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Contemporary Production Theory: The Product Side

Much of the theory of the firm in product space is not nearly as well developed as the theory of the firm in factor space. For example, both general and agricultural economists have devoted considerable effort to developing functional forms representing production processes in factor space, but the companion effort in product space has been very limited. This chapter discusses some problems in the modification for use in product space of functional forms commonly used in factor space. Extensions to the theory of the firm in product space are developed by using factor space and duality theory as the basis.

Key terms and definitions

Contemporary Production Theory
Duality
Product Space
Translog Function
Product Space Elasticity

25.1 Introduction

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An equation for a production process involving *n* inputs and a single output is:

†25.1 $$y = f(x_1,...,x_n)$$

with an isoquant representing a fixed constant output arising from possible combinations of the x_i :

†25.2 $$y° = f(x_1,...,x_n)$$

In product space, the analogous equation linking the production of *m* outputs to the use of a single input (or bundle of inputs, is

†25.3 $$x = h(y_1,.., y_m).$$

The production possibilities function representing possible combinations of the y_i that can be produced from a fixed quantity of a single input (or input bundle, with the quantities of each input being held in fixed proportion to each other) is:[1]

†25.4 $$x° = h(y_1,..., y_m)$$

Considerable effort has been devoted to the development of explict specifications for equation †25.1 (Fuss and McFadden, Diewert, 1971). Most attempts at developing explicit forms of †25.3 have consisted of simple modifications of explicit forms of †25.1 , by replacing the x_i with y_i and y_2, and substituting the quantity of x in the product space model, a single input (or input vector $\mathbf{x} = \{x_1°,...,x_n°\}$) for $y°$ in the factor space model. The standard presentation of the neoclassical theory of the firm usually specifies isoquants in factor space with a diminishing (or possibly constant) marginal rates of substitution. The standard presentation in product space specifies product transformation functions with an increasing (or possibly constant) rate of product transformation. This suggests that the parameters of and even the explicit form of *h* (equation †25.3) needed to generate product transformation functions consistent with neoclassical theory might be quite different from the parameters and form of *f* (equation †25.1).

25.2 Duality in Product Space

In product space, the total revenue function is analogous to the cost function in factor space. Suppose that products (a) are either supplemental or competitive but not complementary with each other for the available resource bundle $x°$, and (b) rates of product transformation between output pairs are non-decreasing. These assumptions are analogous in product space to the free disposal and non increasing marginal rate of substitution assumptions (McFadden, pp. 8-9) in factor space.

In factor space, if there is free disposal of inputs, and non increasing marginal rates of substitution, then the cost function that is dual to the underlying production function $c(y;\mathbf{v}) = \min[\mathbf{v}'\mathbf{x}: f(\mathbf{x}) \geq y]$

(i) *exists.* This is true because any continuous function defined on a closed and bounded set achieves its minimum within the set.

(ii) *is continuous.*

(iii) *is non-decreasing* for each price in the input price vector **v**.

(iv) *is homogeneous of degree one* in all variable input prices. This implies that if all input prices double, so also will total variable cost, and

(v) *is concave* in each input price for a given level of output (y^*).

Detailed proofs of (i)-(v) can be found in McFadden, 1978, pp. 10-13. The isoquant maps needed for the existence of a corresponding dual cost function are not necessarily more plausible in an applied setting than other isoquant maps, but rather are a matter of mathematical convenience. For example, the Cobb-Douglas, CES and Translog production functions all are capable of generating isoquant maps consistent with these assumptions, under the usual parameter restrictions.

Given the product space function

†25.27 $$x = g(y_1, y_2, \ldots, y_m),$$

the corresponding total revenue function that maximizes total revenue for a given input bundle $x°$ is:

†25.28 $$r = \max[\mathbf{p}'\mathbf{y}; g(\mathbf{y}) \leq x°].$$

If conditions (a) and (b) are met, then equation †25.28

(vi) *exists*

(vii) *is continuous*

(viii) *is non-decreasing in each price* in the product price vector **p**

(ix) *is linearly homogeneous in all product prices* $\{p_1, \ldots, p_m\}$ (and in all outputs $\{y_1, \ldots, y_m\}$). A doubling of all product prices or a doubling of all outputs will double revenue.
and

(x) *is convex in each output price* for a given level of input $x°$ (Hanoch, p. 292).

The product transformation functions needed for the existence of a corresponding dual revenue function are not necessarily more plausible in an economic setting than other product transformation functions, but are rather a mathematical convenience. A Cobb-Douglas like funtion in product space will not generate product transfomation functions consistent with (a) and (b), while under certain parameter assumptions, a CES-like or translog like function in product space will generate product transformation functions consistent with these assumptions.

25.3 Cobb-Douglas-Like Product Space

Consider first a Cobb-Douglas like analogy in product space. A Cobb-Douglas like two product one input model suggested by Just, Zilberman and Hochman (p. 771) from Klein is:

†25.5 $$y_1 y_2^{\beta} = A x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$$

Now suppose there is but one input to the production process. That is

†25.6 $$A x^{\alpha_1} = y_1 y_2^{\beta}$$

Solving for input x

†25.7 $$x = (1/A)^{1/\alpha_1} y_1^{1/\alpha_1} y_2^{\beta/\alpha_1}$$

The parameters α_1 and β would normally be non-negative, since additional units of y_1 or y_2 can only be produced with additional units of the input bundle, and additional units on the input bundle produce additional units of outputs y_1 and y_2.

Rewriting †25.7 in a slightly more general form:

†25.8 $$x = B y_1^{\phi_1} y_2^{\phi_2}$$

However, with positive parameters, in no case will equations †25.7 and †25.8 generate product transformation curves concave to the origin, for the Cobb-Douglas like function is quasi-concave for any set of positive parameter values.

Given the general single-input, two-output product transformation function:

†25.9 $$x = h(y_1, y_2)$$

For an increasing rate of product transformation:

†25.10 $$h_{11} h_2^2 + h_{22} h_1^2 - 2 h_{12} h_1 h_2 > 0$$

For a Cobb-Douglas like function in product space equation †25.10 with a positive ϕ_1 and ϕ_2 is equal to:

†25.11 $$(-\phi_1 \phi_2^2 - \phi_2 \phi_1^2) y_1^{3\phi_1 - 2} y_2^{3\phi_2 - 2} < 0$$

A Cobb-Douglas-like function in product space cannot generate product trans-formation functions consistent with neoclassical theory and the usual constrained optimization revenue maximization conditions.

25.4 CES-Like Functions in Product Space

The CES production function in two input factor space is:

†25.12 $$y = C[8_1 x_1^{!D} + 8_2 x_2^{!D}]^{!1/D}$$

Just, Zilberman and Hochman also suggest a possible CES-like function in product space. A version of this function with one input and two outputs is:

†25.13 $$x = C[8_1 y_1^{!n} + 8_2 y_2^{!n}]^{!1/n}$$

The five familiar cases (Chapter 12 and in Henderson and Quandt; and Debertin, Pagoulatos and Bradford) with respect to the CES production function assume that the parameter D lies between !1 and +4. Isoquants are strictly convex when D > !1. When D = !1, isoquants are diagonal lines. When D = +4, isoquants are right angles convex to the origin.

For a CES-like function in product space, the rate of product transformation (*RPT*) is defined as:

†25.14 $$RPT = !dy_2/dy_1$$

†25.15 $$dy_2/dy_1 = !(8_1/8_2)(y_2/y_1)^{(1+n)}$$

The product transformation functions generated from the CES-like function in product space are downsloping so long as 8_1 and 8_2 are positive, irrespective of the value of the parameter n.

Differentiating †25.15 :

†25.16 $$d^2y_2/dy_1^2 = !(1+n)(!8_1/8_2)y_2^{1+n}y_1^{!(2+n)}$$

Since y_1, y_2, 8_1, $8_2 > 0$, the sign on †25.16 is dependent on the sign on !(1+n). In factor space, the values of D that are of interest are those that lie between !1 and +4, for these are the values that generate isoquants with a diminishing marginal rate of substitution on the input side. If the value of n is exactly !1, then the product transformation functions will be diagonal lines of constant slope $8_1/8_2$ [since $(y_2/y_1)^{\circ} = 1$] and products are perfect substitutes.

However, as was indicated in chapter 15, the CES-like function can generate product transformation functions with an increasing rate of product transformation. The five CES cases outlined by Henderson and Quandt in factor space include only values of D that lie between !1 and +4. In product space, the values of n that lie between !1 and !4 generate product transformation functions with an increasing rate of product transformation, since equation †25.16 is negative when n < !1. As n 6 !4, the product transformation functions approach right angles, concave to the origin. Small negative values for n generate product transformation functions with a slight bow away from the origin. As the value of n becomes more negative, the outward bow becomes more extreme.

In the limiting case, when n 6 !4, y_2 is totally supplemental to y_1 when y_1 exceeds y_2; conversely y_1 is totally supplemental to y_2 when y_2 exceeds y_1. This is equivalent to the joint product (beef and hides) case.[2] If n is a fairly large negative number (perhaps < !5) there exist many combinations of y_1 and y_2 where one of the products is "nearly" supplemental to the other. As n 6 ! 1, the products become more nearly competitive throughout the possible combinations, with the diagonal product transformation functions when n = ! 1 the limiting case. Regions of product complementarity are not possible with a CES-like product transformation function.

Product transformation functions exhibiting a constant or an increasing rate of product transformation must necessarily intersect the y axes. Thus, there is no product space counterpart to the asymptotic isoquants generated by a Cobb-Douglas type function in factor space.

25.5 Alternative Elasticity of Substitution Measures in Product Space

Diewert (1973) extended the concept of an elasticity of substitution (which he termed elasticity of transformation) to multiple product-multiple input space. Hanoch suggests that the elasticity of substitution in product space can be defined analogously to the elasticity of substitution in factor space. In the case of product space, revenue is maximized for the fixed input quantity $x°$, is substituted for minimization of costs at a fixed level of output $y°$(p. 292) in factor space. The elasticity of substitution in two product one input space (Debertin) is defined as:

†25.17 $,_{sp}$ = % change in the product ratio y_2/y_1 ÷ % change in the *RPT*

or as

†25.18 $N_{sp} = [d(y_2/y_1)/dRPT][RPT/(y_2/y_1)]$.

Another way of looking at the elasticity of substitution in product space is in terms of its linkage to the rate of product transformation for CES-like two-product space. Suppose that $Y = y_2/y_1$, or the output ratio. The rate of product transformation for CES-like product space is defined as

†25.19 $RPT = Y^{(1+n)}$

The elasticity of substitution in product space (equation †25.18) can be rewritten as:

†25.20 $(d\log Y)/(d\log RPT)$.

Taking the natural log of both sides of †25.19 yields

†25.21 $\log RPT = (1+n)\log Y$

Solving †25.21 for log Y and logarithmically differentiating

†25.22 $(d\log y)/(d\log RPT) = 1/(1+n)$

Assuming that n < ! 1, the elasticity of substitution in product space for a CES- like function is clearly negative, but 6 0 as n 6 ! 4.

The concept of an elasticity of substitution in product space is of considerable economic importance, for it is a pure number that indicates the extent to which the products which can be produced with the same input bundle can be substituted for each other. Assuming competitive equilibrium, the inverse product price ratio p_2/p_1 can be substituted for the *RPT*, and equation †25.18 can be rewritten as:

†25.23 $$N_{sp} = [d(y_2/y_1)/d(p_1/p_2)][(p_1/p_2)/(y_2/y_1)]$$

In two-input factor space, Equation †25.23 is rewritten as

†25.24 $$N_s = [d(x_2/x_1)/d(w_1/w_2)][(w_1/w_2)/(x_2/x_1)]$$

As McFadden has indicated, there is no natural generalization of of the of †25.24 when whe number of factors is greater than 2. The elasticity of substitution will vary depending on what is assumed to be held constant. However, the Allen, Morishima (Koizumi), and Shadow (McFadden) Elasticities of Substitution all collapse to †25.23 when n equals 2. Similarly, there is no natural generalization of product space elasticity of substitution when the number of products exceeds two.

In the case of farming, the elasticity of substitution in product space is a pure number that indicates the extent to which the farmer is able to respond to changes in relative product prices by altering the product mix. An elasticity of substitution in product space near zero would indicate that the farmer is almost totally unable to respond to changes in product prices by altering the mix of products that are produced. An elasticity of substitution in product space of - 4 indicates that the farmer nearly always would be specializing in the production one of the two commodities with the favorable relative price. As relative prices change toward the other commodity, a complete shift would be made to the other commodity.

For most agricultural commodities, the elasticity of substitution in product space would be expected to lie between 0 and ! 4, indicating that to a certain degree, farmers will respond to changes in relative product prices by altering the product mix. Commodities which require very similar inputs would be expected to have very large negative elasticities of product substitution. Examples include Durum wheat versus Hard Red Spring wheat in North Dakota, or corn versus soybeans in the corn belt. Conversely, two dissimilar commodities requiring very different inputs would be expected to have elasticities of substitution approaching zero, and a change in relative prices would not significantly alter the output combination.

A representation of equation †25.23 in m product space when m>2 is

†25.25 $$,_{sp} = [d\log y_k \ ! \ d\log y_i]/[d\log p_i \ ! \ d\log p_k]$$

Equation †25.25 is representative of a two-output, two-price (or *TOTP*) elasticity of product substitution analogous to the two input two price (*TTES*) elasticity of substitution in factor space, with the quantities of outputs other than i and k held constant.

Other elasticity of product substitution concepts can be defined, each of which is analogous to a similar concept in factor space. For example, the one output one price (or *OOOP*) concept is Allen-like and symmetric:

†25.26 $$,_{spa} = \$(d\log y_i)/(d\log p_k)$$

The one-output, one-price (*OOOP)* concept in factor space is proportional to the cross price input demand elasticity evaluated at constant output. Similarly, the *OOOP* concept is proportional to the cross output price product supply elasticity evaluated at a constant level of input use. An own price *OOOP* can also be defined, that is proportional to the own price elasticity of product supply. In factor space, the Allen elasticity of substitution is proportional to the cross price input demand elasticity evaluated at constant output. Normally, as the price of the *j*th input increases, more of the *i*th input, and less of the *j*th input would be used in the production process, as input x_i is substituted for input x_j, evaluated at constant output. Thus, the sign on the Allen elasticity of substitution in factor space is normally positive if inputs substitute for each other.

However, in product space, the Allen like elasticity of substitution is proportional to the cross output price product elasticity of supply evaluated at a constant level of input use. Normally, as the price of the *j*th output increases, the amount of the *j*th output produced would increase, and the amount of the *i*th output produced would decrease, the opposite relationship from the normal case in factor space. Thus, while the Allen elasticity of substitution in factor space would normally have a positive sign, the Allen like elasticity of substitution would normally have a negative sign in product space. The negative sign is also consistent with the sign on the product elasticity of substitution for the CES-like function derived earlier.

In the *n* input setting, Hanoch (p. 290) defines the optimal (cost minimizing) share for input x_j as a share of total variable costs as:

†25.27 $$S_j = w_j x_j / C$$

where

$C = \Sigma w_i x_i$

w_i = the price ofthe *i*th input

y = a constant

Invoking Shephard's lemma,

†25.28 $$\partial C/\partial w_j = x_j.$$

Equation †25.27 representing the optimal share of total cost for the *j*th input can then be rewritten as:

†25.29 $$S_j = d\log C/d\log w_j$$

In the *n* input case, the Allen elasticity of substitution (A_{ij}) between input x_i and x_j evaluated at a constant input price w_j is defined as:

†25.30 $$A_{ij} = (1/S_j)(E_{ij})$$

where

$E_{ij} = d\log x_i/d\log w_j$, the cross-price elasticity of demand for input x_j with respect to the *j*th input price.

By substituting †25.29 into †25.30 , equation †25.30 may be rewritten as (Hanoch, p. 290):

†25.31 $A_{ij} = d\log x_i/d\log C = A_{ji} = d\log x_j/d\log C$,

since the inverse of the Hessian matrix for the underlying production function *f* is symmetric. In this contect the Allen elasticity of substitution is the elasticity of x_i with respect to total cost C for a change in another price p_j (Hanoch).

These relationships may be derived analogously on the product side. Define the revenue maximizing revenue share (R_k*) for output y_k treating the input $x°$ (or input vector bundle **x**°) constant as

†25.32 $R_k^* = p_k y_k^*/R$,

where

p_k = the price of the *k*th output

$R = \Sigma p_i y_i$, $i = 1, ..., m$

y_k^* = the revenue-maximizing quantity of output y_k from the fixed input bundle **x**°.

Invoking the revenue counterpart to Shephard's lemma (Beattie and Taylor, p. 235)

†25.33 $\partial R/\partial p_k = y_k$.

Equation †25.33 representing the share of total revenue for optimal quantity of the *k*th output can then be rewritten as:

†25.34 $R_k = d\log R/d\log p_k$

In the *m* output case, the Allen like elasticity of substitution (or transformation) (A_{ik}^p) in product space between input x_i and x_j evaluated at a constant input price w_j is defined as:

†25.35 $A_{ik}^p = (1/R_k)(E_{ij}^p)$

where

$E_{ij}^p = d\log y_i/d\log p_k$,

the cross-price elasticity of supply for output y_i with respect to the kth product price.

By substituting †25.34 into †25.35 , equation †25.35 may be rewritten as

†25.36 $A_{ij}^p = d\log y_i/d\log R = A_{ki} = d\log y_k/d\log R$, since the inverse of the Hessian matrix for the underlying function *h* in product space is symmetric. In this context the Allen like elasticity of substitution in product space is the elasticity of y_i with respect to total revenue R, for a change in another price p_k, holding the quantity of the input (or input bundle) constant.

Yet another way of looking at the Allen like elasticity of substitution in product space is by analogy to the Allen elasticity of substitution defined in factor space defined in terms of the cost function and its partial derivatives. The Allen elasticity of substitution between the *i*th and *j*th input (A_{ij}^{f}) in factor space can be defined as in terms of the cost function and its partial derivatives:

†25.37 $$A_{ij}^{f} = (CC_{ij})/(C_i C_j)$$

where

$$C = h(w_1, ..., w_n, y^*)$$

$$C_i = \partial C/\partial w_i$$

$$C_j = \partial C/\partial w_j$$

$$C_{ij} = \partial^2 C/\partial w_i \partial w_j$$

The corresponding revenue function definition in product space is:

†25.38 $$A_{ij}^{p} = (RR_{ij})/(R_i R_j)$$

where

$$R = h(p_1,...,p_n, x^*)$$

$$R_i = \partial R/\partial w_i$$

$$R_j = \partial R/\partial w_j$$

$$R_{ij} = \partial^2 R/\partial w_i \partial w_j$$

The two-output, one-price (or *TOOP*) elasticity of product substitution is analogous to the two-output, one-price, or Morishima elasticity of substitution in factor space. The Morishima like elasticity of substitution in product space (Koizumi) is:

†25.39 $$\sigma_{spm} = (d\log y_i - d\log y_k)/d\log p_k.$$

Like its factor-space counterpart, the Morishima-like elasticity of substitution in product space is nonsymmetric.

Fuss and McFadden (p. 241) note that in factor space, each elasticity of substitution can be evaluated based on constant cost, output or marginal cost. In product space, the total revenue equation is analogous to the cost equation in factor space. Hence, each elasticity of substitution in factor space may be evaluated based on constant total revenue, marginal revenue, or level of input bundle use.

Generalization of the various product elasticity of substitution measures to *m* outputs involves making assumptions with regard to the prices and/or quantities of outputs other than the *i*th and *j*th output. A shadow-like elasticity of substitution in product space is, like its factor space counterpart (McFadden), a long-run concept, but in this case, all quantities of outputs other than *i* and *j* are allowed to vary.

25.6 Translog-Like Functions in Product Space

The second-order Taylor's series expansion of log y in log x_i, or translog production function (Christensen Jorgenson and Lau), has received widespread use as a basis for the empirical estimation of elasticities of substitution in factor space. The slope and shape of the isoquants for the translog production function are dependent on both the estimated parameters of the function and the units in which the inputs are measured. Given the two input translog production function:

†25.40 $$y = Ax_1^{"_1}x_2^{"_2}e^{(_{12}\log x_1 \log x_2 + (_{11}(\log x_1)^2 + (_{22}(\log x_2)^2}$$

The important parameter in determining the convexity of the isoquants is (12. Imposing the constraint that (11= (22 = 0, equation †25.29 may be rewritten as:

†25.41 $$y = Ax1^{"_1}x2^{"_2}e^{(_{12}\log x_1 \log x_2}$$

or as:

†25.42 $$\log y = \log A + "_1\log x_1 + "_2\log x_2 + (_{12}\log x_1\log x_2$$

Berndt and Christensen (p. 85) note that when $(_{ij}$… 0, there exist configurations of inputs such that neither monotonicity or convexity is satisfied. In general, the isoquants obtained from †25.42 will be convex only if $(_{12}$ $ 0. In addition, since the natural log of x_i is negative when 0< xi<1, so the isoquants may have regions of positive slope even when $(_{12}$>0, depending on the units in which the x_i are measured. It is also possible to obtain convex isoquants for the translog production function when $(_{12} < 0$, depending on the magnitude of x_1 and x_2, which is units dependent.

The parameter $(_{12}$ is closely linked to the elasticity of substitution in factor space. If $(_{12}$ = 0, the function is Cobb-Douglas. Small positive values of $(_{12}$ will cause the isoquants to bow more sharply inward than is true for the Cobb-Douglas case.

Imposing the same constraint that $2_{ii} = 0$, a two-output translog function in product space can be written as

†25.43 $$\log x = \log B + \$_1\log y_1 + \$_2\log y_2 + 2_{12}\log y_1\log y_2$$

In two-product space, the parameter 2_{12} would normally be expected to be negative, just as in factor space, $(_{12}$ would be expected to be normally positive.

25.7 Translog Revenue Functions

The indirect two output translog revenue function that represents the maximum amount of revenue obtainable for any specific quantity of the input $x°$, allowing the size of the input bundle to vary is:

†25.44 $$\log R^* = \log D + *_1\log p_1 + *_2\log p_2 + *_{11}(\log p_1)^2 + *_{22}(\log p_{22})^2$$
$$+ *_{12}\log p_1\log p_2 + n_{1x}\log p_1\log x + n_{2x}\log p_2\log x + n_x\log x + n_{xx}(\log x)^2$$

Every point on the translog revenue function in product space is optimal in the sense that every point is a point on the output expansion path, which represents the maximum amount of revenue obtainable from a given level of resource use $x°$.

Beattie and Taylor (p. 235-6) derive the revenue counterpart to Shephards lemma. They show that

†25.45 $$\partial R^*/\partial p_j = y_j.$$

Thus, if the firm's revenue function is known, systems of product supply equations can be derived by differentiating the revenue function and performing the substitution indicated by †25.45 . Factor prices are treated as fixed constants in such an approach.

Differentiating †25.44 with respect to the *j*th product price, say p_1, yields:

†25.46 $$d\log R^*/d\log p_1 = {*}_1 + 2{*}_{11}\log p_1 + {*}_{12}\log p_2 + n_{1x}\log x.$$

Economic theory imposes a number of restrictions on the values that the parameters of equation †25.46 might assume in the *m* output case. These restrictions are similar to those imposed on the parameters of cost share equations in factor space.

First, total revenue from the sale of *m* different products is

†25.47 $$R = \Sigma R_i \; i = 1, ..., m$$

Thus, if the revenue from $m - 1$ of the revenue share equations is known, the remaining revenue share is known with certainty, and one of the revenue share equations is redundant.

Young's theorem holds in product just as it does in factor space. Thus, ${*}_{ij} = {*}_{ji}$, which is the same as the symmetry restriction in factor space.

Any revenue function should be homogeneous of degree one in all product prices. The doubling of all product prices should double total revenue. This implies that

†25.48 $$\Sigma {*}_i = 1$$

and

†25.49 $$\Sigma {*}_{ij} = 0$$

One might also draw the analogy to the Brown and Christensen assertion that in factor space, the cost function represents constant returns to scale technology. In product space, the corresponding assumption is that there is a constant increase in revenue associated with an increase in the size of the input bundle. This implies

†25.50 $$dR^*/dx = {*}_x = 1$$

†25.51 $$\Sigma {*}_{ix} = 0 \text{ for } i = 1, ..., n$$

†25.52 $${*}_{xx} = 0$$

These assumptions are as plausible in product space as the analogous assumptions are with regard to indirect cost functions in factor space.

It is also possible to think in terms of an analogy to a Hicks' like technological change in product space. In product space, technological change occurring over time may favor the production of one commodity at the expense of another commodity. If, as the state of technology improves over time, and no shift is observed in the proportions of the y_i to y_j over time, then the technology is regarded as Hicks like neutral in product space. Technology that over time shifts the output-expansion path toward the production of the jth commodity, then the technology is regarded as Hicks like favoring for product y_j. If technological change causes the output expansion path to shift away from the production of commodity y_i, then the technological change could be referred to as y_i inhibiting technological change.

Brown and Christensen derive the constant-output Allen elasticities of substitution in factor space from the formula:

†25.53 $$\sigma_{ij} = (\theta_{ij} + S_iS_j)/S_iS_j$$

where

S_i, S_j = the cost shares attributed to factors *i* and *j*, respectively.

θ_{ij} = the restricted regression coefficient from the $\log r_i \log r_j$ term in the cost share equation, where r_i and r_j are the corresponding factor prices for inputs *i* and *j*.

The estimated parameter θ_{ij} is usually positive, indicating that inputs *i* and *j* are substitutes, not complements within the *n* dimensional factor space.

The analogous formula for deriving the Allen-like elasticities of substitution in product space is

†25.54 $$\sigma_{ijp} = (\delta_{ij} - R_iR_j)/R_iR_j$$

As indicated earlier, the parameter δ_{ij} will usually be negative, and the Allen-like elasticity of substitution in product space (σ_{ijp}) for most commodities is negative.

25.8 Empirical Applications

Many possibilities exist for empirical analysis linked to agriculture based on the models developed in this chapter. One of the simplest approaches would be to estimate revenue share equations for major commodities in U.S. agriculture for selected time periods (following the approach used by Aoun for estimating cost share equations for agricultural inputs in factor space) and derive various elasticity of substitution measures in product space. These revenue share parameter estimates would be used to estimate product elasticity of substitution measures for the various major agricultural commodities in the United States. Such an empirical analysis could stress the implications for current agricultural policy in terms of determining how farmers alter their product mix over time in the face of changing government price support programs such as those contained in the 1986 farm bill.

The Hicks-like technological change approach appears to be promising as well. As technological change occurs for a specific agricultural commodity, presumably that commodity is favored relative to others in a product space model. For example, has technological change over the past thirty years tended to favor the production of soybeans relative to other grains? Such an approach might be useful in assessing the economic impacts of genetic improvements in specific crops or classes of livestock.

Another possibility is to estimate changes in the product space elasticity of substitution measures over time. Some thirty years ago Heady and others discussed the impacts of specialized versus flexible facilities using a product space model. One way of looking at a facility specialized for the production of a specific commodity is that it represents product space in which the elasticity of substitution is near zero. A flexible facility is represented by a product space elasticity of substitution that is strongly negative.

Note

[1] There is considerable disagreement in the literature with regard to terminology relating to the firm capable of producing more than one product. Henderson and Quandt argue that the term joint product should be used in any instance where a firm produces more than one output, even in instances where the products can be produced in varying proportions. The convention followed in many agricultural production economics texts is to use the term joint product to refer only to those products that must be produced in fixed proportions with each other such as beef and hides. If products must be produced in fixed proportions with each other, then relative prices will not infuence the output mix. The term multiple products is used to refer to any situation where more than one output is produced, regardless of whether or not the outputs are produced only in fixed proportion with each other. 2 The concept of an elasticity of substitution in product space is one mechanism for resolving the problems with the joint and multiple product terminology. The output elasticity of substitution is zero when outputs must be produced in fixed proportions (joint) with each other. The output elasticity of substitution is -4 when products are perfect substitutes for each other. A CES-like product space function encompasses a series of intermediate cases for which the product transformation function is downsloping but concave to the origin and the value for the product space elasticity of substitution lies between 0 and -4.

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