society.

A production possibilities curve thus represents the possible alternative efficient sets of outputs from a given set of resources. A simple equation for a production possibilities curve is

†15.1 $X^{\circ} = g(B, G)$

where X° = fixed quantity of resources available to the society
B = amount of butter that is produced
G = amount of guns that are produced

A series of production possibilities curves could be drawn, each representing a slightly different value for the resource bundle X. Production possibilities curves representing smaller resource bundles would lie inside, or interior to, production possibilities curves representing larger resource bundles. Like isoquants, production possibilities curves representing different size input bundles would never touch each other.

15.2 Production Possibilities at the Farm Level

The product-product model of agricultural production is a firm ! level version of the production possibilities curve. The production possibilities curve at the firm level is called a *product transformation curve*. The resource base for the farm is a bundle of inputs that could be used to produce either of two outputs. The farmer must choose to allocate the available bundle of inputs between the alternative outputs.

A society faces a problem in attempting to determine how best to allocate its resource bundle between guns and butter, for it cannot rely entirely on market signals. Consumers as individuals would each demand all consumer goods and no defense. But in the aggregate, the society may need protection from other warring nations, so market signals are useless in determining how much of a society's resources should be allocated to the production of guns or butter.

The farmer, or for that matter, any firm, faces a much simpler problem. Firm owners can rely on the market to provide an indication of the proportions of the input bundle that should be allocated to each alternative use. The market provides these signals through the price system. The relative prices, or price ratios, provide important information to the farm firm with respect to how much of each output should be produced.

The other piece of information that a farmer needs to know is the technical coefficients that underlie the production function transforming the input bundle into each alternative output. Just as a family of production functions underlie an isoquant map, so do they underlie a series of product transformation curves or functions. And the law of diminishing returns has as much to do with the outward bow of the product transformation curve as it did with the inward bow of the isoquants.

Consider a farmer who has available 10 units of an input bundle x. Each unit of the input bundle consists of the variable inputs required to produce either corn or soybeans. The proportions of each input in the bundle are equivalent to the proportions defined by the expansion path for the commodity. Since the two commodities require very nearly the same set of inputs, suppose that each unit of the bundle is exactly the same regardless of whether it is being used in the production of corn or soybeans. (This is a bit of a simplification in that no two commodities do require exactly the same inputs in the same proportion. Corn requires nitrogen and seed corn. Soybeans require little if any nitrogen and seed soybeans. Overlook this problem for the moment.)

The farmer is faced with hypothetical production function data (Table 15.1). The farmer faces a constraint that no more than 10 units of the input bundle *x* be used. The data for the soybean production function are presented starting with the greatest amount of input first. Each row of Table 15.1 may thus be looked upon as the quantity of each output produced from a total of 10 units of the input bundle. The production function for both corn and soybeans is subject to the law of diminishing returns. Each additional unit of the input bundle produces less and less additional output. The farmer cannot circumvent the law of diminishing returns in the production of either corn or soybeans.

Table 15.1 Production Function for Corn and Soybeans from a Variable Input Bundle x

Units of *x* Applied to Corn	Yield on an Acre (bushels)	Units of *x* Applied to Soybeans	Yield on an Acre (bushels)	Point
0	0	10	55	A
1	45	9	54	B
2	62	8	52	C
3	87	7	49	D
4	100	6	45	E
5	111	5	40	F
6	120	4	34	G
7	127	3	27	H
8	132	2	19	I
9	135	1	10	J
10	136	0	0	K

The greatest yields result when the farmer allocates all of the input bundle to the production of one of the possible outputs, but then none of the alternative output is produced. Suppose that the farmer initially allocates all 10 units of *x* to the production of corn and receives 136 bushels per acre. This point is depicted at A on Figure 15.2. By allocating, instead, 1 of the 10 units of *x* to the production of soybeans instead of corn, the farmer gives up but 1 bushel of corn. In return, 10 bushels of soybeans are received. What is happening is that the unit of the input bundle is being taken away from corn production in a very nonproductive region of the corn production function, where the *MPP* of *x* for corn is very low. The unit of the bundle is applied to the production function for soybeans in a very productive region of the soybean production function, where the *MPP* of *x* for soybeans is very high. Figure 15.2 illustrates some of the other options represented by the tabular data. Each additional unit of *x* taken from corn production results in a greater and greater loss in yield. As these additional units of *x* taken from corn production are applied to soybeans, each additional unit of *x* produces fewer and fewer additional soybeans. If a line is drawn that connects each of these points, the product transformation curve of function for the farmer results. The bowed-out shape of the production possibilities curve is a direct result of the law of diminishing returns, as evidenced by the declining marginal productivity of *x* in the production of each output.

If the production functions for both outputs do not have diminishing marginal returns, then the product transformation curve would not be bowed outward but would have a constant downward slope. The product transformation curve would be bowed inward if both underlying production functions had increasing marginal returns, or increased at an increasing rate.

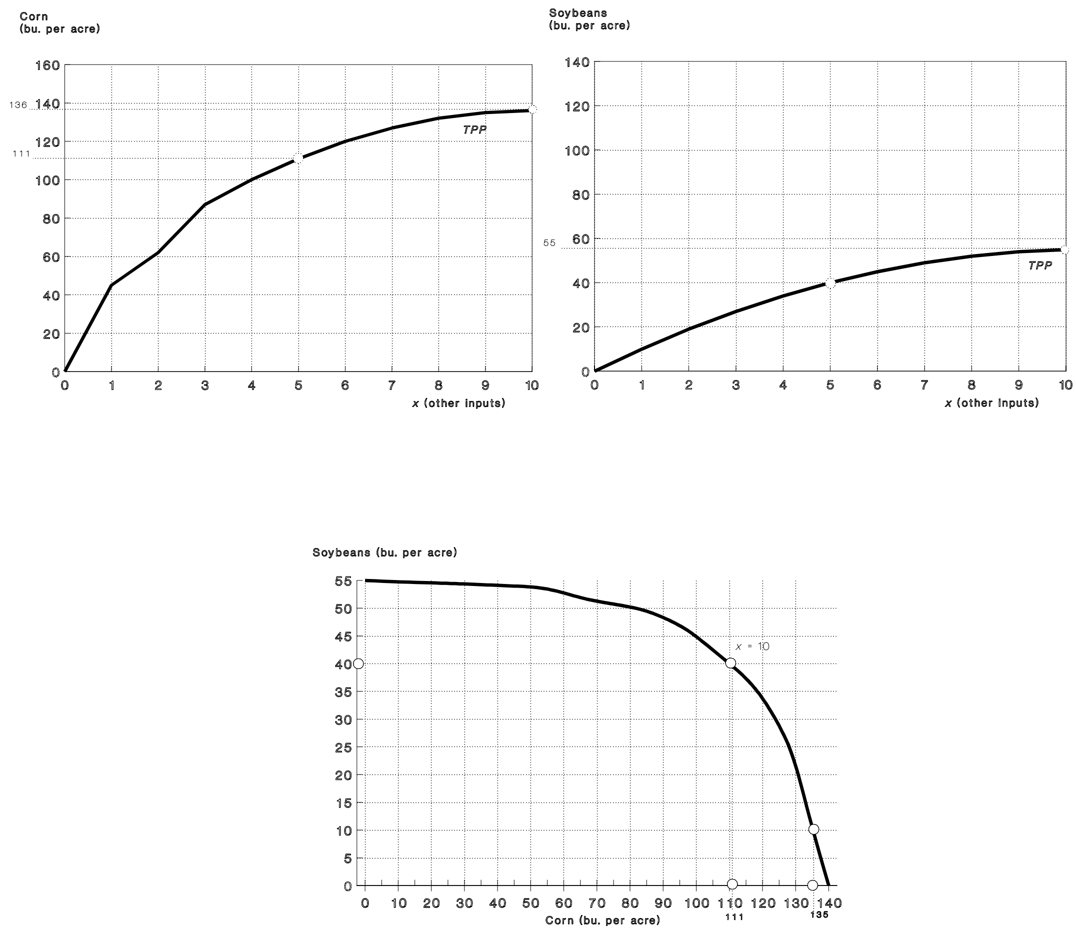


Figure 15.2 Deriving a Product Transformation Function from Two Production Functions

15.3 General Relationships

There exists a close association between the shape of a product transformation function and the two underlying production functions. Suppose that the equation for the product transformation curve is given by

†15.2 $$x = g(y_1, y_2)$$

where x is the input bundle and y_1 and y_2 are alternative outputs, such as corn and soybeans in the earlier example.

This is clearly not a production function, for it tells the amount of the input bundle that will be used as a result of varying the quantity of y_1 and y_2 that are produced. Note how similar this function is to the earlier function representing a product transformation curve for a society, but the function g is clearly not the same as the now familiar production function f.

Following the procedure outlined earlier for taking the total differential of a function, we have

†15.3 $$dx = (\partial g/\partial y_1)dy_1 + (\partial g/\partial y_2)dy_2$$

The partial derivatives $\partial g/\partial y_1$ and $\partial g/\partial y_2$ can readily be interpreted. The function g is actually x, and the equation could have been written as $x = x(y_1, y_2)$. (Again, x equals x of y_1 and y_2, not x equals x times y_1 and y_2.) Each partial derivative represents the change in the use of the input bundle x that arises from a change in the production of one of the outputs and is an inverse marginal product. The partial derivative $\partial g/\partial y_1$ is $1/MPP_x$ in the production of y_1, assuming that y_2 is constant. This might be called $1/MPP_{xy_1}$. The partial derivative $\partial g/\partial y_2$ is $1/MPP_x$ in the production of y_2, assuming that y_1 is held constant. This might be called $1/MPP_{xy_2}$. The equation for the total differential could then be rewritten as

†15.4 $$dx = (1/MPP_{xy_1})dy_1 + (1/MPP_{xy_2})dy_2$$

The basic assumption underlying a specific product transformation function is that the quantity of the input bundle x does not change. The product transformation function thus provides the alternative quantities of y_1 and y_2 that can be produced from a fixed amount of x. Hence dx, the change in x along a product transformation function, is zero. The total differential may then be rewritten as

†15.5 $$0 = (1/MPP_{xy_1})\, dy_1 + 1/MPP_{xy_2})\, dy_2$$

†15.6 $$!\ (1/MPP_{xy_1})\, dy_1 = (1/MPP_{xy_2})\, dy_2$$

†15.7 $$!\ (1/MPP_{xy_1}) = (1/MPP_{xy_2})(dy_2/dy_1)$$

†15.8 $$!\ (1/MPP_{xy_1})/(1/MPP_{xy_2}) = dy_2/dy_1$$

†15.9 $$!\ MPP_{xy_2}/MPP_{xy_1} = dy_2/dy_1$$

The expression dy_2/dy_1 represents the slope of the product transformation curve at a particular point. (The slope between a pair of points could be called $\Delta y_2/\Delta y_1$.) The slope of a product transformation function has been called different things by various economists. The term most often used is the *rate of product transformation* (*RPT*). The *RPT* is the slope (or in some textbooks, the negative slope) of the product transformation function and indicates the rate at which one output can be substituted for or transformed to the production of the other output as the input bundle is reallocated.

For the derivative dy_2/dy_1, y_1 is substituting and y_2 is being substituted. The derivative dy_2/dy_1 is the rate of product transformation of y_1 for y_2, or $RPT_{y_1y_2}$. Some textbooks define the $RPT_{y_1y_2}$ as the negative of dy_2/dy_1, so that the rate of product transformation is positive when the product transformation function is downward sloping. The derivative dy_1/dy_2 is $RPT_{y_2y_1}$.

Along a product transformation function, the $RPT_{y_1y_2}$ is equal to the negative ratio of individual marginal products

†15.10 $RPT_{y_1y_2} = !\,MPP_{xy_2}/MPP_{xy_1}$

(If the rate of product transformation is defined as $!\,dy_2/dy_1$, it is equal to MPP_{xy_2}/MPP_{xy_1}.) The rate of product transformation for each point in the tabular data can be calculated with this rule (Table 15.2).

15.4 Competitive, Supplementary, Complementary and Joint Products

Given a fixed amount of the resource bundle x, one output must be forgone in order to produce more of the other output. Therefore, under ordinary circumstances, the $RPT_{y_1y_2}$ will be negative. Hence the two outputs are competitive with each other. Two outputs are said to be *competitive* when the product transformation function is downward sloping.

†15.11 $dy_2/dy_1 < 0$ implies competitive products.

An output y_1 is said to be *supplementary*, if some positive level of production of the output y_1 is possible without any reduction in the output of y_2. Supplementary outputs imply either a zero or infinite rate of product transformation, depending on which output appears on the horizontal axis. This suggests that

†15.12 $dy_2/dy_1 = 0$ or $dy_2/dy_1 = 4$

Table 15.2. The Rate of Product Transformation of Corn for Soybeans from a Variable Input Bundle *x*.

Units of *x* Applied to Corn	Yield per Acre (bushels)	*MPP* of *x* in Corn Production	Units of *x* Applied to Soybeans	Yield per Acre (bushels)	*MPP* of *x* in Bean Production	*RPT* of Corn for Soybeans
0	0		10	55		
		45			1	1/45 = 0.022
1	45		9	54		
		17			2	2/17 = 0.118
2	62		8	52		
		15			3	3/15 = 0.200
3	87		7	49		
		13			4	4/13 = 0.308
4	100		6	45		
		11			5	5/11 = 0.455
5	111		5	40		
		9			6	6/9 = 0.667
6	120		4	34		
		7			7	7/7 = 1.00
7	127		3	27		
		5			8	8/5 = 1.60
8	132		2	19		
		3			9	9/3 = 3.00
9	135		1	10		
		1			10	10/1 = 10.0
10	136		0	0		

An example of a supplementary enterprise sometimes cited is a farm flock of chickens. The farm wife's labor would be wasted were it not for the chicken flock. The chicken flock does not reduce the output from remaining enterprises on the farm. This example is not very popular with women's groups. Neither is it a very good example. Even if the farm wife's labor were wasted, chickens take other inputs such as feed, that would reduce the output from the other enterprises. A good example of a supplementary enterprise is difficult to find. Usually, the enterprise is supplementary only with respect to certain types of inputs contained within the input bundle, in this example, the housewife's labor.

An output y_1 is said to be *complementary*, if production of y_1 causes the output of y_2 to increase. The rate of product transformation is positive at least for certain combinations of y_1 and y_2. In other words;

†15.13 $dy_2/dy_1 > 0$ for certain production levels for y_1 and y_2

An often cited example of a complementary enterprise is a legume in a rotation. The legume increases production of grain crops in alternate years. But it is not entirely clear that such a rotation would necessarily increase the total output of crops over a horizon of several years, and the farmer may produce more output by using chemical fertilizers instead of the legume. Good examples of complementary farm enterprises are difficult to find. Again, these examples are usually called complementary only with respect to a few of the inputs contained in the bundle needed for production.

Joint products, narrowly defined, are those that must be produced in a fixed ratio to each other. As a result, the product transformation function will either be a single point or a right angle. The classical example is the production of beef and hides. Only one hide can be produced per beef animal, no more and no less. The elasticity of product substitution between beef and hides is zero.

Another example is the production of wool and lamb. Although these may appear to be joint products, much like beef and hides, some sheep tend to produce more wool, whereas others are favored for the production of meat. Over time a farmer might substitute a wool breed for a meat breed and produce more wool but less lamb. Or the meat breed might be substituted for the wool breed to produce more meat and less wool. So substitution could take place over time but within a narrow range of possibilities. It would not be possible to raise a sheep that produced all lamb and no wool, or all wool but no lamb.

Figure 15.3 illustrates some possible product transformation functions representing competitive, supplementary, and complementary products. Two outputs are normally competitive everywhere on the product transformation function. It is possible for two outputs to be supplementary or complementary over only a portion of the transformation function.

15.5 Product Transformations from Single-Input Production Functions

It is often possible to develop a specific transformation relationship between two products by working with the underlying single-input production functions. Suppose that the two single input production functions are given by

†15.14 $y_1 = 2x_{y_1}$

†15.15 $y_2 = 3x_{y_2}$

†15.16 $x_{y_1} + x_{y_2} = x$

where y_1 and y_2 are alternative outputs and x_{y_1} and x_{y_2} represent the quantities of x used in the production of y_1 and y_2, respectively. The sum of these quantities must be equal to x, the total amount available. Solving the first and second equations for x_{y_1} and x_{y_2} and substitution into the third equation yields

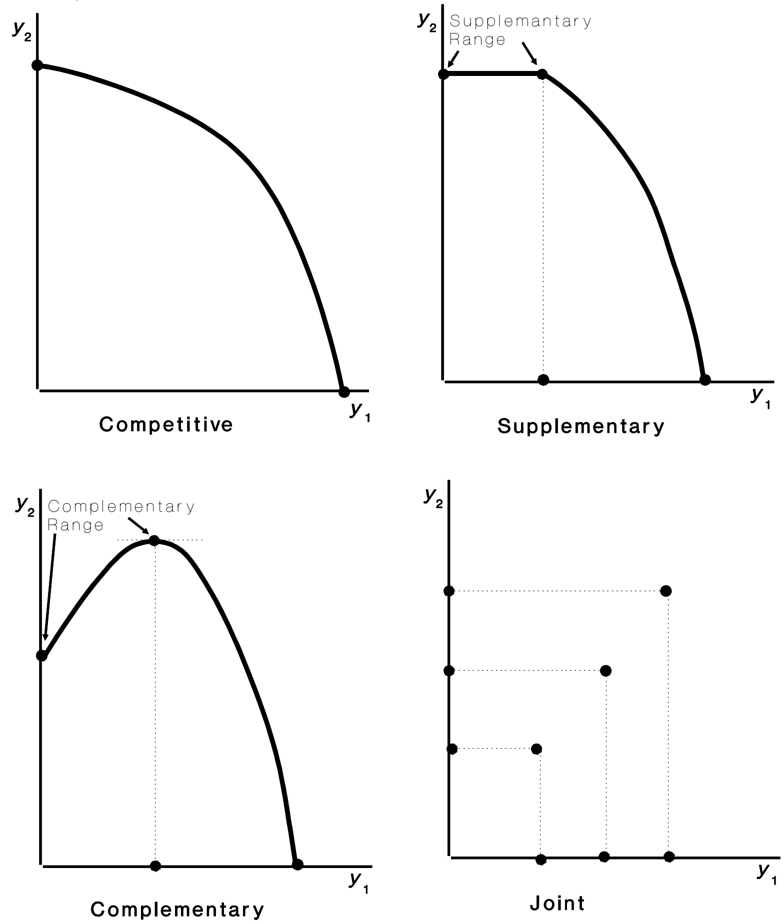


Figure 15.3 Competitive, Supplementary, Complementary and Joint Products

†15.17 $x_{y_1} = y_1/2$

†15.18 $x_{y_2} = y_2/3$

Therefore,

†15.19 $y_1/2 + y_2/3 = x$

If x is fixed at a particular value, this becomes an equation for the product transformation curve. The total differential of equation †15.19 is

†15.20 $dx = 1/2dy_1 + 1/3dy_2$

Along a product transformation function, there is no change in x, and dx is zero

†15.21 $dy_2/dy_1 = RPT_{y_1y_2} = (!\ 1/2)/(1/3) = !\,3/2$

The product transformation function has a constant downward slope of ! 3/2. The slope arises directly from the fact that the underlying single-input function exhibit constant marginal returns to the input bundle x.

Now consider a slightly more general form for the underlying production functions

†15.22 $y_1 = bx_{y_1}$

†15.23 $y_2 = cx_{y_2}$

†15.24 $x_{y_1} + x_{y_2} = \mathrm{x}$

where b and c are positive constants and other terms are as previously defined. Solving equations †15.22 and †15.23 for x_{y_1} and x_{y_2} and substituting into equation †15.24 results in

†15.25 $(1/b)y_1 + (1/c)y_2 = x$

The total differential of equation †15.25 is

†15.26 $dx = (1/b)dy_1 + (1/c)dy_2 = 0$

Rearranging yields

†15.27 $dy_2/dy_1 = RPT_{y_1y_2} = !\ c/b$

Again the rate of product transformation is constant and equal to the negative ratio of the marginal products for the two underlying production functions.

Now consider the case where the underlying production functions are

†15.28 $y_1 = x_{y_1}^{0.5}$

†15.29 $y_2 = x_{y_2}^{0.33}$

†15.30 $x_{y_1} + x_{y_2} = x$

Solving equations †15.28 and †15.29 for x_{y_1} and x_{y_2} yields

†15.31 $x_{y_1} = y_1^{1/0.5} = y_1^2$

†15.32 $x_{y_2} = y_2^{1/0.33} = y_2^3$

The equation for the underlying product transformation function is

†15.33 $y_1^2 + y_2^3 = x$

The rate of product transformation of y_1 for y_2 ($RPT_{y_1y_2}$) can be derived by taking the total differential of equation †15.33

†15.34 $dx = 2y_1\, dy_1 + 3y_2^2\, dy_2 = 0$

†15.35 $dy_2/dy_1 = !\,2y_1/3y_2^2$

The slope of the product transformation curve will vary and depend on the specific values of y_1 and y_2 as well as the parameters of the underlying production functions for y_1 and y_2.

A still more general formulation assumes that a general multiplicative production function exists for the production of both outputs

†15.36 $y_1 = Ax_{y_1}{}^a$

†15.37 $y_2 = Bx_{y_2}{}^b$

†15.38 $x = x_{y_1} + x_{y_2}$

Solving equations †15.36 and †15.37 for x_{y_1} and x_{y_2} and inserting into equation †15.38 yields

†15.39 $x_{y_1}{}^a = y_1/A = y_1 A^{!1}$

†15.40 $x_{y_1} = y_1{}^{1/a} A^{!1/a}$

†15.41 $x_{y_2}{}^{\mathrm{b}} = y_2/B = y_2 B^{!1}$

†15.42 $x_{y_2} = y_2{}^{1/b} B^{!1/b}$

Substitute equations †15.40 and †15.42 into equation †15.38 . The equation for the resultant product transformation function is

†15.43 $x = y_1{}^{1/a} A^{!1/a} + y_2{}^{1/b} B^{!1/b}$

The total differential of equation †15.43 is

†15.44 $dx = A^{(!1/a)}(1/a)y_1{}^{[(1!a)/a]}\, dy_1$

$+ B^{(!1/b)}(1/b)y_2{}^{[(1!b)/b]}dy_2 = 0$

A general expression for the $RPT_{y_1y_2}$ is obtained by setting dx in equation †15.44 equal to zero and solving for dy_2/dy_1

†15.45 $dy_2/dy_1 = \,![A^{!1/a}(1/a)y_1{}^{(1!a)/a}]/[B^{!1/b}\,(1/b)y_2{}^{(1!b)/b}]$

$= \,!\,[B^{1/b}\, by_1{}^{[(1!a)/a]}]/[A^{1/a}\, ay_2{}^{(1!b)/b}]$

The rate of product transformation is explicitly linked to the parameters of the two underlying production functions.

The process of solving the production function for y_1 and y_2 in terms of x involves inversion of the production function. The production function for each output must be solved for x in terms of the output. The production functions used here were chosen primarily because they could easily be inverted. Suppose that the production functions for y_1 and y_2 were

†15.46 $y_1 = ax + bx^2$

†15.47 $y_2 = bx + dx^2$

Such functions are not easily inverted. For certain values of the parameters a, b, and d, the inverse functions do not exist. It is difficult to solve for the product transformation function in any instance where the underlying production functions exhibit negative marginal product for certain values of x. The inverse is a correspondence but not a function.

15.6 Product Transformation and the Output Elasticity of Substitution

An output elasticity of substitution could be defined analogous to an elasticity of substitution on the input side. The definition of the output elasticity of substitution is the percentage change in the output ratio divided by the percentage change in the rate of product transformation. The value for the elasticity of product transformation would provide a clue as to the shape of the product transformation function, just as an elasticity of substitution on the input side provides an indication of the shape of an isoquant.

Products that could be substituted for each other without incurring the law of diminishing marginal returns would have a product transformation function with a constant negative slope. This would result in an infinite elasticity of substitution on the product side. Products that could be produced only in fixed proportions would have a right angle product transformation function and a zero elasticity of substitution on the product side.

The common cases would lie between these two extremes, and elasticities of product substitution in the two-output case would normally lie between zero and infinity. Some formulas for the elasticity of product substitution (e_{ps}) are

e_{ps} = percentage change in the output ratio (y_2/y_1) divided by the percentage change in the rate of product transformation

†15.48 $$= [)\,(y_2/y_1)/y_2/y_1]/()\,RPT_{y_1y_2}/RPT_{y_1y_2})$$

At the limit, when $) = d$

†15.49 $$e_{ps} = [d(y_2/y_1)/y_2/y_1]/dRPT_{y_1y_2}/RPT_{y_1y_2}$$

$$= [d(y_2/y_1)/d(RPT_{y_1y_2})][RPT_{y_1y_2}/(y_2/y_1)]$$

The development of algebraic formulas representing the product transformation relationship has not taken place to the extent that two-input production functions have been developed. Klein proposed a function

†15.50 $$x = Ay_1^{a}y_2^{b}$$

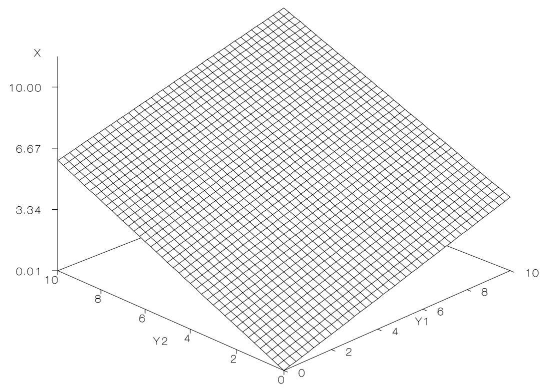
where A, a and b are parameters. The function looks very similar to a Cobb Douglas type of production function. Just, Zilberman and Hochman presented a CES type of function for the output side

†15.51 $$x = B[R_1 y_1^{!<} + R_2) y_2^{!<}]^{!1/<}$$

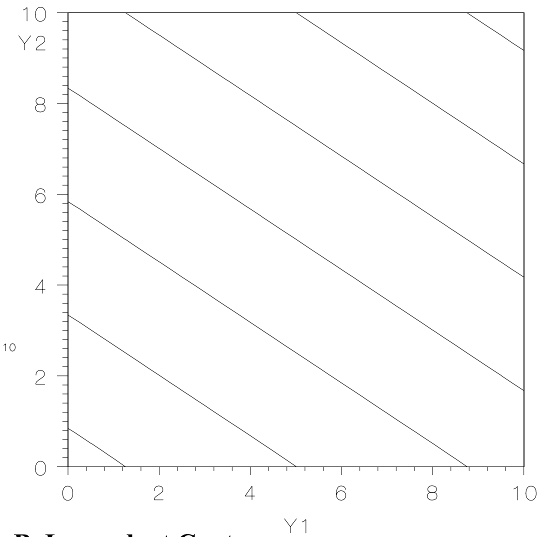
However, under the usual parameter assumptions, neither equations †15.50 or †15.51 would generate product transformation functions concave to the origin, consistent with neoclassical theory. Equation †15.51 will generate product transformation functions if < < ! 1.

Figure 15.4 illustrates the isoproduct surfaces and contours for the CES type of function for four alternative values for < < ! 1.

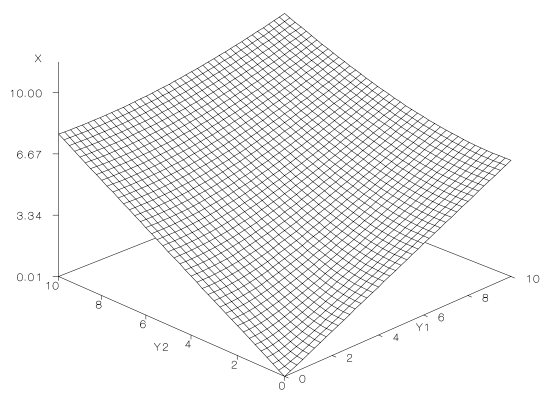
Case 1: < 6 ! 1, e_{ps} 6 ! 4. At the limit the isoproduct contours consist of lines of constant slope, and the production function is a hyperplane (without curvature, diagrams A and B). The rate of product transformation is constant everywhere and equal to the negative of the slope of the isoproduct contours.



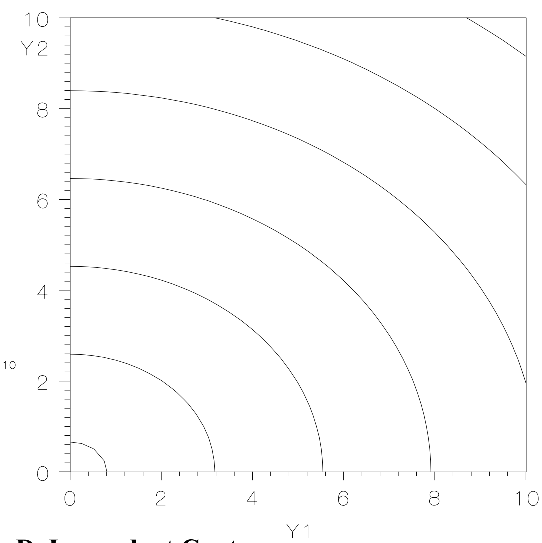
A Surface < approaches -1



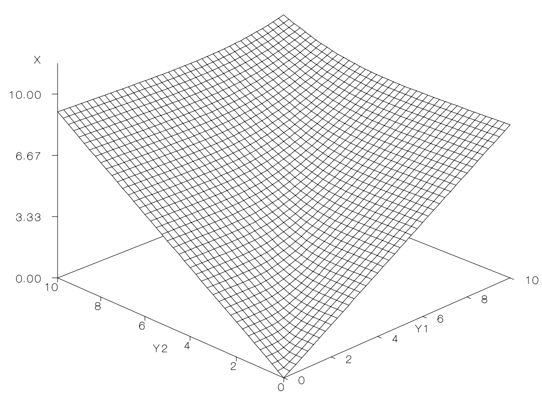
B Isoproduct Contours



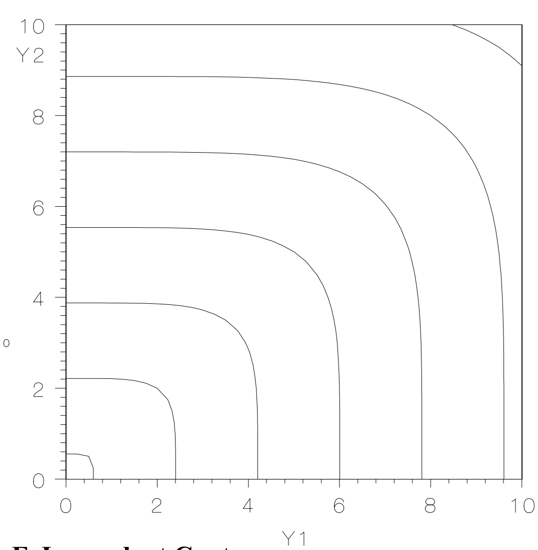
C Surface < = -2



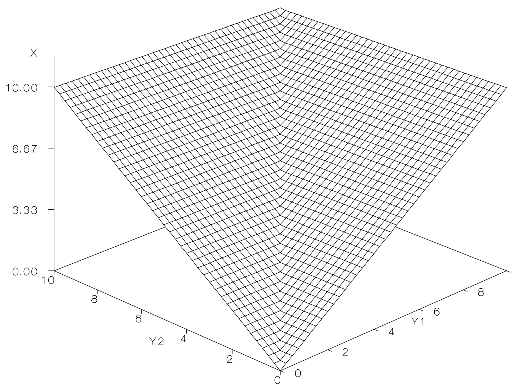
D Isoproduct Contours



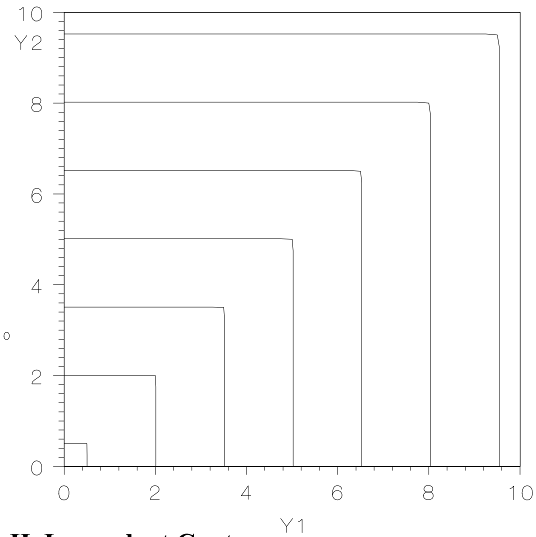
E Surface < = -5



F Isoproduct Contours



G Surface < = -200



H Isoproduct Contours

Figure 15.4 Isoproduct Surfaces and Isoproduct Contours for a *CES* type of Function, < < -1

Case 2: < = -2, e_{ps} = -1. Isoproduct contours become concave to the origin and intersect the axes. The rate of product transformation is no longer constant, but increases (in absolute value) from left to right along a specific isoproduct contour.
The isoproduct surface now has a concave curvature.

Case 3: < = -5, e_{ps} = -.25. Isoproduct contours become increasingly concave to the origin and the surface , when viewed from the top, appears more concave than in case 2.

Case 4: < 6 !4 , e_{ps} 6 0. As < becomes more negative the elasticity of substitution in product space also becomes more negative. At the limit, e_{ps} the elasticity of substitution approaches 0 from the negative side. Isoproduct contours form right angles. Figures G and H were generated assuming a value for < of !200. The surface forms an inverted pyramid. Compare G and H with Diagrams I and J of Figure 12.2.

The sum of the parameters R_1 and R_2 control scale effect in product space. If $R_1 + R_2 > 1$, Isoproduct contours representing constant incremental increases in input bundle (*x*) use will be positioned closer and closer together. If $R_1 + R_2 = 1$ Isoproduct contours will be equally spaced. If $R_1 + R_2 < 1$, isoproduct contours along a ray from the origin will be placed farther and farther apart.

The position of the isoproduct contours also depends on the relative magnitude (ratio) of $R_1 + R_2$, and each isoproduct contour will be positioned closest to the axis representing the largest R.

Policy Applications

Like its factor space counterpart, the elasticity of substitution in product space is of considerable importance for policy applications and empirical analysis. Suppose, first of all, that two commodities that a farmer produces are not substitutes at all. Hence, e_{ps} approaches 0. An example would be two unrelated crops, for example, brocolli and soybeans, that require very different inputs. The farmer would continue to produce the two commodities in approximately the same proportions irrespective of their relative prices.

Now consider the opposite extreme, an instance where the isoproduct contours have a constant *RPT*, and the e_{ps} approaches !4. As a result, even the slightest shift in relative prices would cause a huge (at the limit, total) shift in the production of one output. In North Dakota, for example, hard red spring wheat requires virtually identical inputs to the production of durum wheat. However, the two wheats are put to quite different uses, the hard red spring wheat for making bread, and the durum wheat for making pasta products. Durum wheat makes inferior bread and hard red spring wheat, although occasionally percentage blended with durum wheat in pasta production, makes inferior, glue-like, pasta. As a result, the relative prices for the two wheats can be quite different. North Dakota wheat producers do indeed make substantial shifts in acreages of the two wheats, based on relative prices at planting time, indicating that the elasticity of substitution in product space for these two wheats approaches !4.

Grain producers in the corn belt face a slightly different situation is making a decision between corn and soybean production. While these two crops use a similar complement of resources, there are a few differences. For example, there are differences in the required harvesting equipment, and corn requires nitrogen whereas soybeans, a legume, normally does not. As a result, one would expect that farmers would shift to a degree from corn to soybean production or from soybean to corn production, as the relative prices for corn and soybeans changed, but clearly the shift is not complete based on the relative price ratios alone. This would correspond with an intermediate case, in which the elasticity of substitution in product space is negative, but not infinite.

Empirical analysis employing a function such as equation 15.1 could provide valuable information about elasticities of substitution faced by farmers when attempting to choose among possible products. The could be used as a guide in making agricultural policy. With knowledge of product space elasticities of substitution, a federal policy maker, attempting to set support prices for commodities such as wheat and corn would be better able to determine the responsiveness of farmers in acreage and production as a result f changing relative prices.

15.7 Concluding Comments

This chapter has developed the physical relationships underlying the product-product model. The product transformation curve is the production possibilities curve on a firm, rather than society level. The slope of the product transformation function is closely tied to the marginal products of the single-input production functions that underlie the transformation of input into outputs. An expression for an output elasticity of substitution can be derived, but specific equations representing input use in the production of alternative outputs have not been developed to the extent that single-output production functions using alternative inputs have been developed by economists and agricultural economists.

Problems and Exercises

1. Assume the following production function data

Units of x Applied to y_1	Output of y_1	Units of x Applied to y_2	Output of y_2
0	0	10	50
1	20	9	49
2	30	8	47
3	38	7	44
4	45	6	40
5	51	5	35
6	56	4	29
7	60	3	22
8	63	2	14
9	64	1	5
10	64.5	0	0

If only 10 units of input *x* are available, graph the production possibilities (product transformation) curve from this production function data.

2. Suppose that at a particular point, the *MPP* of x in the production of y_1 is positive but the *MPP* of x in the production of y_2 is negative (stage III of production). What would be the slope of the product transformation function? Explain.

3. Assume the following production functions for x in the production of y_1 and y_2. Find the rate of product transformation of y_1 for y_2.

$$y_1 = x^{0.25}$$
$$y_2 = x^{0.33}$$

4. What do competitive, supplementary, complementary, and joint enterprises each imply about the shape of the production functions that underlie the product transformation functions?

References

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